A consistent description of Neutron Stars with Quark Cores

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We consider the equation of state of quark matter at finite density in mean field theory, through an effective chiral lagrangian whose parameters (coupling constants) are all fixed by hadronic data. Between three to seven times nuclear density, for charge neutral quark matter in β equilibrium, we find the ground state to be a neutral pion condensate. With increasing baryon density we then expect nuclear matter, followed by pion condensed quark matter at intermediate density, and finally the diquark colour-flavour CFL condensate. These are all states with chiral spontaneous symmetry breaking (SSB).

We find another remarkable feature and this is that the scalar (pseudoscalar) coupling, λ, has a crucial and unexpected influence on the physics of neutron stars. Neutron stars with pion condensed quark matter cores exist only in a small window, between, $5.7 < \lambda < 6.45$. Interestingly, this range is consistent with the value of λ derived from π, π scattering data and such stellar cores may carry magnetar strength magnetic fields.

I. INTRODUCTION

Neutron stars have been a subject of abiding interest for several decades. There are a variety of astrophysical phenomena that arise from the physics of neutron stars. These include supernovae, pulsars, accreting binary X-ray sources and Magnetars, which have superstrong magnetic fields. Most of these phenomena require us to understand the physics of matter at very high density, which govern the mass and the size of neutron stars. In other words, one needs to have a clear understanding of the equation of state (EOS) of superdense matter. Although much effort has gone into this enterprise over the last four decades it still remains poorly understood. Why?

Central densities of neutron stars are high, more than $\sim$5 times nuclear density $\rho_{\rm nuc} = 0.17$ fm^{-3}. For a single species, neutrons, this naively translates into a fermi gas with typical fermi momentum, $k_f^{\rm N} \sim 600$ MeV. On the other hand nucleons have structure and a typical size of the order of a fermi (200 MeV)$^{-1}$. It is clear that at such high densities nucleons (neutrons) cannot be treated as elementary. They are composite and resolved. They are colour singlet bound states of three valence quarks. At such high densities, therefore, treating nucleons as point particles interacting via two body (or more) forces will be inadequate. Yet most available equations of state adopt this approach and therefore fail to capture the correct physics at high density.

On the other hand, if we use quarks as the elementary degrees of freedom, we are presently bound by the fact that only perturbative calculations can be done for QCD. This implies that calculations can be done in QCD only at very high density when the theory is approximately in an asymptotically free (AF) phase. However, at intermediate and low density (close to nuclear density), where a nucleonic description is valid, we cannot use perturbative QCD as the coupling becomes strong and the physics non perturbative and intractable. This is the dilemma.

There are attempts to model the physics by a two phase structure – a quark matter core with a hadronic/nuclear exterior shell and crust. Since there is no simple way to link the two phases without using separate parameters for both, this description is somewhat arbitrary. Further, the nature of the quark matter state is not clear – for example, if it is in a spontaneous chiral symmetry broken state.

Can we find a single theory that connects both these domains? For this we use an Effective Chiral Lagrangian, L, that receives broad support from many contexts. This lagrangian has quarks, gluons and a chiral multiplet of $[\vec{\pi}, \sigma]$ that flavour-couples only to the quarks [1–6].

$$\begin{aligned} \mathrm{L} \;=\; & -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} - \sum \overline{\psi} \left(\not{D} + g_y(\sigma + i\gamma_5 \vec{\tau}.\vec{\pi}) \right) \psi \\ & -\frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}(\partial_\mu \vec{\pi})^2 - \frac{1}{2}\mu^2(\sigma^2 + \vec{\pi}^2) \\ & -\frac{\lambda^2}{4}(\sigma^2 + \vec{\pi}^2)^2 + \mathrm{const} \end{aligned} \quad (1)$$

The masses of the scalar (psuedoscalar) and fermions follow from the minimization of the potentials above. This minimization yields

$$\mu^2 = -\lambda^2 < \sigma >^2 \quad (2)$$

It follows that

$$m_\sigma^2 = 2\lambda^2 < \sigma >^2 \quad (3)$$

Experimentally, in vacuum, $< \sigma >= f_\pi$, the pion decay constant. This theory is an extension of QCD in that it

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additionally couples the quarks to a chiral multiplet, ($\vec{\pi}$ and σ) [1–4].

We now summarize some interesting physics that supports this Lagrangian at the mean field level (MFT)[4, 7].

In this chiral, effective L, we do not consider vector gluon mean fields which would spontaneously break colour symmetry and lorentz invariance. Thus, at the level of Mean Field Theory (MFT) our model reduces to a linear sigma model with quarks.

(i) It provides a model in which the nucleon is realized as a soliton with quarks being bound in a skyrmion configuration for the chiral field expectation values (EV) [1, 4, 7]. This model with composite nucleons gives a good account of the static properties of nucleons and nucleon-nucleon interaction potentials [8].

The model provides a natural explanation for the 'Proton spin puzzle' [9]. Such a Lagrangian also seems to naturally produce the Gottfried sum rule [10].

The Nambu Jona Lasinio model, that can be recast as a chiral sigma model [11], at the level of MFT, also yields a quark soliton nucleon. This gives structure functions for the nucleon which are close to the experimental ones.

(ii) In a finite temperature mean field theory such an effective Lagrangian also yields screening masses that match with those of a finite temperature QCD simulation with dynamical quarks [12]. This work does not show any parity doubling for the hadronic states.

(iii) The theory gives a qualitatively consistent description for the transition from hadronic matter to quark matter at high density and temperature [4, 5, 13, 14].

This L has a single dimensional parameter, f_π, that is the pion decay constant, and three couplings, g_3, the QCD coupling, g_y, the Yukawa coupling between quarks and mesons, that will be determined from the nucleon mass and the meson-meson coupling, λ, which, for this model, can be determined from meson meson scattering [15]. No further phenomenological input will be used.

Once the couplings of L are determined from the hadronic sector, the same effective lagrangian describes the physics of the quark matter sector.

We now consider the question of the scales in QCD and the scale of validity of this effective L.

i) Constituent quarks vs Current quarks: To begin with let us consider the two main features of the strong interactions (QCD) at low energy. These are a) that quarks are confined as hadrons and b) chiral symmetry is spontaneously broken (SSB) with the pion as an approximate Goldstone boson. There is no specific reason that these two phenomena should occur at an identical temperature scale, though QCD lattice simulations show that for flavour $SU_2(L) \times SU_2(R)$ they are close. The problem in giving an unequivocal answer to this question is that we are yet to find a solution to many of the non-perturbative aspects of QCD.

An interesting question arises: Is the quark matter in a chiral SSB state with constituent quarks or is it, as is usually assumed, in a chirally restored state with current quarks?

At finite density [6, 13], we shall see in what follows, that quark matter is in a chiral SSB state. Also, if the chiral symmetry restoration (energy/temperature) scale was lower than the confinement scale we would expect hadrons to show parity doubling below the confinement scale but above the chiral SSB scale. This is not seen in finite temperature lattice simulations.

ii) Compositeness scale: Actually, QCD can have multiple scales [5]. Apart from a confinement scale and a chiral symmetry restoration scale we also have a compositeness scale for the pion. We find, somewhat in analogy with the top quark (large Yukawa coupling) composite higgs picture, that we can get a compositeness scale for the scalars (pseudoscalars) in this model by using Renormalisation Group (RNG) evolution. This is given by the scale at which the wavefunction renormalisation for the mesons - the coefficient of the kinetic term - vanishes. Once this term vanishes the meson fields are no longer bonafide degrees of freedom and can be eliminated using their field equation. We find that this scale, for the mesons, is inversely proportional to the running Yukawa coupling and thus naively vanishes when the Yukawa coupling blows up. For our theory such a ballpark scale falls between 700–800 MeV This also gives us an approximate idea of the range of validity of our effective Lagrangian, as, above the compositeness scale we lose the meson degrees of freedom.

An independent approach in setting a limit to the range of validity of non asymptotically free (e.g. Yukawa) theories, like ours, is the vacuum instability to small length scale fluctuations (or large momenta in quantum loop corrections), discovered by one of us [16]. The scale at which this occurs is of the same order as above. This is not very surprising since it is connected to non-AF character of the Yukawa coupling [16, 17].

This discussion is to support the use of our effective lagrangian up to a threshold scale in energy - the compositeness scale.

Given these facts we use the Mean Field Theory to describe quark matter in the density regime bounded from above by the compositeness scale.

The plan of the paper is as follows. In Sec. 2 we discuss how we fix the couplings for our lagrangian from the hadronic sector. In Sec. 3 we present the equation of state (EOS) for 3 flavour quark matter using the lowest energy ground state we have found - the neutral pion condensed phase with chiral SSB. In Sec. 4 we use our quark matter EOS and the EOS for nuclear matter given by Akmal et al (APR) [18] to make neutron stars. In Sec. 5 we discuss the phase diagram of QCD at finite density. We comment on other ground states and on the comparison between the pion condensed phase with the colour superconducting phase. Sec. 6 summarizes our results.

II. PRELIMINARIES

A. The couplings of L

The nucleon, in our linear sigma model with quarks, is a colour singlet bound state of three valence quarks in a skyrme background. The general solution follows on varying the skyrme configuration π and σ and quark fields, that occur in the soliton field equation, independently. The quark soliton so obtained is projected to give a nucleon with good spin and isospin [1](b).

The mass, M, of the nucleon depends just on f_π (which is 93 MeV), g_y and λ. The dependence on λ is marginal, so the only parameter that the mass depends on is the Yukawa coupling, g_y. Fixing $M = M_{\rm nucleon}$, yields a generally accepted value for $g_y = 5.4$ [1](b).

Recently, Schechter et al [15] made a fit to scalar channel scattering data to see how it may be fitted with increasing $\sqrt{s}$ (centre of mass energy), using chiral perturbation theory and several resonances. They, further, looked at this channel using just a linear sigma model. Their results indicate that for centre of mass energy $\sqrt{s} < 800$ MeV, a reasonable fit to the data can be made using the linear sigma model with a 'tree' level sigma mass close to 800 MeV, or equivalently, $\lambda \sim 6$. We may add that the 'tree' level mass is a parameter for the 'tree' level L, and is equivalent to setting a value for λ - it is not the physical mass of the sigma (see [15] for details).

The range of validity of the linear sigma model is also consistent with the range of validity of our effective L. This completes the determination of all the parameters of our L from the hadronic sector.

III. THE GROUND STATE FOR 3 FLAVOUR QUARK MATTER

For neutron stars it is the ground state of charge neutral quark matter in β equilibrium at given density that is needed for the equation of state. We find, in what follows, the ground state to be the neutral pion condensate. What is also new is that the couplings of L are fixed from the hadronic sector.

For the equation of state we need to calculate the free energy density at a given density. We first review the calculation for the energy density from a previous paper [6], which will be used to calculate the free energy.

In [6] we considered different patterns of symmetry breaking for the, ($\vec{\pi}$ and σ), fields and calculated the respective ground state energies. In particular, we considered two cases

i) the two and three-flavour (see below), π_0, pion condensed phase, where the ($\vec{\pi}$ and σ) expectation values are in a stationary wave configuration, with a wavevector, $\vec{q}$ (see below).

and ii) the space uniform symmetry broken state, which follows on putting $\vec{q} = 0$. At high density this state goes through a chiral restoration and essentially resembles conventional strange quark matter (SQM), that is chirally restored quark matter (CRQM).

The π_0 condensed ground state (PC) is found to have the lowest energy in the chiral limit [6, 13] (see table). We refer the reader to [6], where we have set up the machinery to describe this ground state which is built on non trivial symmetry breaking in the presence of the π_0 condensate. This is given as follows,

For the $SU(3)$ flavour case we have a singlet ξ_0 and an $SU(3)$ octet ξ_a of scalar fields and a singlet ϕ_0 and an $SU(3)$ octet ϕ_a of pseudoscalar fields, with the expectation values [6],

$$< \xi_0 > \; = \; \sqrt{3/2}F(1 + 2\cos(\vec{q}.\vec{r}))/3 \tag{4}$$

$$< \xi_8 > \; = \; -\sqrt{3}F(1 - \cos(\vec{q}.\vec{r}))/3 \tag{5}$$

$$< \phi_0 > \; = \; 0 \tag{6}$$

$$< \phi_3 > \; = \; F(\sin(\vec{q}.\vec{r})) \tag{7}$$

while all other fields have zero expectation value. On putting $\vec{q} = 0$, we get the vacuum (space uniform) symmetry broken state. This yields the simple mass relation for the strange quark, $M_s = g_y F + m_s$, where m_s is the current mass and, $F = \sqrt{<\vec{\pi}>^2 + <\sigma>^2}$, is the chiral order parameter (F = f_π, at zero density).

The ground state energy is obtained by summing all occupied single quasiparticle states, in the presence of the pion condensate, for the u and d quarks up to their fermi energy. The quasiparticle states in the presence of this condensate have a spin-isospin alignment which gives the ground state a magnetic dipole moment. To this we add the sum over the plane wave states for the strange quarks of mass, M_s, up to the fermi energy. Besides, we have, the gradient energy and the potential functional contributions from the meson sector. Charge neutrality requires us to include electrons as well. β-equilibrium is imposed and this implies several chemical potential relations between the different species (see [6]).

The ground state energy and the baryon density depend on the two variational parameters, the order parameter or the expectation value, F, and the condensate momentum, $|\vec{q}|$.

Ref.[6] provides the expressions for the baryon density, n_b, and the total energy density, ϵ, of the PC in terms of the u,d,s quark fermi energies/chemical potentials.

For the EOS we need to construct the Gibbs free energy at a fixed baryon density.

$$\Omega = \epsilon - n_b \mu_b \tag{8}$$

The baryon chemical potential is defined as

$$\mu_b = \partial\epsilon/\partial n_b \tag{9}$$

After meeting all the neutrality and equilibrium conditions above for fixed F and q, we can write all the above variables as a function of a single variable, μ_u.

We then minimize Ω independently with respect to F and q. The energy per baryon, E_b, etc then follow.

TABLE I: Charge neutral, 3-flavour, beta-equilibrium pion condensed phase with $m_\sigma = 800$ MeV. The columns are: u-quark chemical potential (μ_u in MeV), baryon density (n_b in fm^{-3}), energy per baryon (E_b in MeV), electron density (n_e in fm^{-3}), ratio of densities of d-quark and u-quark (n_d/n_u), that of s-quark and u-quark (n_s/n_u), the order parameter (F in MeV) and magnitude of the vector q.

μ_u	n_b	E_b	n_e	n_d/n_u	n_s/n_u	F	q
280.0	0.2972	984.94	.2303E-02	1.916	.0318	37.0406	2.5945
300.0	0.3645	981.48	.1602E-02	1.731	.2269	31.9655	2.6149
320.0	0.4591	994.07	.1141E-02	1.599	.3640	28.6471	2.9703
340.0	0.5409	1008.88	.1173E-02	1.564	.4004	30.4335	3.0216
360.0	0.6700	1043.21	.9455E-03	1.499	.4672	28.8195	3.6217
380.0	0.7628	1062.14	.1063E-02	1.482	.4847	31.6743	3.4574
400.0	0.8967	1104.94	.4413E-03	1.345	.6245	23.6066	3.9497
420.0	1.0472	1134.64	.1283E-02	1.463	.5040	36.1496	3.9839
440.0	1.1774	1168.76	.4890E-03	1.317	.6529	26.8871	4.0720
460.0	1.3225	1216.52	.1794E-03	1.218	.7529	19.3370	4.3125
480.0	1.5149	1246.21	.3896E-03	1.267	.7033	26.9453	4.3670
500.0	1.6900	1290.81	.2212E-03	1.211	.7597	23.0806	4.4865

The results are presented in the tables below and in Figure 1a. Fig 1b gives the the EOS for all the phases considered so far.

As promised in the introduction

i) We note that the SSB neutral pion condensed ground state is always lower energy than the CRQM state, indicating that quark matter is in the chiral symmetry broken state.

ii) As is clear from the table we have stayed in range of validity, $\mu < 700$ MeV, of the our effective L.

iii) Another feature of this π_0 condensate is that we have a spin isospin polarization, 'u' and 'd' quark quasi-particles have opposite spin and opposite charge. We can then get a net magnetic moment in the ground state, as the magnetic moments of the u and d quarks add [6] .

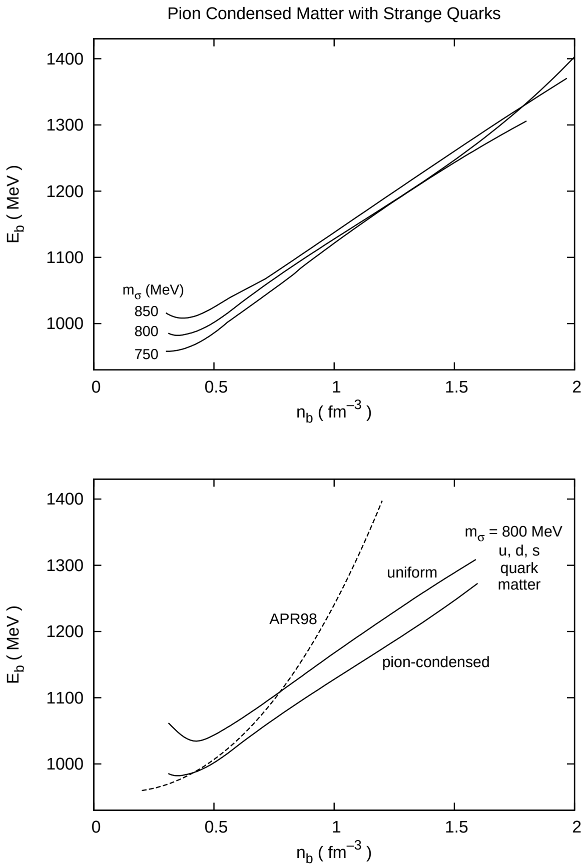

FIG. 1: (a) Upper panel: energy per baryon vs baryon number density for 3-flavour pion-condensed phase for three values of assumed tree-level mass of the scalar meson σ. Charge neutrality and beta equilibrium are imposed. One-gluon exchange interaction is included using the prescription of Baym [19]. (b) Lower panel: comparison of the equations of state of the 3-flavour space uniform phase and pion-condensed phase for $m_\sigma = 800$ MeV, with APR [18].

IV. NEUTRON STARS FROM OUR EOS

In order to construct stars with this pion-condensed (PC) matter, we note that this state is thermodynamically stable only at densities above twice the nuclear density, so it is necessary to extend the equation of state to lower densities by interfacing with a nuclear equation of state. To describe the nuclear regime we use the APR [18] equation of state. We treat the phase transition between these two states as a first order one with a density discontinuity, where the pressures and the chemical potentials of both phases match. We determine this phase transition via the regular Maxwell construction (fig. 2). We find that the density discontinuity is small, giving us the confidence that even if the transition occurs via a mixed state [20] the results will be similar to those obtained with a discontinuous transition. At subnuclear densities, the equation of state is smoothly matched to those of Negele and Vautherin [21], Baym, Pethick and Sutherland [22], and Feynman, Metropolis and Taylor [23].

To construct the neutron star, we solve the Tolman-Oppenheimer-Volkoff (TOV) hydrostatic equilibrium equation [24] with the above equation of state. For a given m_σ, the PC core exists only if the central density of the star exceeds the APR-PC transition, which would happen above a threshold stellar mass M_T. With increasing m_σ, the APR to PC phase transition moves up to higher densities, and M_T increases correspondingly. At $m_\sigma > 850$ MeV ($\lambda = 6.45$), the PC core cannot form since M_T exceeds the maximum mass of the neutron star, which in this model works out to be about 1.6 $M_\odot$ (see fig. 3). On the other hand, at $m_\sigma < 750$ MeV ($\lambda = 5.7$) the Maxwell construction between the PC and the

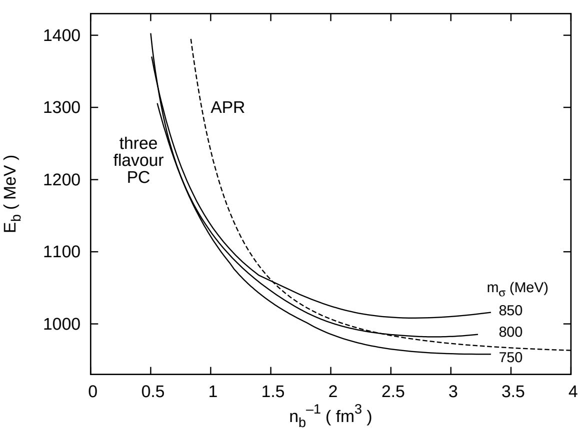

FIG. 2: The Maxwell construction: Energy per baryon plotted against the reciprocal of the baryon number density for APR equation of state (dashed line) and the 3-flavour pion-condensed (PC) phase, for three different values of m_σ (solid lines). A common tangent between the PC phase and the APR phase in this diagram gives the phase transition between them. The slope of a tangent gives the negative of the pressure at that point, and its intercept gives the chemical potential. As this figure indicates, the transition pressure moves up with increasing m_σ, and at m_σ below $\sim$ 750 MeV a common tangent between these two phases cannot be obtained.

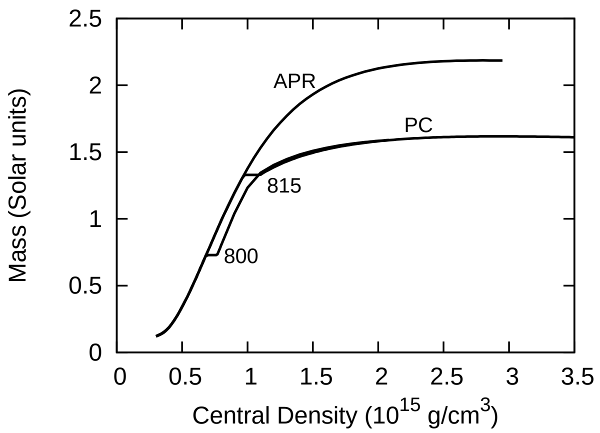

FIG. 3: Mass vs central density of neutron stars with magnetic pion condensed cores. The upper line corresponds to pure nuclear matter with APR [18] equation of state. The lower lines (labelled PC) indicate stars with pion-condensed quark core, displayed for two values of m_σ, 800 and 815 MeV. The density discontinuity at the phase transition is clearly visible.

APR state is no longer possible. Neutron stars with PC cores can therefore exist only if m_σ is in the range 750–850 MeV. The magnetic moment of such PC cores could lead to magnetic fields as strong as $\sim 10^{15}$ G at the surface of the neutron star and may be responsible for the strong fields found in magnetars.

V. DISCUSSION

A. quark matter ground states

(i) At issue is the question if the neutral pion condensate we have considered is the lowest energy ground state. It is to be noted from the lecture notes of Baym [19], that the neutral pion condensate is the preferred ground state over the charged pion condensate for charge neutral nuclear matter in β equilibrium, in the non relativistic limit.

Let us first consider the question as to how the charged pion condensate ground state compares with our ground state. Reference [13] finds these two condensates are related by a chiral rotation and are degenerate in energy, but this is only for isospin symmetric matter - that is in the absence of both charge neutrality and β equilibrium.

We have considered this question for charge neutral quark matter in β equilibrium analytically and find that the neutral pion condensate is the preferred ground state over the charged pion condensate, for non-relativistic and point-like ($g_A = 1$) quarks. This is important as the charged pion condensate has no dipole magnetism [13].

(ii) The K condensate in nucleon matter is a strong candidate for the ground state. In nuclear matter the term that gives rise to this is the chiral symmetry breaking, sigma term, which is proportional to the nucleon mass and first order in the symmetry breaking expansion parameter (or m_s). In the case of quark matter, with point like quarks, such a term is not proportional to the nucleon mass and is second order in m_s and is thus unlikely to play a defining role. We think that this may rule against K condensates in quark matter as opposed to nuclear matter, though we have not carried out this calculation.

(iii) It is worth pointing out that all these condensate states have lower energy than the chirally restored CRQM state, are chiral symmetry broken states.

B. Quark matter at even higher density

At very high density, QCD gluon interactions become weak and enter the asymptotically free regime. It is well known that the quark fermi seas are unstable to the formation of the diquark condensate state, no matter how weak the gluon interaction is.

There is an important issue which then arises; at what density does the pion condensed quark matter state transit into the diquark condensate state? In this section we review some work [25, 26] that addresses this question using the NJL model and which has implications for our work.

We have argued that the linear sigma model is a valid model till centre of mass energies/scales of less than, 700-800 MeV, the right procedure would be to take this model to describe physics up to this scale. But our model has only chiral condensates.

It is well known that there is an identity between the NJL model and the linear sigma model [27], and thus the NJL can be mapped to our linear sigma model [26] and the ground state thereof. The NJL model, which has a chiral symmetric four fermion interaction can, however, accommodate both chiral (quark-antiquark colour singlet) condensates and diquark condensates.

A comparison of these two states has been done by Sadzikowski [25, 26, 28] in the context of a NJL chiral symmetric model, for the case of 2 flavours – $SU(2)_L \times SU(2)_R$. What is done is at the level of mean field theory. The NJL model has four fermion interactions in terms of the quark bilinears corresponding to the σ and π field quantum numbers, with a common dimensional coupling, G. If we are interested in a ground state carrying sigma and/or pion condensates we can replace these quark bilinears by the corresponding σ and π EV's in the MFT. This yields the ground state energy of the space uniform SSB and the PC states. Further, these works calculate the ground state energy of the the diquark condensate which is got from the NJL four fermi interaction by Fierz transformation.

Although these results [25, 26, 28] are not for exactly the same parameters they provide a good sense of the physics. In this case the PC is the preferred ground state till μ well above 400 MeV. As the tables suggest such values of μ, correspond to baryon density 5–6 times nuclear density - the central density in our stars. This makes the PC a likely state in our neutron star cores.

Of course, these works deal with the two-flavour case – where the colour diquark condensate is a chiral singlet. Realistically, we must consider 3 flavours, since the quark chemical potential is much greater than the strange quark mass. In this case we are very likely to have the CFL state as the lowest energy state. The criterion for this is given in [29] and is $\Delta > m_s^2/4\mu$, which is easily satisfied. Furthermore, in this case the diquark condensate is a colour-flavour condensate which has both chiral SSB and colour SSB, albeit in a manner different to the PC.

With increasing baryon density we then expect the following hierarchy. At nuclear density and above we have nuclear matter, followed by neutral pion condensed quark matter and finally a transition to the diquark CFL state which all have chiral SSB. These considerations indicate that, at any finite density ($T = 0$) chiral symmetry remains spontaneously broken. A similar result has also been obtained in 1+1 dimension [30].

VI. RESULTS

i) For neutron stars it is charge neutral quark matter in β equilibrium at given density that is needed for the equation of state. We have found that the neutral pion condensate is the ground state for such quark matter. This is the first such calculation for this EOS and what is also new is that all the parameters of L are fixed from the hadronic sector.

ii) We have made a strong case for the spontaneous breaking of chiral symmetry at all baryon density (at T = 0).

iii) We have constructed neutron stars with pion condensed quark matter cores and found that such cores occur only under very particular constraints on the value of the scalar (pseudoscalar) coupling, $5.7 < \lambda < 6.45$ (or equivalently, when the 'tree' level sigma mass in this model is in a small window, 750 - 850 MeV). This window is consistent with, π, π scattering data.

iv) We also find that at the central density of such stars the pion condensed state is the most likely and that the quark cores can carry high magnetic fields.

Is this a coincidence that a single parameter in our effective L, the 'tree' level mass of the sigma or the value of λ, plays a crucial role? Is it fortuitous that the tree level sigma mass set by scattering experiments sits in a small window that simultaneously rules out SQM as the absolute ground state of matter [6] and also can provide us with neutron stars that can have magnetic PC cores?

The problem in sustaining a PC core with a nuclear exterior is that we have a stiff exterior with a soft interior – a rather unstable situation. It is thus not so surprising that very particular conditions must obtain for this to occur.

Clearly to get quantitative results we have to work with a specific model. In this case we have worked with a specific effective L that is built on the two symmetries of the strong interaction, chiral symmetry and colour symmetry. Further, all its couplings are determined from experimental hadronic data. In this case we are working from low density (energy) to higher density (energy), till the compositeness scale. This is different from the vast literature on diquark condensation (CFL), which examines the problem from the high density end where is no experimental data. Though, we cannot claim any sanctity for our L, we have provided physical justification for it, its range of validity and determined its parameters from hadronic physics. Finally, proof can be provided only from testing our specific results.

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