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## A critical damage criterion for creeping solids

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Creep damage is the progressive reduction in the material's ability to resist stress and is manifested as an increase in creep rate during tertiary creep eventually resulting in failure. Monkman–Grant relationship [1] that has contributed significantly to the development of creep fracture models [2], relates minimum creep rate  $\dot{\epsilon}_m$  and rupture life  $t_r$  as  $\dot{\epsilon}_m t_r = \text{constant} = C_{MG}$ , for which, we provided a physical basis in terms of first order kinetics for creep [3–6]. In the present study, Monkman–Grant constant  $C_{MG}$  is henceforth referred to as 'Monkman–Grant Ductility (MGD)' that is the contribution of secondary creep strain to strain to failure,  $\epsilon_f$  (Fig. 1). Monkman–Grant relation does not describe evolution of damage and its coupling to the deformation rate. The two approaches that describe this coupling are 'Continuum Creep Damage Mechanics (CDM)' by Kachanov–Rabotnov [7, 8] and 'Materials Properties Council (MPC)-Omega' by Prager [9]. CDM treats damage as an internal state variable and an important outcome is the creep damage tolerance factor  $\lambda$  [10, 11] defined as ratio of  $\epsilon_f$  to MGD (Fig. 1) i.e.,  $\lambda = \epsilon_f / (\dot{\epsilon}_m t_r)$  and is a constant.  $\lambda$  is suggested [10] to be a better measure of creep ductility as it assesses the susceptibility of a material to localized cracking at strain concentrations [12], and thus can be considered as a material performance characteristic. Ashby and Dyson [11] provided a physical basis to CDM approach and demonstrated that each damage micromechanism, when acting alone, results in a characteristic value of  $\lambda$ . According to MPC-Omega approach [9], creep rate  $\dot{\epsilon}$  from its initial value  $\dot{\epsilon}_0$  increases with strain  $\epsilon$  as  $\dot{\epsilon} = \dot{\epsilon}_0 \exp(\Omega_p \epsilon)$ , where  $\Omega_p$  is reciprocal of MGD, i.e.,  $\Omega_p = 1/(\dot{\epsilon}_m t_r)$  for conditions showing negligible primary creep. The total damage coefficient  $\Omega_p$  is the rate at which material's ability to resist stress is degraded by strain and is a material performance characteristic that is related to creep damage tolerance, i.e., higher the  $\Omega_p$ , lesser is the resistance to creep damage.

In this paper, we identify that MGD is the critical strain at which true tertiary creep damage sets in and further introduces the concept of  $t_{MGD}$  (Fig. 1) as the time at which MGD is reached along the creep curve and the total secondary creep ductility is exhausted.  $t_{MGD}$  can be obtained as the time at which  $\epsilon - \epsilon_p = \text{MGD}$ ,  $\epsilon_p$  is the limiting primary creep strain, and for negligible  $\epsilon_p$  as shown in Fig. 1,  $t_{MGD}$  is the time to reach MGD. Damage evolves along the creep curve and for any damage mechanism, we put forward that  $t_{MGD}$  is the time at

which damage attains a critical level, beyond which its accelerated growth leads to failure. For a typical case of cavitation mechanism, it is shown that time to reach critical cavity size  $t_{CCS}$  matches well with  $t_{MGD}$ . We propose a *critical damage criterion* based on CDM as well as MPC-Omega method in terms of a unique relationship between  $t_{MGD}$  and  $t_r$  that depends only on  $\lambda$ . Further, we demonstrate its universal applicability for a wide range of materials and discuss its implications to engineering creep design.

In order to validate our proposition that  $t_{MGD}$  is the time at which damage attains a critical level, we analyse the mechanistic data reported by Davis and Williams [13] on  $\alpha$ -iron at various temperatures (815–978 K) and stress levels (17.24–68.95 MPa). They observed a two stage tertiary creep behaviour; in the first stage, tertiary creep strain-time followed a  $t^{4/3}$  law, whereas the second stage obeyed an exponential law. They concluded that the end of first stage corresponds to the time at which creep cavities attain a critical size and the second stage is the 'true tertiary creep'. Accordingly, the time corresponding to the end of first stage is designated as time to attain critical cavity size  $t_{CCS}$ . Creep curves that include secondary and tertiary creep regimes were generated using creep data as well as the tertiary creep strain-time relations given by Davis and Williams [13].  $\epsilon_f$  was obtained as the sum of  $\epsilon_p$ , MGD and  $\epsilon_t$  (cf. Fig. 1), where  $\epsilon_t$  was calculated as  $\epsilon_t = A \exp[\beta(t_r - t_{ot})]$  and the constant reported value [14] of  $\epsilon_p$  was taken as 0.051. From the generated creep curves,  $\lambda$  (discussed later) and  $t_{MGD}$  were determined;  $t_{MGD}$  was obtained as the time at which  $\epsilon - \epsilon_p = \text{MGD}$ . The plot of  $t_{CCS}$  vs.  $t_{MGD}$  in Fig. 2 demonstrates that  $t_{CCS}$  matches well with  $t_{MGD}$  and validates our proposition of  $t_{MGD}$  as the time at which creep damage attains a critical level. We like to extend such a validation for other damage mechanisms, but it has not been possible due to the lack of mechanistic data. Further, when two coupled mechanisms operate as in the case of nickel base superalloys [12], we suggest that  $t_{MGD}$  might correspond to the time at which cavitation intervenes the strain softening mechanism causing true tertiary creep damage.

Based on CDM approach, recently we proposed [15] a new relationship between  $t_{MGD}$  and  $t_r$  in terms of  $\lambda$ , and a brief description is given here. According to CDM, the evolution of deformation and damage is expressed in terms of internal state damage variable  $\omega$  and for uniaxial stressing [12, 16]  $\dot{\epsilon} = \dot{\epsilon}_0 (\sigma/\sigma_0)^n$

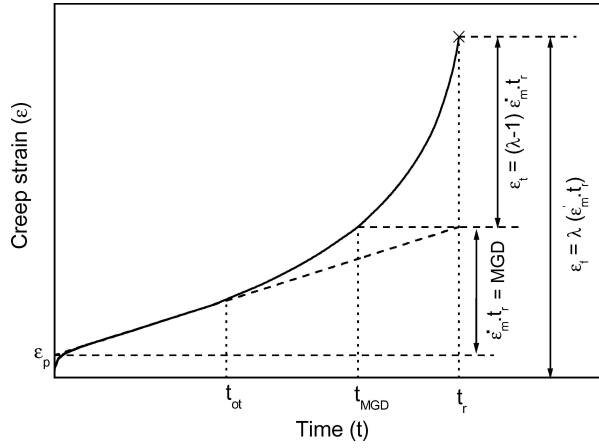


Figure 1 Schematic creep curve with negligible primary creep strain  $\varepsilon_p$  illustrating time to reach Monkman-Grant ductility  $t_{MGD}$ , time to onset of tertiary creep  $t_{ot}$ , damage tolerance factor  $\lambda$  and limiting tertiary creep strain  $\varepsilon_t$ .

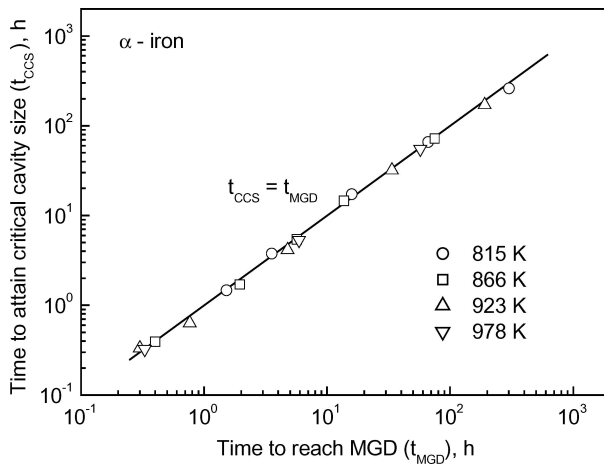


Figure 2 Demonstrates  $t_{CCS} \approx t_{MGD}$  for creep cavitation micromechanism in  $\alpha$ -Fe. Solid line is according to  $t_{CCS} = t_{MGD}$  and symbols correspond to the experimental data.

$[1/(1 - \omega)]^m$  and  $\dot{\omega} = \dot{\omega}_0 (\sigma/\sigma_0)^m [1/(1 - \omega)]^m$ , where  $\dot{\varepsilon}_0$  and  $\dot{\omega}_0$  are temperature dependent rate constants at a stress  $\sigma_0$ ;  $n$  and  $m$  are constants. It is assumed that  $\omega = 0$  when the material is in its undamaged state and  $\omega = 1$  at rupture. Integration of these coupled equations at constant stress, for  $m + 1 > n$  gives relation between strain fraction  $\varepsilon/\varepsilon_f$  and time fraction  $t/t_r$  as  $\varepsilon/\varepsilon_f = 1 - (1 - t/t_r)^{1/\lambda}$ , where  $\lambda = (m + 1)/(m - n + 1)$ . For negligible  $\varepsilon_p$  (Fig. 1),  $\varepsilon_t = (\lambda - 1)\dot{\varepsilon}_m t_r$ , since  $\lambda = \varepsilon_f/(\dot{\varepsilon}_m t_r)$  and  $\varepsilon_t = \varepsilon_f - (\dot{\varepsilon}_m t_r)$ . At  $t = t_{MGD}$ , creep strain  $\varepsilon = MGD = \dot{\varepsilon}_m t_r$ , and on substituting this in the equation relating  $\varepsilon/\varepsilon_f$  and  $t/t_r$ , and on rearrangement we get  $\varepsilon_t/\varepsilon_f = (1 - t_{MGD}/t_r)^{1/\lambda}$ . In this equation, substituting for  $\varepsilon_t$  and  $\lambda$ , we obtain the *critical damage criterion* in terms of a universal relationship between  $t_{MGD}$  and  $t_r$  as

$$\frac{t_{MGD}}{t_r} = 1 - \left(\frac{\lambda - 1}{\lambda}\right)^\lambda = \text{constant} = f_{CDM}, \quad (1)$$

where  $f_{CDM}$  can be determined knowing the value of  $\lambda$ . Unlike  $t_{MGD}$  and  $t_r$ ,  $t_{MGD}/t_r$  is independent of stress and temperature. We call the physically based

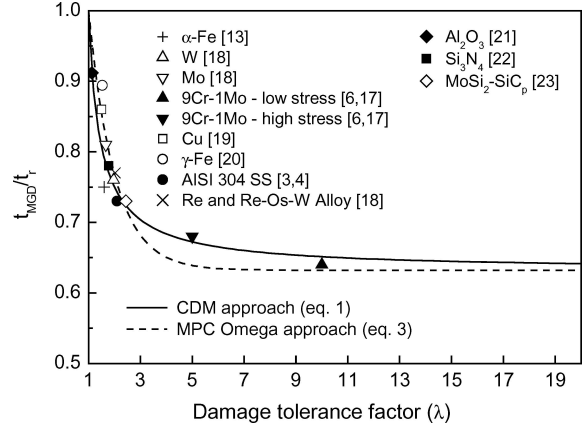


Figure 3 Validity of critical damage criterion (Equation 1) for various materials. Solid line is according to Equation 1 based on CDM approach, whereas broken line is according to Equation 3 based on MPC-Omega method. Symbols correspond to  $f_{EXP}$  values obtained from double logarithmic plot of  $t_{MGD}$  vs.  $t_r$  for different materials. Data for W is taken from Fig. 5.9, Mo from Fig. 8.13, Re and Re-Os-W alloy from Fig. 6.5 and Fig. 6.17B, respectively, from Ref. 18.

Equation 1 as *critical damage criterion* because damage attains critical value at  $t_{MGD}$  when the criterion  $t_{MGD} = f_{CDM} t_r$  is met, whereas  $t_{MGD}$  is the time at which creep damage attains a critical level. The theoretical plot of  $t_{MGD}/t_r$  vs.  $\lambda$  following Equation 1 is shown in Fig. 3, where  $t_{MGD}/t_r$  decreases with increasing  $\lambda$  and saturates at 0.63. This is in order since Equation 1 is of the functional form  $y = 1 - [1 - (1/x)]^x$  and in the limit  $x \rightarrow \infty$ ,  $y = (1 - 1/e) = 0.63$ .

Along the lines of CDM approach, we also deduce the critical damage criterion based on MPC-Omega method [9]. For the sake of brevity, we start from the equation given by Prager (i.e., Equation 12 in Ref. [9]) for any time  $t$  and at  $t_r$  as

$$\frac{1}{\dot{\varepsilon}_m \Omega_p} (e^{-\Omega_p \varepsilon} - e^{-\Omega_p \varepsilon_f}) = (t_r - t), \quad (2)$$

where  $\Omega_p = 1/\dot{\varepsilon}_m t_r$  and from the definition of  $\lambda$ ,  $\varepsilon_f = \lambda/\Omega_p$ . Substituting for  $\Omega_p$  and  $\varepsilon_f$  as well as the condition  $\varepsilon = MGD = \dot{\varepsilon}_m t_r$  at  $t = t_{MGD}$  in Equation 2 and on rearrangement, the relationship between  $t_{MGD}$  and  $t_r$  in terms of  $\lambda$  can be deduced as

$$\frac{t_{MGD}}{t_r} = 1 - e^{-1} + e^{-\lambda} = \text{constant} = f_{MPC}. \quad (3)$$

In Equation 3, when  $\lambda \gg 1$ ,  $t_{MGD}/t_r = (1 - 1/e) = 0.63$  and is in accordance with Equation 1 for large values of  $\lambda$ . Using Equation 3,  $f_{MPC}$  can be calculated knowing the value of  $\lambda$ . The theoretical plot of Equation 3 is also shown in Fig. 3 illustrating that  $t_{MGD}/t_r$  decreases with increasing  $\lambda$  and saturates at 0.63. It is evident from Fig. 3 that both the Equations 1 and 3 are identical in implying the uniqueness of the relationship between  $t_{MGD}$  and  $t_r$ . Thus MPC-Omega method further strengthens the proposed damage criterion (i.e., Equation 1) and reinforces that  $t_{MGD}/t_r$  depends only on  $\lambda$ .

We demonstrate the validity of critical damage criterion for a wide range of materials. Analysis of results

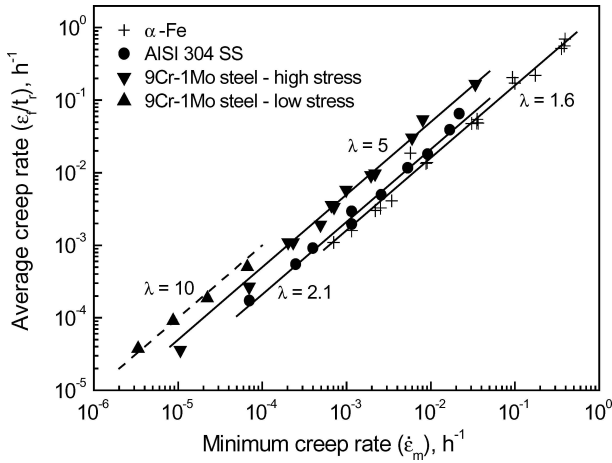


Figure 4 Illustrates the constancy of  $\lambda$  for  $\alpha$ -Fe, 9Cr-1Mo steel and AISI 304 stainless steel.  $\lambda$  is determined as the intercept at  $\dot{\epsilon}_m = 1$  in the double logarithmic plots of  $\epsilon_f/t_r$  vs.  $\dot{\epsilon}_m$ , since  $\epsilon_f/t_r = \lambda \dot{\epsilon}_m$ .

on 9Cr-1Mo ferritic steel [6, 17] and AISI 304 stainless steel [3, 4] showing the constancy of  $\lambda$  is presented in Fig. 4. Unlike 304 stainless steel ( $\lambda = 2.1$ ), 9Cr-1Mo steel exhibited separate constant values of  $\lambda = 10$  and 5 for low and high stress regimes, respectively. Published tensile creep data [13, 18–23] for various materials such as pure metals, ceramics and composite of intermetallic silicide were also analysed for  $\lambda$ ,  $t_{MGD}$  and  $t_r$ .  $\lambda$  was found to be constant for a given material and this for a typical case of  $\alpha$ -Fe [13] is shown in Fig. 4. For different materials, logarithmic plots of  $t_{MGD}$  vs.  $t_r$  obeyed  $t_{MGD} \propto t_r$  (i.e.,  $t_{MGD} = f_{EXP} t_r$ ) and the observed  $f_{EXP}$  values are shown as symbols in Fig. 3. The plot of  $t_{MGD}$  vs.  $t_r$  in Fig. 5 demonstrates the validity of damage criterion (Equation 1) for various materials and  $f_{CDM}$  values ranged from 0.65 for 9Cr-1Mo steel to 0.91 for  $Al_2O_3$ . Unlike Fig. 3, the difference in  $f_{CDM}$  values is not seen in Fig. 5 due to logarithmic representation of the plot. We emphasise that the damage mechanism specific nature of damage criterion comes from  $f_{CDM}$  which is related to  $\lambda$ , as any change in damage mechanism changes the value of  $\lambda$  [11].

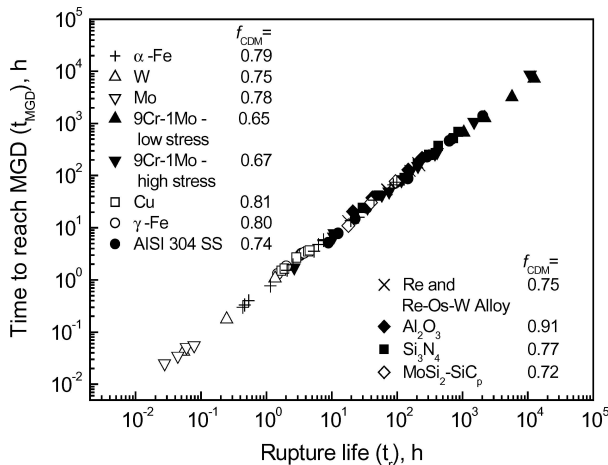


Figure 5 Variation of  $t_{MGD}$  with  $t_r$  demonstrating the validity of critical damage criterion (Equation 1) with  $f_{CDM}$  values calculated using respective values of  $\lambda$  for different materials. Symbols represent experimental data obeying Equation 1.

The damage criterion has important implications. First is that  $t_{MGD}$  conceptually divides creep curve into two parts (Fig. 1), and this is something similar to plastic instability in tension dividing the stress-strain curve into uniform and non-uniform deformation regimes. The second is its importance to engineering creep design. The ratio  $t_{MGD}/t_r$  saturating to  $\sim 2/3$  (Fig. 3) provides a physical basis for the factor of safety of 67% employed on stress to cause rupture in  $10^5$  hr to arrive at the design allowable stress [24, 25]. Further, as critical damage sets in and minimum ductility is assured up to  $t_{MGD}$ , we suggest that the stress to cause  $t_{MGD}$  in  $10^5$  hr can be considered as a new design criterion. The proposed damage criterion also has its implication to Robinson's life fraction damage rule [25, 26], which states that under non steady stress and temperature conditions, failure occurs when  $\sum \Delta t_i/t_{ri} = 1$ , where  $\Delta t_i$  is the time spent at any given stress and temperature and  $t_{ri}$  is the rupture life under those conditions. When  $\sum \Delta t_i/t_{ri}$  either equals or exceeds unity, it can be conveniently used as a conservative basis for life prediction. For situations when it predicts non conservative values of remnant life when  $\sum \Delta t_i/t_{ri} < 1$ , we propose a modified life fraction rule as  $\sum \Delta t_i/t_{MGDi} = 1$ , which is conservative. It can be shown that  $\sum \Delta t_i/t_{MGDi} = 1$  leads to  $\sum \Delta t_i/t_{ri} < 1$  as  $t_{MGD} = f_{CDM} t_r$  (i.e.,  $\sum \Delta t_i/t_{MGDi} = (1/f_{CDM}) \sum \Delta t_i/t_{ri} = 1$  and  $\sum \Delta t_i/t_{ri} = f_{CDM} < 1$ ), since it is reasonable to assume that the  $f_{CDM}$  remains constant (i.e.,  $\lambda$  is constant) in a given stress-temperature domain.

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