On an SO(5) unification attempt for the cuprates

G. Baskaran† and P.W. Anderson
Joseph Henry Laboratories of Physics
Princeton University
Princeton, NJ 08540
(February 1, 2005)

Zhang [1] in his paper ‘A unified theory based on SO(5) symmetry of superconductivity and antiferromagnetism, Science 275 1089 (97)’ has argued that there exists an approximate global SO(5) symmetry in the low temperature phase of the high Tc cuprates. This suggestion contains a five component order parameter: three components correspond to a spin singlet, charge of the Mott insulating state; while the last two components correspond to antiferromagnetic order in the vicinity of the Mott insulating state; while the last two components correspond to a spin 1, charge zero particle-hole pair condensed at the center of mass momentum (π, π), corresponding to antiferromagnetic order in the vicinity of the Mott insulator. He identifies the operator 

\[ \pi \] 

as one of the generators which rotates the 5-vector between the spin and charge direction. The doping density apparently acts like a pseudo magnetic field for the order parameter.

For an exact SO(5) symmetry breaking scheme the generator π should annihilate the vacuum (as it will be a zero momentum Goldstone mode creation operator) or its spectral function should have a pole of finite residue at zero energy. However, if the pole has a finite but small energy, small compared to transition temperatures, one can claim an approximate symmetry. Such a state above the top of the two-particle continuum is referred to as an “antibound” state. Demler and Zhang [2] (DZ) claim that the spectral function of the π operator has a pole around 41 meV for the cuprate YBCO based on a fermi liquid t-matrix analysis, and identify the pole (i.e. the antibound state) with a resonance in neutron spectroscopy.

There are many criticisms which can be made of these ideas, from mathematical ones to contradictions of the model with experimental features of the cuprates, to deep general questions of physics. The latter are more conclusive and less arguable, but perhaps initially it is essential to demonstrate the implausibility of this work so we shall start at the simplest and work up. First let us list our objections:

1) (due to M. Greiter [3]) The Hubbard gap is ignored, the chemical potential misidentified and issues of projections are not discussed.

2) The model Hamiltonian omits a large term \( t' \), which is experimentally observed and which displaces the resonance and possibly removes it.

3) The exchange terms in the Hamiltonian have little resemblance to the correct ones, and the antibound state from magnetic interactions disappears once this error is corrected.

4) The conventional Hubbard Hamiltonian omits longer range coulomb interactions, which are not important in the usual low energy physics but are very much so here. (We learned that Greiter also has brought up this point in his second comment [3]).

5) The antiferromagnetic and superconducting phases each derive from a more fundamental thermodynamic phase, the Mott insulator and the metal respectively. The Mott insulator and the metal can not be related to a quantum critical point, by Elitzur’s theorem, since they differ by a local gauge symmetry. These phases hence have no locally stable, homogeneous intermediate phases, and can not be deformed continuously into each other, certainly not by an operator as simple as an SO(5) rotation.

We elaborate our points now. 1) Greiter has brought up the importance of the upper Hubbard band and the problems related to treating the chemical potential in a mean field fashion without invoking projections. We do not wish to go into the Greiter - Demler, Zhang [3] discussion on the issue of chemical potential. The following is our comment on the projection issue. The spectral function of the π operator will have a range from zero (the ground state) to an energy of the order of \( U \). The average energy of the state created by π operator is unimportant. The issue is whether the spectral function has a low energy pole with finite residue, and if so, whether the π operator projected (renormalized) to this pole continues to be a generator obeying the old SO(5) algebra or not. The projected operator may be defined as

\[ \pi_\alpha = P_\alpha \pi P_\alpha \] 

where \( P_\alpha = |\alpha > < \alpha| \) is the projection operator; and the state \( |\alpha > \) is the many body \((N + 2)\)-particle state corresponding to the pole. Projections to subspaces can change the commutation relations. For example, in the case of the Mott insulator, the π operator projected to the lower Hubbard band creates only spin and hence it is incapable of rotating between the charge and spin sectors of SO(5).

2) The known fermi surfaces of cuprates are inconsistent with the bipartite model used by DZ, which contains only nearest neighbor hopping. The correct form of dispersion, as has also been noted by DZ, is
with \( t' \) of the order of \( \frac{1}{2} t \). The \( t' \) term does not commute with the operator \( \pi \) and spreads its spectrum over 0.1 eV. Any resonance must be outside this range, presumably an antibound state above it. It is also easy to show that the spreading of the two particle spectrum at \((\pi, \pi)\) reduces the already small antibinding energy \( (0.1J \approx 10 \text{ to } 15 \text{meV}) \). This is because the spin triplet antibinding state has a p-symmetry in the relative orbital co-ordinate forced by antisymmetry, and a p-wave bound state \( \frac{3}{2} \) (to be precise \( p_x - p_y \) symmetry) does not effectively make use of short range attraction in view of the node at the origin.

That spreading of the two particle band (corresponding to relative motion) will destroy the two particle antibound state can be already inferred from DZ’s results directly. A change of the center of mass momentum by only about \( \frac{\delta \mu}{\mu} \) is enough to remove the antibinding energy (figure 1 in DZ). This happens when the two particle band width is \( \approx 0.1J \), comparable to the antibinding energy at \((\pi, \pi)\). This is a consequence of the fact that in a tight binding situation nearest neighbor attraction will give a p-wave binding only when its strength is of the order of the band width.

3) The Hamiltonian (equation 1 of DZ) used by DZ includes a nearest neighbor exchange term in a Hubbard model. This is, rather confusingly, included in order to capture the higher order effects in the model. This is, rather confusingly, included in order to give a p-wave binding only when its strength is of the order of the band width.

The general idea is to propose that thermodynamic phases are characterized by an order parameter such as superconducting pair wave functions or an (anti)ferromagnetic moment. One imagines a critical point at \( T = 0 \), at which this order parameter changes continuously from one type to another (or ceases to exist) as some parameter - in the case of cuprates, the doping - is varied. One has then a multi-dimensional space of some sort, containing an order parameter vector of finite dimensionality and this vector rotates from one sector of this space to another at the critical point. The associated \( T = 0 \) quantum critical fluctuations spread its influence into a range of finite \( T \) and doping.

This picture does not take into account the basic physics of the two states. The antiferromagnet is a Mott insulator, and it is an antiferromagnet because it is a Mott insulator, not vice versa. Superexchange is a consequence of the insulating state (see PWA [1]). The insulating state has a large gap for charge fluctuations, and the antiferromagnetic order parameter forms in the subspace formed by bare spins.

The superconductor, on the other hand, forms in a metal, and is a property of low-energy quasiparticle-like excitations near a fermi surface. The fermi surface is meaningless in the insulator, but is crucial to the superconductor.

The dominant transition, then, is between metal and insulator, each being states described by an infinite dimensional order parameter \([\mu]\). The transition between these two states can not be continuous, and is not; the insulator has a discontinuity in chemical potential, \( \mu = \frac{2 \pi}{\beta \text{meV}} \) and is therefore a cusp of the free energy. States
of intermediate doping cannot be homogeneous and are not. When doping is done by mobile charges (laser doping, or oxygen doping in $La_2CuO_{4+\delta}$) the system segregates. In summary, the order parameter characterizing antiferromagnetism and superconductivity relate to subspaces of Hilbert space which are algebraically inequivalent and can not be mapped on each other; thus neither can the order parameters themselves. There is no underlying quantum critical point expressing an essential continuity between the two phases.

I. ACKNOWLEDGEMENT

We would like to acknowledge Steve Strong for bringing to our attention the preprint by Martin Greiter and Steve Strong and David Clarke for discussions. This work was supported by the National Science Foundation Grant DMR - 9104873.

‡ on leave of absence from: Institute of Mathematical Sciences, Madras 600 113, India

[1] Shou-Cheng Zhang, Science 275 1089 (97)
[5] DZ get the energy of the $\pi$ resonance $E_\pi$ by adding this antibinding energy $E_b \approx 0.1J$ to the center of the two particle ($\pi, \pi$) band which is assumed to be around $0.2J$ above the ground state; $E_\pi \approx 2\mu + E_b = 0.2J + 0.1J \approx 40\text{meV}$.

[6] That the $\pi$ operator creates a state that has a $p_x-p_y$ symmetry can easily be seen by rewriting $\pi$ in terms of relative momenta as follows

$$\pi = \sum (\cos k_x - \cos k_y) c_{-k \pi \uparrow}^\dagger c_{k \pi \uparrow}^\dagger$$

$$= \sum (\sin k_x - \sin k_y) c_{q \pi \uparrow}^\dagger c_{-q \pi \uparrow}^\dagger$$

[7] Pauli blocking of the two particle states reduce this spread. However, a spread as little as 10 meV seems to be enough to remove the antibound state.

[8] It is easily seen that the term $\frac{1}{2}n_i n_j$ coming from the $S_z$ components of $S_i, S_j$ which causes binding (see equation 3 of DZ) in leading order in $J$ is canceled by our term $\frac{1}{2}n_i n_j$.