1969Ap&SS...4..218K

ATMOSPHERES OF EXTREME METAL-DEFICIENT STARS

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(Received 23 December, 1968)

Abstract. Atmospheric models have been constructed for effective temperatures 4000°, 4500° and 5000° and for hydrogen-to-metal ratios of 1, 10^2, 10^3 and 10^4 times the solar values, and for surface gravities of 2×10^4 and 2×10^2. The effect of metal deficiency on the atmospheric structure of these stars are studied.

1. Introduction

The study of metal-deficient stars has assumed great importance in recent years. This is because of the fact that the study of metal-deficient stars might give a clue to the origin of elements in stars and also to the study of stellar evolution. So far, a number of metal-deficient stars have been studied (Aller and Greenstein, 1960; Baschek, 1959; Pagel and Powell, 1966; Johnson *et al.*, 1968; Johnson and Mitchell, 1968; Krishna Swamy, 1966, 1968a). In general one finds that the metal deficiency in stars varies by factors between 10 and 100 as compared to that of solar value. Few stars show metal deficiency by factors larger than 100 (Wallerstein *et al.*, 1963; Pagel, 1965). Recently Greenstein (1968) finds that there may be stars whose metal deficiency may be as much as a factor of 1000 or more with respect to that of solar value. It is therefore of significance to make a theoretical analysis of the atmospheres of extreme metal-deficient stars. This paper is concerned with such a study.

2. Model Atmosphere Calculation

Here we will discuss briefly the various quantities that are required for a solution of the hydrostatic equilibrium equation (for details see Krishna Swamy, 1966a, b)

$$\mathrm{d}P/\mathrm{d}\tau = g/\bar{K}\,, \tag{1}$$

where τ is the optical depth, p the gas pressure, g the surface gravity and $\bar{K}$ the mean absorption coefficient per gram of the stellar substance.

(a) Effective temperature: The model atmospheres are constructed for three values of effective temperature corresponding to 4000 K, 4500 K and 5000 K.

(b) Surface gravity: We construct model atmospheres for two values of surface gravity, namely 2×10^4 and 2×10^2 corresponding to main sequence and giant stars respectively.

(c) Composition: We use $X = 0.75$ and the atmospheric models are constructed

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for hydrogen-to-metal ratios (H/M) of 1, 10^2, 10^3, and 10^4 times the solar value. The abundances of the individual elements are fixed relative to hydrogen using the solar abundances as standard (GOLDBERG *et al.*, 1960).

(d) Sources of Opacity: For the calculation of the Rosseland mean, we have considered the following: continuous absorption by H, H^-, H_2^-, and H_e^- and scattering by H and H_2.

(e) Equation of state: In the equation of state, we have included hydrogen in the stages H_2, H_2^+, H^-, H and H^+. Helium is considered only in the neutral state, since the ionization of helium is negligible in the temperature range of interest. Detailed ionization equilibria of nine representative metallic elements have been considered in the neutral and first ionization states.

(f) In the radiative zone, the following analytical $(T-\tau)$ relation is used (KRISHNA SWAMY, 1966a, 1967, 1968b).

$$T^4 = \tfrac{3}{4} T_e^4 \left[\tau + 1.39 - 0.815 e^{-2.54\tau} - 0.025 e^{-30\tau}\right], \tag{2}$$

where τ is the Rosseland mean optical depth. The model atmosphere in the convective zone is computed under the framework of the mixing-length theory of BÖHM-VITENSE

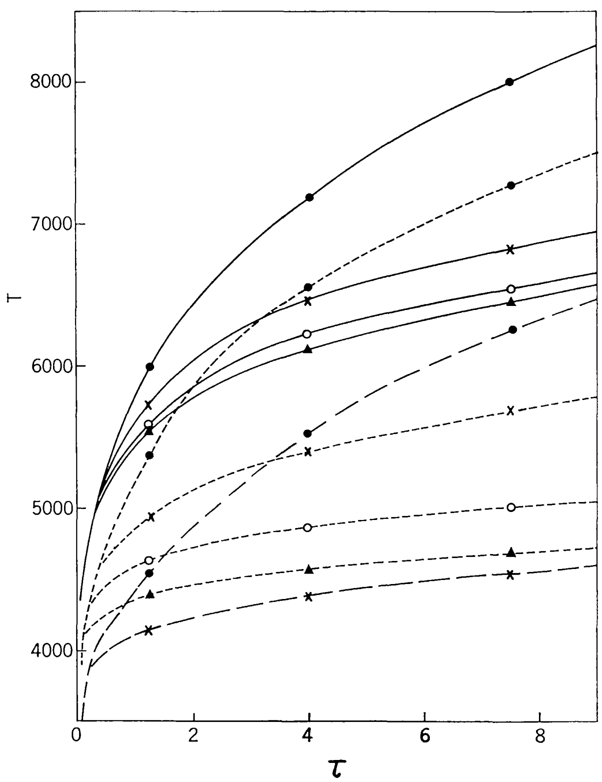

Fig. 1. Temperature is plotted as a function of τ for different effective temperatures and hydrogen-to-metal ratios for $g = 2 \times 10^4$ continuous line, dashed line and long dashed line refer to $T_e = 5000$, 4500 and 4000 K respectively. Curves with dots, crosses, circles and triangles refer to $A = 1$, 10^2, 10^3 and 10^4 respectively.

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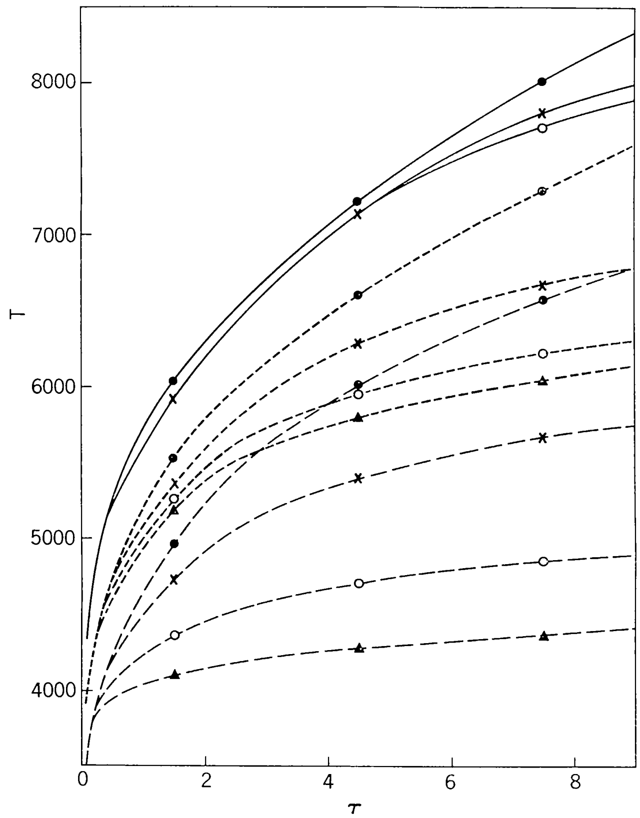

Fig. 2. Same as Figure 1, but for $g = 2 \times 10^2$.

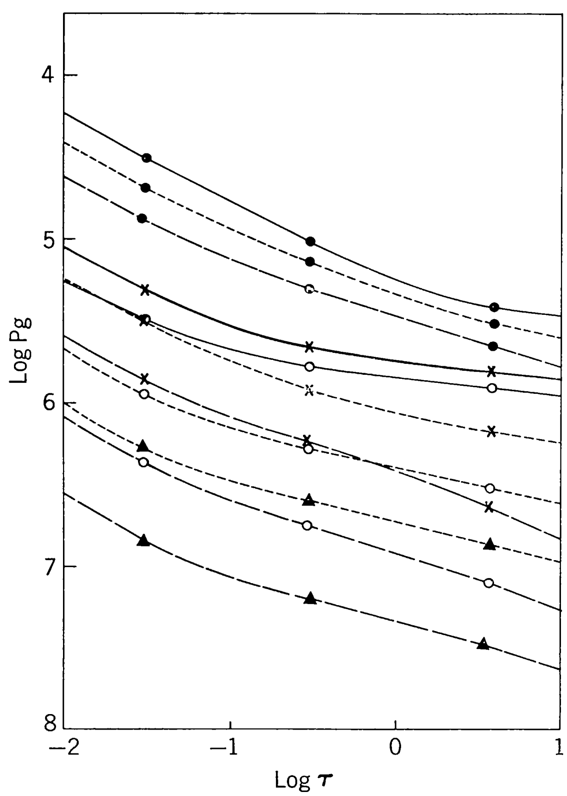

Fig. 3. Gas pressure is plotted as a function of $\log \tau$ for $g = 2 \times 10^4$. Other symbols have the same meaning as that of Figure 1.

(1958) (for details see KRISHNA SWAMY, 1969). In the calculation of quantities like, adiabatic gradient, specific heat at constant pressure etc., the various stages of hydrogen and metals that were considered in the equation of state are explicitly taken into account.

3. Results and Discussion of the Model Atmospheres

(a) Temperature distribution: In Figures 1 and 2, we show the resulting temperature distributions for the hydrogen-to-metal ratios of 1, 10^2, 10^3, and 10^4 times the solar value and for various effective temperatures. Figures 1 and 2 refer to the cases $g = 2 \times 10^4$ and 2×10^2 respectively. One can see clearly from these figures, the branching of the curves depending on the hydrogen-to-metal ratio. For the same metal deficiency the effect of convection on the temperature structure of the model atmosphere is larger for a lower effective temperature star in comparison to that of higher effective temperature star. Also the effect of convection on the temperature distribution of the model atmosphere is larger for $g = 2 \times 10^4$ compared to that of $g = 2 \times 10^2$.

(b) Pressure distribution: The variation of gas pressure and electron pressure as a function of optical depth is shown in Figures 3–6 for various effective temperatures and hydrogen-to-metal ratios. We give in Tables I and II the fraction of hydrogen atoms and hydrogen molecules with respect to the total hydrogen atoms in the model atmospheres. In the tables A refers to hydrogen-to-metal ratio of the star to that of hydrogen-to-metal ratio for the sun. It may be noted that hydrogen exists mostly in

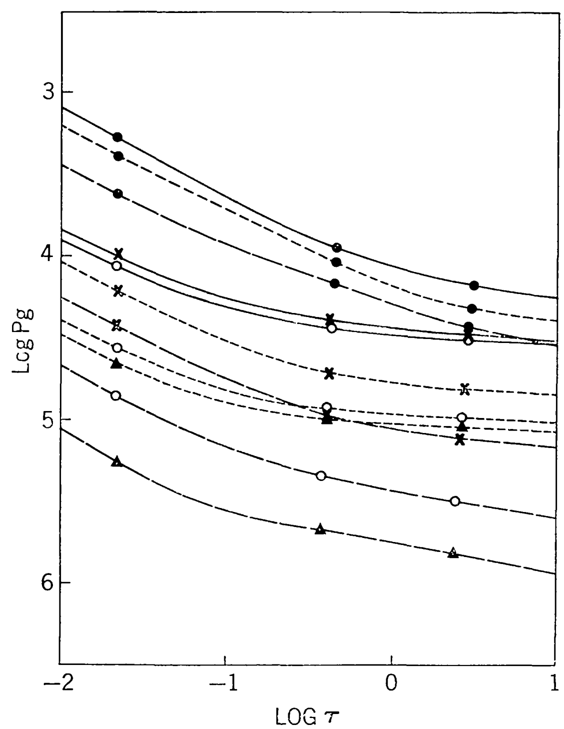

Fig. 4. Same as Figure 3, but for $g = 2 \times 10^2$.

the molecular form in extreme metal-deficient stars. Therefore the contribution of hydrogen molecules to the total gas pressure increases with metal deficiency. Here again, hydrogen molecules are relatively more important for the case $g = 2 \times 10^4$ compared to that of $g = 2 \times 10^2$.

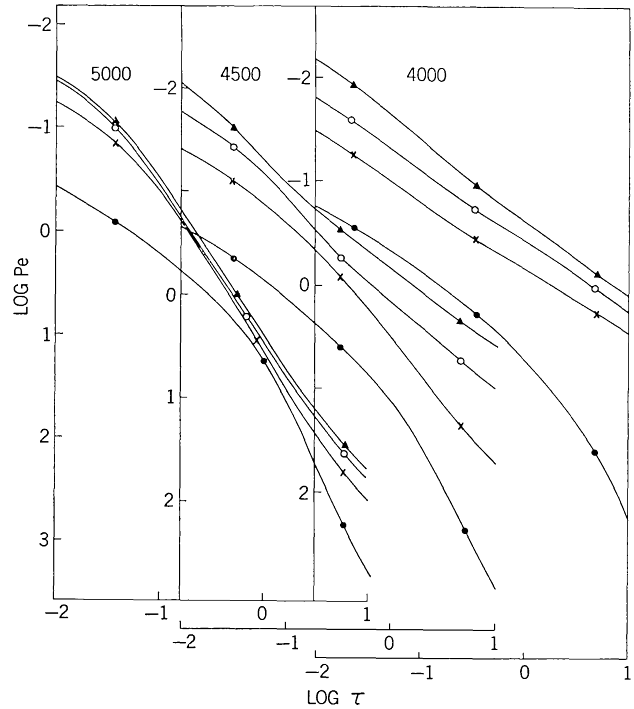

Fig. 5. Electron pressure is plotted as a function of log τ for $g = 2 \times 10^4$. Curves with dots, crosses, circles and triangles refer to $A = 1$, 10^2, 10^3 and 10^4 respectively.

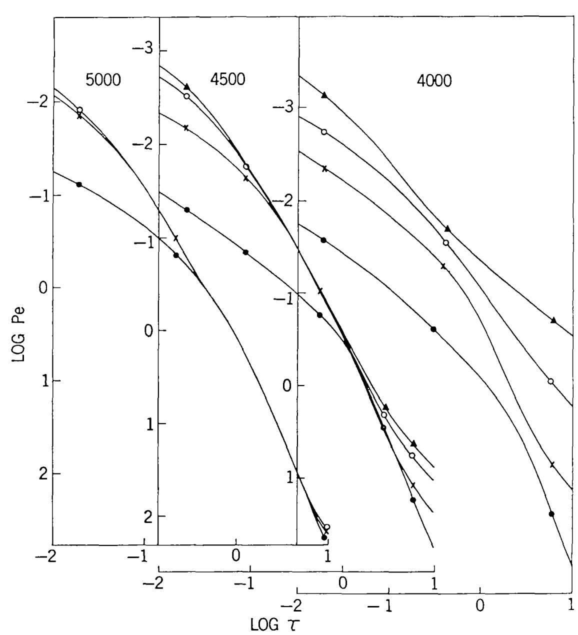

Fig. 6. Same as Figure 5, but for $g = 2 \times 10^2$.

(c) Logarithmic temperature gradients: In Figures 7–10, we show a plot of the actual temperature gradient (∇) and the adiabatic temperature gradient (∇ad) as a function of optical depth for various effective temperatures and hydrogen-to-metal ratios. As can be seen from Tables I and II, the fraction of hydrogen atoms existing

TABLE I

Fraction of hydrogen atoms and hydrogen molecules for the case $g = 2 \times 10^4$

τ	$A=1$		$A=10^2$		$A=10^3$		$A=10^4$	
	FHI	FH_2	FHI	FH_2	FHI	FH_2	FHI	FH_2
$T_e = 5000$ K:								
0.01	0.991	0.005	0.944	0.028	0.914	0.043	0.908	0.046
0.05	0.990	0.005	0.941	0.030	0.915	0.043	0.910	0.045
0.10	0.992	0.004	0.955	0.022	0.939	0.031	0.935	0.032
0.50	0.997	0.002	0.985	0.006	0.980	0.010	0.975	0.012
$T_e = 4500$ K:								
0.01	0.942	0.029	0.737	0.131	0.554	0.223	0.411	0.295
0.05	0.943	0.029	0.736	0.132	0.551	0.224	0.414	0.293
0.10	0.958	0.021	0.788	0.106	0.622	0.189	0.480	0.258
0.50	0.987	0.006	0.910	0.045	0.770	0.111	0.520	0.235
1.00					0.795	0.102	0.540	0.220
$T_e = 4000$ K:								
0.01	0.655	0.172	0.272	0.364	0.159	0.420	0.095	0.452
0.05	0.674	0.163	0.283	0.359	0.165	0.417	0.098	0.451
0.10	0.746	0.127	0.343	0.329	0.204	0.398	0.121	0.440
0.50	0.906	0.050	0.485	0.260	0.300	0.350	0.185	0.406
1.00			0.520	0.240	0.330	0.330	0.200	0.400

TABLE II

Fraction of hydrogen atoms and hydrogen molecules for the case $g = 2 \times 10^2$

τ	$A=1$		$A=10^2$		$A=10^3$		$A=10^4$	
	FHI	FH_2	FHI	FH_2	FHI	FH_2	FHI	FH_2
$T_e = 5000$ K:								
0.01 to 0.50		$<1\%$		$<1\%$		$<1\%$		$<1\%$
$T_e = 4500$ K:								
0.01			0.974	0.013	0.947	0.027	0.934	0.033
0.05		$<1\%$	0.975	0.013	0.950	0.025	0.940	0.030
0.10			0.982	0.009	0.966	0.017	0.960	0.020
0.50			0.996	0.002	0.991	0.004	0.989	0.006
$T_e = 4000$ K:								
0.01	0.958	0.021	0.796	0.102	0.640	0.180	0.477	0.261
0.05	0.963	0.019	0.812	0.094	0.653	0.174	0.479	0.260
0.10	0.975	0.012	0.865	0.068	0.729	0.136	0.564	0.218
0.50	0.995	0.003	0.962	0.019	0.880	0.060	0.750	0.120

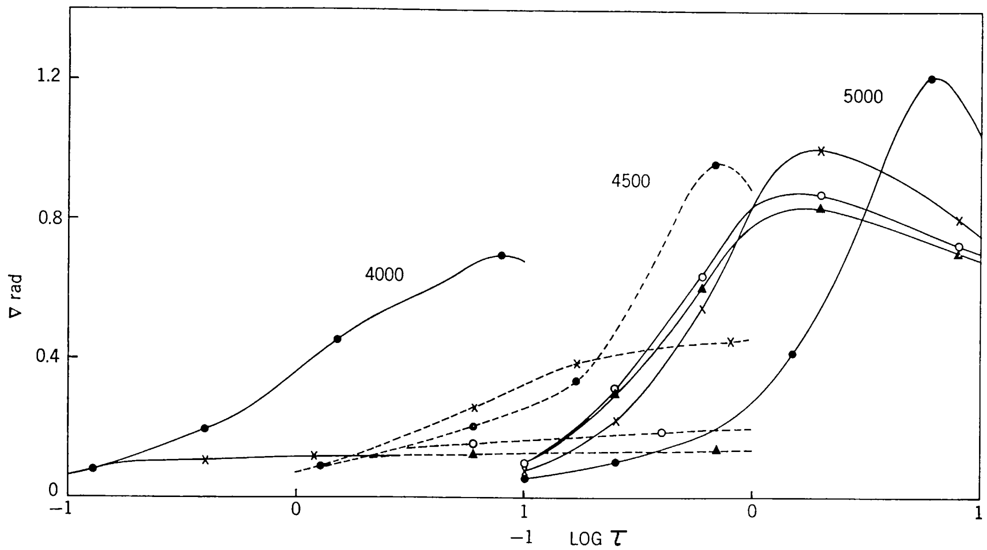

Fig. 7. Plot of ∇rad as a function of $\log\tau$ for $g = 2 \times 10^4$. Other symbols have the same meaning as that of Figure 5.

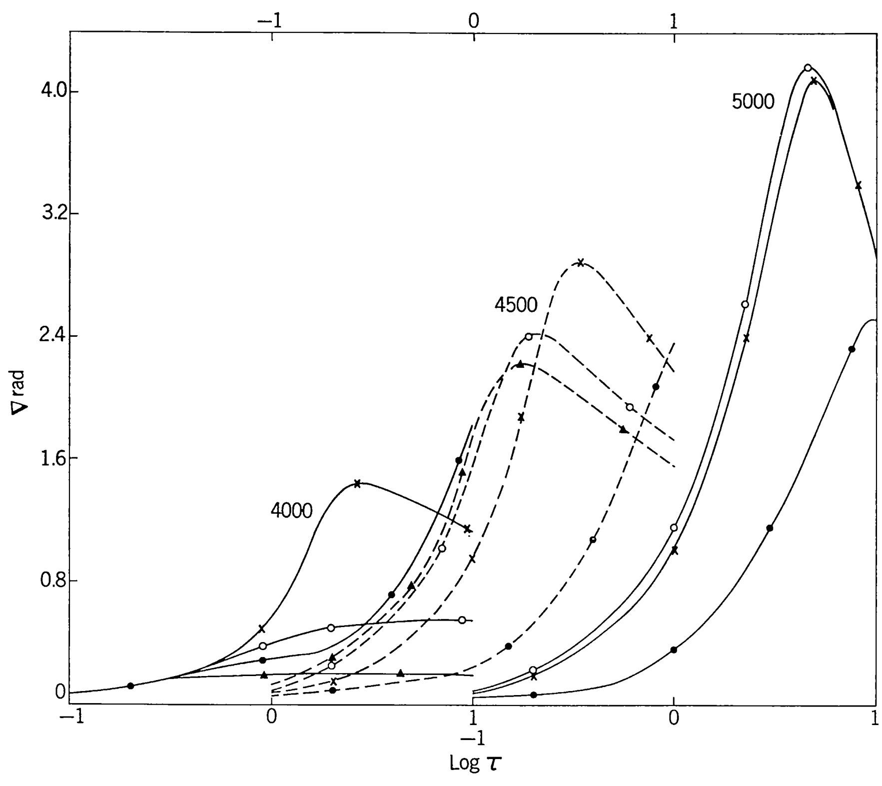

Fig. 8. Same as Figure 7, but for $g = 2 \times 10^2$.

in the molecular form decreases with a decrease in the surface gravity. Therefore, for the same effective temperature the dissociation of hydrogen molecules should give a lower adiabatic gradient for the case $g = 2 \times 10^4$ compared to that of $g = 2 \times 10^2$. Figures 9 and 10 show that this is the case.

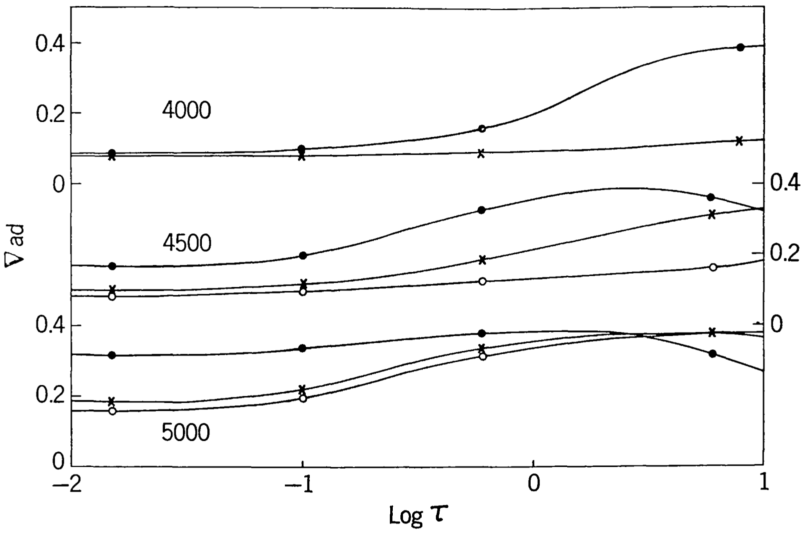

Fig. 9. Plot of ∇ad as a function of logτ for $g = 2 \times 10^4$. Other symbols have the same meaning as that of Figure 5.

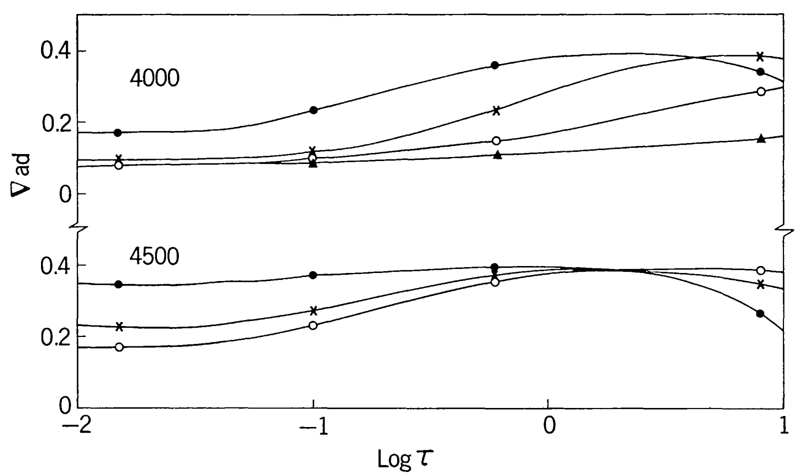

Fig. 10. Same as Figure 9, but for $g = 2 \times 10^2$.

(d) Convective velocity: In Figures 11 and 12, we show a plot of

$$\Delta V = V[T_e, A] - V[T_e, A = 1] \tag{3}$$

as a function of optical depth for different effective temperatures and hydrogen-to-metal ratios. Equation (3) essentially gives the difference in the convective velocities between metal-deficient star and the normal star for the same effective temperature.

(e) Opacity: In Tables III–VI, we give the contribution of Rayleigh scattering by hydrogen atoms and molecules to the total opacity as a function of wavelength. The importance of Rayleigh scattering by hydrogen atoms and molecules in metal-deficient stars may be noted from these tables.

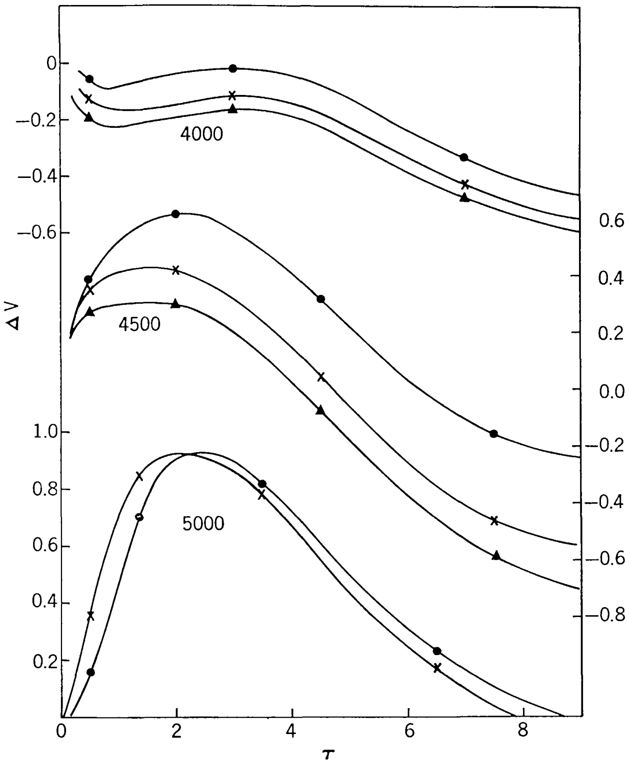

Fig. 11. Plot of differential velocity, Δv, as a function of τ for $g = 2 \times 10^4$. Curves with dots, crosses and triangles refer to $A = 10^2$, 10^3 and 10^4 respectively.

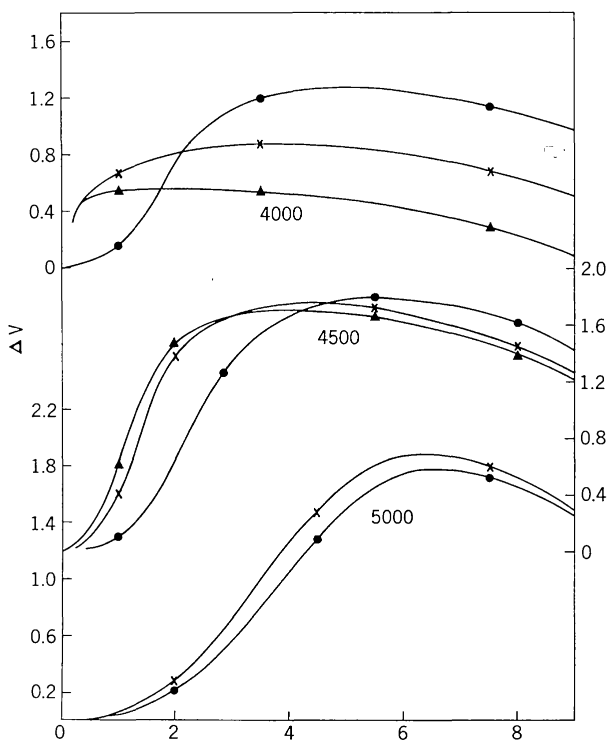

Fig. 12. Same as Figure 11, but for $g = 2 \times 10^2$.

TABLE III

Contribution of Rayleigh scattering by hydrogen atoms and hydrogen molecules to the total opacity at $\tau = 0.05$ for the case $g = 2 \times 10^4$

λ (μ)	$A = 1$		$A = 10^2$		$A = 10^3$		$A = 10^4$	
	σ_H	σ_{H_2}	σ_H	σ_{H_2}	σ_H	σ_{H_2}	σ_H	σ_{H_2}
$T_e = 4500$ K:								
0.14	0.861	0.030	0.814	0.168	0.675	0.317	0.548	0.447
0.20	0.526	0.018	0.734	0.152	0.647	0.304	0.536	0.436
0.24	0.297	0.010	0.617	0.128	0.599	0.281	0.511	0.416
0.28	0.157	0.006	0.466	0.097	0.519	0.243	0.465	0.379
0.40	0.028		0.135	0.028	0.223	0.104	0.250	0.204
0.60	0.004		0.020	0.004	0.040	0.019	0.054	0.044
0.80			0.006		0.011	0.005	0.016	0.013
1.00					0.004		0.007	0.005
$T_e = 4000$ K:								
0.14	0.700	0.194	0.401	0.586	0.254	0.740	0.158	0.840
0.20	0.430	0.120	0.373	0.544	0.250	0.726	0.157	0.837
0.24	0.247	0.069	0.328	0.480	0.239	0.696	0.155	0.826
0.28	0.131	0.036	0.263	0.384	0.221	0.642	0.152	0.806
0.40	0.023	0.006	0.089	0.130	0.125	0.364	0.122	0.652
0.60	0.003		0.014	0.021	0.028	0.081	0.048	0.256
0.80			0.003	0.005	0.008	0.024	0.017	0.089
1.00						0.010	0.009	0.040

TABLE IV

Contribution of Rayleigh scattering by hydrogen atoms and hydrogen molecules to the total opacity at $\tau = 0.05$ for the case $g = 2 \times 10^2$

λ (μ)	$A = 1$		$A = 10^2$		$A = 10^3$		$A = 10^4$	
	σ_H	σ_{H_2}	σ_H	σ_{H_2}	σ_H	σ_{H_2}	σ_H	σ_{H_2}
$T_e = 4500$ K:								
0.14	0.979	0.002	0.983	0.015	0.969	0.029	0.963	0.036
0.20	0.886		0.971	0.015	0.960	0.029	0.950	0.035
0.24	0.740		0.944	0.014	0.948	0.028	0.940	0.035
0.28	0.559		0.889	0.013	0.912	0.028	0.911	0.034
0.40	0.161		0.575	0.008	0.683	0.021	0.701	0.026
0.60	0.024		0.152	0.002	0.229	0.007	0.242	0.008
0.80	0.007		0.046		0.075	0.002	0.080	0.003
1.00			0.021		0.034		0.036	
$T_e = 4000$ K:								
0.14	0.961	0.021	0.879	0.117	0.764	0.234	0.614	0.383
0.20	0.875	0.019	0.865	0.115	0.758	0.233	0.612	0.383
0.24	0.744	0.016	0.841	0.112	0.751	0.230	0.610	0.381
0.28	0.565	0.013	0.794	0.106	0.735	0.226	0.605	0.380
0.40	0.167	0.003	0.512	0.068	0.607	0.186	0.559	0.349
0.60	0.026		0.134	0.018	0.253	0.078	0.349	0.219
0.80	0.007		0.040	0.005	0.090	0.028	0.162	0.101
1.00	0.003		0.018	0.002	0.042	0.013	0.082	0.051

TABLE V

Contribution of Rayleigh scattering by hydrogen atoms and hydrogen molecules to the total opacity for the case $g = 2 \times 10^4$

$\lambda\ (\mu)$	$A=1$		$A=10^2$		$A=10^3$		$A=10^4$	
	σ_H	σ_{H_2}	σ_H	σ_{H_2}	σ_H	σ_{H_2}	σ_H	σ_{H_2}
$T_e = 4500\,K$:								
	$\tau = 0.20$		$\tau = 0.20$		$\tau = 0.34$		$\tau = 0.35$	
0.14	0.778	0.012	0.874	0.085	0.806	0.148	0.633	0.344
0.20	0.350	0.005	0.700	0.068	0.635	0.117	0.560	0.306
0.24	0.169	0.003	0.505	0.049	0.450	0.083	0.462	0.251
0.28	0.082	0.001	0.320	0.031	0.279	0.052	0.336	0.183
0.40	0.013		0.069	0.007	0.059	0.011	0.091	0.050
0.60	0.002		0.009		0.008	0.001	0.013	0.007
0.80							0.004	
$T_e = 4000\,K$:								
	$\tau = 0.20$		$\tau = 0.33$		$\tau = 0.33$		$\tau = 0.34$	
0.14	0.727	0.083	0.584	0.373	0.414	0.570	0.275	0.719
0.20	0.345	0.039	0.471	0.301	0.382	0.525	0.265	0.692
0.24	0.170	0.019	0.340	0.217	0.329	0.452	0.248	0.650
0.28	0.083	0.009	0.136	0.138	0.256	0.353	0.220	0.574
0.40	0.014	0.002	0.047	0.030	0.081	0.110	0.103	0.268
0.60	0.001		0.006	0.003	0.012	0.017	0.020	0.051
0.80					0.003	0.005	0.006	0.014

TABLE VI

Contribution of Rayleigh scattering by hydrogen atoms and hydrogen molecules to the total opacity for the case $g = 2 \times 10^2$

$\lambda\ (\mu)$	$A=1$		$A=10^2$		$A=10^3$		$A=10^4$	
	σ_H	σ_{H_2}	σ_H	σ_{H_2}	σ_H	σ_{H_2}	σ_H	σ_{H_2}
$T_e = 4500\,K$:								
	$\tau = 0.20$		$\tau = 0.20$		$\tau = 0.20$		$\tau = 0.40$	
0.14	0.965	0.001	0.986	0.006	0.981	0.010	0.975	0.008
0.20	0.805		0.944	0.005	0.935	0.009	0.890	0.007
0.24	0.608		0.866	0.004	0.860	0.009	0.763	0.007
0.28	0.404		0.748	0.004	0.734	0.008	0.589	0.004
0.40	0.093		0.314	0.003	0.300	0.003	0.178	0.001
0.60	0.031		0.056		0.053		0.028	
0.80	0.004		0.016		0.015		0.007	
$T_e = 4000\,K$:								
	$\tau = 0.20$		$\tau = 0.20$		$\tau = 0.30$		$\tau = 0.36$	
0.14	0.956	0.007	0.947	0.047	0.903	0.093	0.827	0.169
0.20	0.788	0.006	0.917	0.046	0.888	0.091	0.813	0.166
0.24	0.587	0.004	0.866	0.043	0.852	0.088	0.781	0.160
0.28	0.383	0.002	0.777	0.039	0.792	0.081	0.725	0.149
0.40	0.086		0.381	0.019	0.464	0.048	0.426	0.087
0.60	0.012		0.076	0.003	0.107	0.011	0.100	0.020
0.80	0.003		0.022	0.001	0.032	0.003	0.030	0.006
1.00	0.001		0.009		0.014	0.001	0.013	0.002

TABLE VII

Flux carried by convection in percentage

τ \ A	$g=2\times10^4$				$g=2\times10^2$			
	1	10^2	10^3	10^4	1	10^2	10^3	10^4
$T_e = 5000$ K:								
0.5		<1	5	7				
1.0	<1	6	25	28		<1	<1	<1
5.0	5	68	76	76	<1	12	17	17
10.0	42	82	85	85	6	56	62	62
$T_e = 4500$ K:								
0.5		20	60	69			<1	<1
1.0	<1	41	77	82		<1	1	5
5.0	4	82	90	90	<1	49	67	73
10.0	45	86	95	95	3	71	80	86
$T_e = 4000$ K:								
0.5	6	59	63	63			30	69
1.0	7	76	76	76		<1	52	79
5.0	7	93	93	93	<1	65	86	93
10.0	43	96	96	96	2	83	91	95

(f) Energy transport by convection: In Table VII, we give the percentage flux carried by convection for various cases considered. It may be noted that for extreme metal-deficient stars, most of the energy is carried by convection. For a given T_e, the efficiency of convection is more for main sequence stars compared to giant stars.

(g) Depth of the radiative layer: Convection starts at shallow optical depths in metal-deficient stars. The larger the metal deficiency, the shallower the depth at which convection starts. It is therefore of interest to calculate the decrease in the radiative layer of the star with metal deficiency. For this purpose, we have calculated the quantity

$$F = \frac{[z(\tau_{\text{conv}}) - z(\tau = 0)] \text{ for } A = 1}{[z(\tau_{\text{conv}}) - z(\tau = 0)] \text{ for } A > 1}, \tag{4}$$

where z refers to the linear depth of the atmosphere. The resulting values of F are given in Table VIII. The values of F given in Table VIII are nearly the same for all the T_e's and g's considered here.

TABLE VIII

Depth of the radiative layer of a star with metal deficiency

A	F
1	1.0
10^2	0.72
10^3	0.62
10^4	0.52

(h) Blanketing effect: It is of significance to calculate the effect of metal deficiency on the colors of the stars (see also CAYREL, 1968). Theoretical colors in the UBV System were calculated following the method of MATTHEWS and SANDAGE (1963). For a fixed T_e, we calculate the following quantities:

$$\Delta(B-V) = (B-V)\,[\text{for } A = 1] - (B-V)\,[\text{for } A > 1], \tag{5}$$

$$\Delta(U-B) = (U-B)\,[\text{for } A = 1] - (U-B)\,[\text{for } A > 1]. \tag{6}$$

Equations (5) and (6) essentially give the differential blanketing in the colors between a normal star and a metal-deficient star. The values of $\Delta(B-V)$ and $\Delta(U-B)$ obtained from Equations (5) and (6) are given in Table IX. It may be mentioned that in these calculations convective effects are taken into account but blocking and back warning effects are not. In Figures 13 and 14, we show the ratio

$$f = \frac{\text{FLUX}(T_e, A > 1)}{\text{FLUX}(T_e, A = 1)} \tag{7}$$

as a function of wavelength.

TABLE IX
Color changes due to metal deficiency

T_e(K)	A	$g = 2 \times 10^4$		$g = 2 \times 10^2$	
		$\Delta(B-V)$	$\Delta(U-B)$	$\Delta(B-V)$	$\Delta(U-B)$
	10^2	−0.086	−0.131	−0.074	−0.120
5000	10^3	−0.122	−0.180	−0.081	−0.130
	10^4	−0.141	−0.201	−0.081	−0.133
	10^2	−0.188	−0.280	−0.167	−0.277
4500	10^3	−0.313	−0.448	−0.274	−0.431
	10^4	−0.412	−0.578	−0.319	−0.494
	10^2	−0.232	−0.373	−0.287	−0.472
4000	10^3	−0.290	−0.483	−0.552	−0.856
	10^4	−0.402	−0.701	−0.759	−1.120

4. Summary

In the present paper to a first approximation, we have constructed atmospheric models of Extreme metal-deficient stars corresponding to surface gravities of main-sequence stars and giant stars for effective temperatures in the range 4000 to 5000 K. We have studied the effect of extreme metal-deficiency on the atmospheric structure of these stars. We have given in the form of figures and tables, the various physical quantities of the model atmospheres. One finds that convection plays a very important role in the atmospheres of extreme metal-deficient stars. One can also see from Table IX that the effect of extreme metal-deficiency on the colors of the stars is quite appreciable.

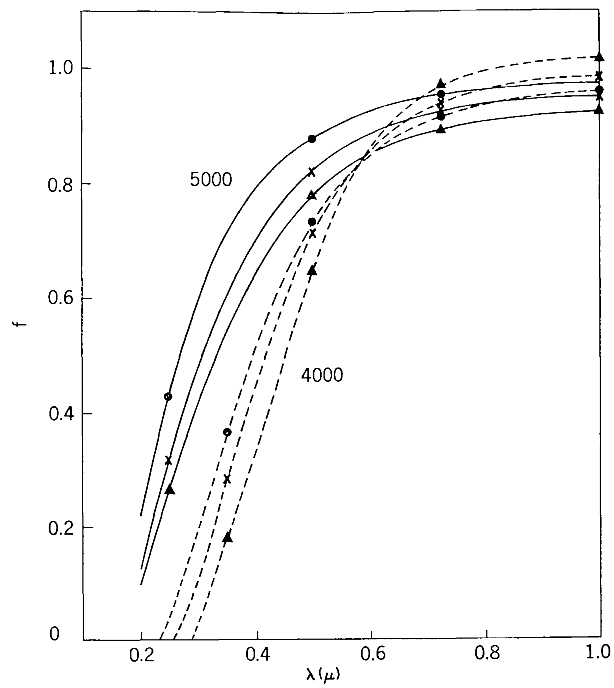

Fig. 13. Plot of differential flux of metal-deficient star with respect to normal star, as a function of λ for $g = 2 \times 10^4$. Dots, crosses and triangles refer to $A = 10^2$, 10^3 and 10^4 respectively.

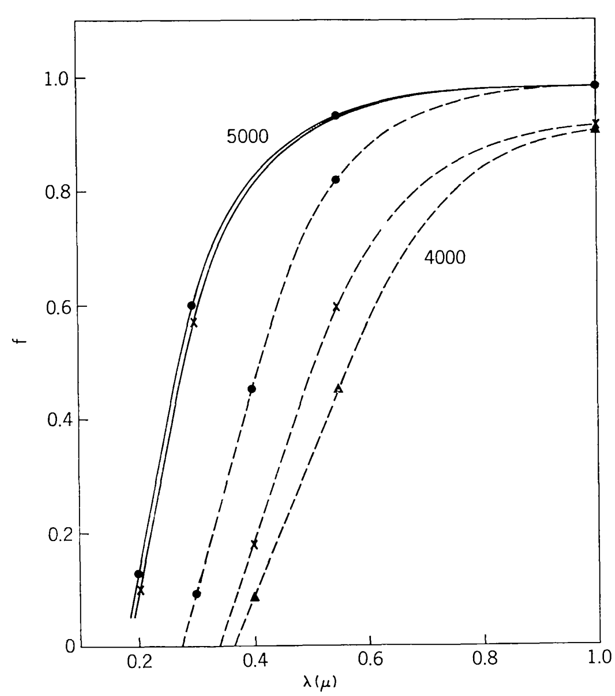

Fig. 14. Same as Figure 13, but for $g = 2 \times 10^2$.

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