IP/BBSR/2001-6

Born-Infeld Chern-Simons Theory: Hamiltonian Embedding, Duality and Bosonization

E. Harikumar[1], Avinash Khare[2*], M. Sivakumar[1†] Prasanta K. Tripathy[2],

[1]*School of Physics, University of Hyderabad,Hyderabad-500 046, INDIA*

[2]*Institute of Physics, Sachivalaya Marg, Bhubaneswar-751 005, INDIA.*

Abstract

In this paper we study in detail the equivalence of the recently introduced Born-Infeld self dual model to the Abelian Born-Infeld-Chern-Simons model in 2+1 dimensions. We first apply the improved Batalin, Fradkin and Tyutin scheme, to embed the Born-Infeld Self dual model to a gauge system and show that the embedded model is equivalent to Abelian Born-Infeld-Chern-Simons theory. Next, using Buscher's duality procedure, we demonstrate this equivalence in a covariant Lagrangian formulation and also derive the mapping between the n-point correlators of the (dual) field strength in Born-Infeld Chern-Simons theory and of basic field in Born-Infeld Self dual model. Using this equivalence, the bosonization of a massive Dirac theory with a non-polynomial Thirring type current-current coupling, to leading order in (inverse) fermion mass is also discussed. We also re-derive it using a master Lagrangian. Finally, the operator equivalence between the fermionic current and (dual) field strength of Born-Infeld Chern-Simons theory is deduced at

*khare@iopb.res.in

†mssp@uohyd.ernet.in

the level of correlators and using this the current-current commutators are obtained.

PACS Numbers: 11.10.Kk, 11.15.-q, 11.10.Ef, 11.40.-q

Keywords: Born-Infeld, Chern-Simons, Hamiltonian Embedding, Duality, Bosonization.

I. INTRODUCTION

The Born-Infeld theory has been an area of intense activity in the recent times because of the important role it plays in the context of string theory [1]. Some time back, after the discovery of D-branes, it was realized that the dynamics of the gauge field excitations on world volume of D-branes is described by the Born-Infeld theory. Various aspects of the Born-Infeld theory have been studied thoroughly both from the string as well as field theoretic point of view. One of the most remarkable properties of the D-branes is that they carry Ramond-Ramond (RR) charges and hence, must couple to the RR states of the closed string. These couplings are incorporated via the Chern-Simons action, which is constructed from the antisymmetric combination of the field strength with the pull back of the RR gauge potential on to the world volume of the brane. The coefficient of the Chern-Simons term gives rise to the RR charge of the brane. Thus dynamics of Dp-brane is described by sum of Born-Infeld and Chern-Simons actions in $p+1$ dimensions. In particular, the $D2$-brane dynamics is described by 3 dimensional Born-Infeld Chern-Simons (BICS) action. It may thus be of some interest to examine the various aspects of BICS theory in $2+1$ dimensions which is what we propose to do in this paper .

Several years ago Deser and Jackiw [2] showed that the self-dual model given by [3]

$$\mathcal{L} = \frac{1}{2} f_\mu f^\mu - \frac{1}{2m} \epsilon_{\mu\nu\lambda} f^\mu \partial^\nu f^\lambda \ , \tag{1.1}$$

is equivalent to the Maxwell-Chern-Simons (MCS) theory given by

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m}{2} \epsilon_{\mu\nu\lambda} A^\mu \partial^\nu A^\lambda \ . \tag{1.2}$$

The equivalence between the two models has been extensively studied in the literature [4,5] and has found application in studying bosonization in $2+1$ dimensions [6–8].

Recently two of us extended this equivalence to the Born-Infeld case [9]. In particular, a self dual model for the Born-Infeld case (SDBI) described by the Lagrangian

$$\mathcal{L} = \beta^2 \sqrt{1 + \frac{1}{\beta^2} f_\mu f^\mu} - \frac{1}{2m} \epsilon_{\mu\nu\lambda} f^\mu \partial^\nu f^\lambda \ , \tag{1.3}$$

was introduced and was shown to be equivalent to the BICS theory [9] as given by

$$\mathcal{L} = \beta^2\sqrt{1 - \frac{1}{2\beta^2}F_{\mu\nu}F^{\mu\nu}} + \frac{m}{2}\epsilon_{\mu\nu\lambda}A^\mu\partial^\nu A^\lambda \ . \tag{1.4}$$

It may be noted that the action corresponding to the BICS Lagrangian is invariant under the $U(1)$ transformation of the vector field

$$A_\mu \rightarrow A_\mu + \partial_\mu\lambda \ , \tag{1.5}$$

while the SDBI Lagrangian (1.3) as well as the corresponding action is not gauge invariant. As expected, the BICS Lagrangian (1.4) reduces to that of the Maxwell-Chern-Simons theory(MCS) (1.2) while the SDBI Lagrangian (1.3) goes over to the SD Lagrangian (1.1) in the limit $\beta^2 \rightarrow \infty$.

This raises the question of extending the previous analysis regarding the equivalence of the MCS and SD models to the Born-Infeld case and in particular study the bosonization issue. Further, it is interesting to enquire the role of non-linearity in the Born-Infeld part of the theory in extending the earlier studies of MCS theory.

At this point we recall that the previous studies of the equivalence relied on converting the second-class constraints in the self-dual model to the first class constraints by applying Batalin, Fradkin and Tyutin (BFT) procedure [10,11]. The inherent non-linearity in Born-Infeld theories makes the application of this method interesting and somewhat nontrivial. There also exists a method due to Buscher [12,13] for obtaining duality equivalence between different theories, which has been applied to the MCS theory and the corresponding self-dual model [14] has been obtained. It should again be interesting to apply this scheme to the BICS theory.

The role of Chern-Simons gauge field in 2+1 dimensional bosonization is well studied for the case of matter field coupled with Chern-Simons gauge field [15,16] and also for a generic current-current interacting theory [17]. Specifically using the connection of the MCS gauge theory to the self-dual model, the MCS gauge theory has been shown to be equivalent to the massive Thirring model up to leading order in (inverse) fermionic mass [7]. It is then natural to enquire if a similar study is also possible in the case of the BICS gauge theory.

In this paper we study three aspects of the BICS-BISD correspondence, which are:

1. Hamiltonian embedding of BISD theory and its equivalence to the BICS theory
2. the equivalence as a duality relation using Buscher's procedure
3. using it to study the bosonization of a massive interacting Dirac theory with a non-polynomial current-current interaction.

We start with the BISD model which has only second-class constraints and convert them to first class constraints and also the Hamiltonian to a gauge invariant one following the generalized BFT scheme. Then we show that the embedded model is equivalent to the BICS theory.

We also show the equivalence between the BICS theory and the BISD model in a manifestly covariant manner, by applying Buscher's procedure, which relies on the presence of a global symmetry in the theory. Here we show that these two theories are related to each other by a duality mapping. We also provide a mapping between the fields at the level of correlators between the two theories.

We next use the equivalence between the BISD and the BICS to show that to the leading order in the inverse fermion mass μ, the latter is a bosonized version of a massive non-polynomial Thirring (BIT) type current-current interacting theory described by the Lagrangian

$$\mathcal{L} = \bar{\Psi}(i\gamma_\mu\partial^\mu - \mu)\Psi + \beta^2\sqrt{1 - \frac{\lambda}{\beta^2} j_\mu j^\mu} \ . \tag{1.6}$$

Here j_μ is the fermionic current $\bar{\Psi}\gamma_\mu\Psi$ and λ and β are dimensionful constants. This is also re-derived starting from a master Lagrangian and operator correspondence between the fermionic current of the BIT theory and (dual) field strength of the BICS theory at the level of correlators is also obtained. We also evaluate the current algebra of the fermionic theory using the operator correspondence. It is found that even though the current algebra is β dependent (of course in the limit $\beta^2 \to \infty$ it agrees with massive Thirring model result) but the bosonization rules are β-independent, hence they are the same for the MCS-Thirring [8] and the BICS-BIT cases.

This paper is organized in the following way. In the next section, we apply the Hamiltonian embedding to the BISD model and show that the embedded model is equivalent to the BICS theory. In section III, we apply Buscher's procedure to show the equivalence of the BICS theory and the BISD model in the Lagrangian formulation. Here we also derive the mapping between the n-point correlators of these two models. In section IV, we derive the mapping between the fermionic BIT theory and the bosonic BICS theory and obtain the bosonization rules as well as the current algebra of the fermionic current. We conclude with discussions in section V. In the Appendix A, using the symplectic quantization scheme, the Dirac brackets of the BICS gauge theory are obtained which are needed for the current algebra evaluation.

We work with $g_{\mu\nu} = diag(1, -1, -1)$ and $\epsilon_{012} = \epsilon^{012} = 1$.

II. HAMILTONIAN EMBEDDING

In this section, we apply the improved Hamiltonian embedding procedure of Batalin, Fradkin and Tyutin (BFT) [18] to the BISD model and convert it into a gauge theory. Then, using the solution of the embedded Hamilton's equation we show the equivalence of the embedded model to the BICS theory. First, we briefly sketch the Improved BFT procedure and then we apply it to the BISD model.

A. Improved BFT Embedding

In the Batalin, Fradkin and Tyutin (BFT) method, first one enlarges the phase space by introducing auxiliary variables Φ_α corresponding to each of the second class constraints $(T_\alpha(P, Q))$ (where P and Q stand for the original canonically conjugate phase space variables) satisfying,

$$\{\Phi_\alpha, \Phi_\beta\} = \omega_{\alpha\beta} \ , \tag{2.1}$$

such that $\det|\omega_{\alpha\beta}| \neq 0$ and $\omega_{\alpha\beta}$ is field independent. Here the new variables Φ_α and the original second class constraints ($T_\alpha(P,Q)$) are of the same Grassman parity. Now we define the first class constraints $\bar{T}_\alpha(P,Q,\Phi_\alpha)$ in the extended phase space, satisfying

$$\{\bar{T}_\alpha, \bar{T}_\beta\} = 0 \ , \tag{2.2}$$

and the solution for which is obtained as

$$\bar{T}_\alpha = \sum_{n=0}^{\infty} T_\alpha(n) \ , \tag{2.3}$$

where n is the order of the term in Φ_α and $T_\alpha(0) = T_\alpha$.

After converting the second class constraints to strongly involutive ones, one proceeds to construct a gauge invariant Hamiltonian $\bar{H}(P,Q,\Phi_\alpha)$ in the extended phase space. This gauge invariant Hamiltonian has to satisfy

$$\{\bar{T}_\alpha, \bar{H}\} = 0 \ , \tag{2.4}$$

whose solution is obtained as an infinite series,

$$\bar{H} = \sum_{n=0}^{\infty} H_n \ , \quad \text{with} \quad H_0 = \ H \ , \tag{2.5}$$

where $H = \bar{H}(P,Q,\Phi_\alpha = 0)$. In the case of the linear theories, it is found that the series (2.3) and (2.5) have only a finite number of terms (i.e., n is finite). However, in the case of the non-linear theories, this series need not terminate. Also, one may not be able to express the series in a closed form and this makes the implementation of the procedure rather complicated. This will be shown to occur to the BISD model in section IIB. In the improved BFT formalism one can circumvent this problem as follows. Corresponding to each of the original phase space variables ϕ, one constructs $\bar{\phi}$ in the enlarged phase space, satisfying

$$\left\{\bar{T}_\alpha, \bar{\phi}\right\} = 0 \ . \tag{2.6}$$

Thus $\bar{\phi}$ is a gauge invariant combination. Now by replacing ϕ with $\bar{\phi}$ in any function $A(\phi)$ of the phase space variable, one can obtain the corresponding involutive function $\bar{A}$ in the enlarged phase space.

We now apply the improved BFT formulation to the BISD model. First we linearize the BISD model by introducing a multiplier field. Then in the linearized form, apart from the original constraints we have constraints coming from the linearizing field. Since we plan to eliminate this field after carrying out the embedding, we concentrate only in the sector of original constraints. We apply a partial embedding of the BISD model using the improved BFT scheme. Using the solutions of the embedded Hamilton's equations, we map the embedded model to the BICS theory.

B. Improved BFT Embedding of Born-Infeld Self Dual Model

We start from the Lagrangian

$$\mathcal{L} = \Phi + \frac{\beta^4}{4\Phi} + \frac{\Phi}{\beta^2} f_\mu f^\mu - \frac{1}{2m}\epsilon_{\mu\nu\lambda} f^\mu \partial^\nu f^\lambda \ . \tag{2.7}$$

Note that by eliminating the auxiliary Φ field from (2.7) by using its equation of motion gives back the standard BISD Lagrangian of Eqn. (1.3).

The primary constraints following from the above Lagrangian (2.7) are

$$\Pi_0 \ \approx \ 0, \quad \Pi_\Phi \ \approx \ 0, \tag{2.8}$$

$$\Omega_i \ = \ (\Pi_i + \frac{1}{2m}\epsilon_{0ij} f^j) \ \approx \ 0 \ . \tag{2.9}$$

Here Π_μ and Π_Φ are the conjugate momenta corresponding to f_μ and Φ respectively. The canonical Hamiltonian is

$$H_c = -\frac{\Phi}{\beta^2} f_i f^i - \Phi - \frac{\beta^4}{4\Phi} - f_0(\frac{\Phi}{\beta^2} f^0 - \frac{1}{m}\epsilon_{0ij}\partial^i f^j) \ . \tag{2.10}$$

The persistence of the primary constraints leads to the following secondary constraints

$$\Omega = \frac{2\Phi}{\beta^2} f_0 - \frac{1}{m}\epsilon_{0ij}\partial^i f^j \approx 0 \ , \tag{2.11}$$

$$\Lambda = 1 - \frac{\beta^4}{4\Phi^2} + \frac{1}{\beta^2} f_i f^i + \frac{1}{\beta^2} f_0 f^0 \approx 0 \ . \tag{2.12}$$

In the above, the constraint Ω_i (2.9) is due to the symplectic structure of the theory and not a true constraint. Following Faddeev and Jackiw [19], we impose this symplectic condition strongly leading to the modified bracket

$$\Big\{f_i(x), f_j(y)\Big\} = -m\epsilon_{0ij}\delta(x-y) \ . \tag{2.13}$$

The auxiliary field Φ is introduced in (2.7) for re-expressing the Lagrangian in Eqn. (1.3) in a convenient form and after embedding the above model, we will eliminate the Φ field. Hence, in converting the second class constraints to the first class ones using the BFT embedding we consider only the remaining constraints Π_0 and Ω.

Following the BFT procedure we enlarge the phase space by introducing a pair of canonically conjugate variables ψ and Π_ψ. Now we modify the constraints Π_0 and Ω such that they have vanishing Poisson bracket among themselves.

The modified constraints read

$$\Omega_0 = \Pi_0 + \psi \ , \tag{2.14}$$

$$\Omega' = \frac{2\Phi}{\beta^2}(f_0 + \Pi_\psi) - \frac{1}{m}\epsilon_{0ij}\partial^i f^j \approx 0 \ , \tag{2.15}$$

and it is easy to see that the Poisson bracket between these two constraints vanishes strongly. The general procedure of BFT requires the construction of the embedded Hamiltonian which is in involution with (2.14) and (2.15). Due to the non-linearity inherent in this theory, the embedded Hamiltonian is an infinite series,

$$H_{emb} = H_c - \Pi_\psi\Omega - \frac{\Phi\Pi_\psi^2}{\beta^2} + \frac{\psi}{2\Phi}\partial_i(f^i\Phi) + ... \ . \tag{2.16}$$

In the improved BFT scheme, which make use of the fact that any function of involutive combination of fields by itself is involutive, the embedded Hamiltonian is constructed by replacing f_0 and f_i by corresponding gauge invariant combinations $\bar{f}_0$ and $\bar{f}_i$ respectively.

Now corresponding to the original phase space variables f_0 and f_i we construct $\bar{f}_0$ and $\bar{f}_i$ which are in strong involution with the modified constraints Ω_0 and Ω'. Thus we get

$$\bar{f}_0 = f_0 + \Pi_\psi \ , \tag{2.17}$$

$$\bar{f}_i = f_i + \frac{\beta^2}{2\Phi}[\partial_i\psi - \psi\partial_i(ln\Phi)] \ . \tag{2.18}$$

Next the embedded Hamiltonian has to be constructed. This is easily done by the improved BFT scheme by replacing the original fields by involutive combinations (2.17, 2.18). Thus using (2.10) we get the embedded Hamiltonian as

$$\mathcal{H}_{emb} = -\frac{\Phi}{\beta^2}\bar{f}_i\bar{f}^i - \Phi - \frac{\beta^4}{4\Phi} + \frac{\Phi}{\beta^2}\bar{f}_0\bar{f}^0 - \bar{f}_0\bar{\Omega} \ , \tag{2.19}$$

where $\bar{\Omega} = (2\frac{\Phi}{\beta^2}\bar{f}^0 - \frac{1}{m}\epsilon_{0ij}\partial^i\bar{f}^j)$. It is easy to see that the modified constraints Ω_0 and Ω'_i (2.14, 2.15) have vanishing Poisson brackets with the embedded Hamiltonian (2.19). Note that this embedded Hamiltonian reduces to (2.10), in the unitary gauge, where the newly introduced fields α and Π_α are set to zero.

The two equations of motion following from the embedded Hamiltonian (2.19) are

$$\frac{2\Phi}{\beta^2}\bar{f}_0 - \frac{1}{m}\epsilon_{0ij}\partial^i\bar{f}^j = 0 \ , \tag{2.20}$$

$$\frac{2\Phi}{\beta^2}\bar{f}_i - \frac{1}{m}\epsilon_{i\mu\nu}\partial^\mu\bar{f}^\nu = 0 \ , \tag{2.21}$$

which can be expressed as

$$\frac{2\Phi}{\beta^2}\bar{f}_\mu - \frac{1}{m}\epsilon_{\mu\nu\lambda}\partial^\nu\bar{f}^\lambda = 0 \ , \tag{2.22}$$

in a covariant way. This equation imply $\partial^\mu(\Phi\bar{f}_\mu) = 0$. A gauge invariant solution of Eqn. (2.22) satisfying the condition $\partial^\mu(\Phi\bar{f}_\mu) = 0$ is

$$\bar{f}_\mu = \frac{\beta^2}{2\Phi}\epsilon_{\mu\nu\lambda}(\partial^\nu A^\lambda - \partial^\lambda A^\nu) \ . \tag{2.23}$$

On substituting this solution for $\bar{f}_\mu$ in Eqn. (2.19), embedded Hamiltonian becomes,

$$\mathcal{H}_{emb} = \frac{\beta^2}{2m^2\Phi}F_{ij}F^{ij} - \frac{\beta^2}{m^2\Phi}F_{0i}F^{0i} - \Phi - \frac{\beta^4}{4\phi} - f_0\tilde{\Omega} \ , \tag{2.24}$$

where $\tilde{\Omega} = (\epsilon_{0ij}F^{ij} - \frac{\beta^2}{m}\partial^i(\frac{F_{0i}}{\Phi}))$. Substituting for $\bar{f}_\mu$ from Eqn. (2.23) in Eqn. (2.22) we get the corresponding equation following from the BICS theory. With the identification

$\frac{1}{m}F_{0i} = -E_i$, and $\frac{1}{\sqrt{2}m}\epsilon_{0ij}F^{ij} = B$, it is easy to see that the above Hamiltonian (2.24) and constraints $\tilde{\Omega}$ and $\tilde{\Lambda}$ ($\tilde{\Lambda}$ is the constraint (2.12) expressed in terms of the solution for $\bar{f}_\mu$) are the ones which follow from the Lagrangian

$$\mathcal{L} = \Phi - \frac{\beta^4}{4\Phi} - \frac{\beta^2}{2m^2\Phi}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2m}\epsilon_{\mu\nu\lambda}A^\mu\partial^\nu A^\lambda \ . \tag{2.25}$$

Eliminating Φ from the above Lagrangian and redefining $\frac{A_\mu}{m} \to A_\mu$ gives the BICS Lagrangian (1.4). This shows the equivalence of the embedded version of the BISD model and the BICS theory.

III. DUALITY EQUIVALENCE

In this section we show that the BICS theory is related to BISD model through duality. The duality equivalence between these two models is obtained here in a Lagrangian formulation.

We derive this duality equivalence using Buscher's procedure [12,13] of constructing dual theories. Basically, this procedure consists of gauging a global symmetry in the theory with a suitable gauge potential. To make it equivalent to the original theory, we constrain the dual field strength of the gauge potential to vanish by means of a Lagrange multiplier. Integrating the multiplier field and the gauge field, original action is recovered. Instead, if one integrates the original field and gauge potential, the dual theory is obtained where the multiplier field becomes the dynamical field. This procedure has been used recently to show the equivalence between a topologically massive gauge theory and different Stückelberg formulations in $3+1$ dimensions [20]. The duality relation obtained here between the BICS theory and BISD model nicely complements the equivalence obtained between these theories in the canonical formulation. We also use it to obtain the mapping between the correlators of these two theories.

We start with the partition function for the BICS theory

$$Z = \int DA_\mu \ exp\left(\ i\int d^3x\mathcal{L}\right) \ , \tag{3.1}$$

where the Lagrangian

$$\mathcal{L} = \beta^2\sqrt{1 - \frac{1}{2\beta^2}F_{\mu\nu}F^{\mu\nu}} + \frac{m}{2}\epsilon_{\mu\nu\lambda}A^\mu\partial^\nu A^\lambda + \frac{1}{2}\epsilon_{\mu\nu\lambda}F^{\mu\nu}J^\lambda \ , \tag{3.2}$$

with J^λ being the source term (which is coupled to dual field strength). As in the last section, we first linearize the Born-Infeld part of the above Lagrangian by introducing an auxiliary field Φ .

Apart from the local $U(1)$ invariance (1.5), the action corresponding to this Lagrangian is invariant under the global shift symmetry of the vector field

$$\delta A_\mu = \epsilon_\mu \ , \tag{3.3}$$

where ϵ_μ is a constant. As discussed earlier, the necessary ingredient for dualization in Buscher's procedure is the presence of a global symmetry. In order to gauge the global shift symmetry (3.3) of A_μ in Chern-Simons term, it is convenient to linearize it using an auxiliary vector field P_μ. Thus we re-express the Lagrangian (3.2) as

$$\mathcal{L} = [1 - \frac{1}{2\beta^2}F_{\mu\nu}F^{\mu\nu}]\Phi + \frac{\beta^4}{4\Phi} + \frac{m}{4}\epsilon_{\mu\nu\lambda}P^\mu F^{\nu\lambda} - \frac{m}{8}\epsilon_{\mu\nu\lambda}P^\mu\partial^\nu P^\lambda + \frac{1}{2}\epsilon_{\mu\nu\lambda}F^{\mu\nu}J^\lambda \ . \tag{3.4}$$

We convert the global symmetry (3.3) of the Born-Infeld Chern-Simons Lagrangian into a local one by introducing a 2-form gauge potential $G_{\mu\nu}$. The source is coupled to (dual) field strength rather than to field itself so as to keep the invariance under the transformation (3.3). Following Buscher's procedure, we constrain the dual field strength of the gauge field to be flat. This is achieved by a multiplier field Θ. Thus we get the Lagrangian

$$\begin{aligned}\mathcal{L} = {} & [1 - \frac{1}{2\beta^2}(F_{\mu\nu} - G_{\mu\nu})(F^{\mu\nu} - G^{\mu\nu})]\Phi + \frac{\beta^4}{4\Phi} + \frac{m}{4}\epsilon_{\mu\nu\lambda}P^\mu(F^{\nu\lambda} - G^{\nu\lambda}) \\ & + \frac{1}{2}\epsilon_{\mu\nu\lambda}(F^{\mu\nu} - G^{\mu\nu})J^\lambda - \frac{m}{8}\epsilon_{\mu\nu\lambda}P^\mu\partial^\nu P^\lambda + \frac{1}{2}\epsilon_{\mu\nu\lambda}\Theta\partial^\mu G^{\nu\lambda} \ . \end{aligned} \tag{3.5}$$

Here, under the local shift of A_μ, the gauge potential also transform as $\delta G_{\mu\nu} = (\partial_\mu\epsilon_\nu - \partial_\nu\epsilon_\mu)$ so that $(F_{\mu\nu} - G_{\mu\nu})$ (and hence the Lagrangian (3.5)) is gauge invariant. Note that under this transformation P_μ and Θ remain invariant.

It is interesting to note that the action corresponding to the Lagrangian (3.5) is also invariant under $\delta P_\mu = \partial_\mu\lambda$ provided the multiplier field Θ undergoes a transformation

$\delta\Theta = -m\lambda/2$ while $\delta A_\mu = 0$ and $\delta G_{\mu\nu} = 0$. Thus we see that Θ undergoes a compensating Stückelberg transformation. Integrating $G_{\mu\nu}$ and A_μ from the partition function leads to the following Lagrangian

$$\begin{aligned}\mathcal{L} = \Phi + \frac{\beta^4}{4\Phi} + \frac{\beta^2}{4\Phi}[(\frac{m}{2}P_\mu + \partial_\mu\Theta)(\frac{m}{2}P^\mu + \partial^\mu\Theta)] \\ + \frac{\beta^2}{4\Phi}J_\mu[J^\mu + mP^\mu + 2\partial^\mu\Theta] - \frac{m}{8}\epsilon_{\mu\nu\lambda}P^\mu\partial^\nu P^\lambda \ . \end{aligned} \tag{3.6}$$

By integrating out Φ from the above Lagrangian the partition function reduces to

$$Z = \int DP_\mu D\Theta \ exp\Big(\ i\int d^3x\mathcal{L}\Big) , \tag{3.7}$$

where

$$\mathcal{L} = \beta^2\sqrt{1 + \frac{1}{\beta^2}(\frac{m}{2}P^\mu + \partial^\mu\Theta)^2 + \frac{1}{\beta^2}J_\mu(J^\mu + mP^\mu + 2\partial^\mu\Theta)} - \frac{m}{8}\epsilon_{\mu\nu\lambda}P^\mu\partial^\nu P^\lambda \ , \tag{3.8}$$

showing the equivalence of the BICS theory to the Stückelberg formulation of the BISD model. The above Lagrangian (3.8) is invariant under

$$\begin{aligned}\delta P_\mu &= \partial_\mu\lambda, \\ \delta\Theta &= -\frac{m}{2}\lambda \ . \end{aligned} \tag{3.9}$$

The field Θ introduced as a multiplier field in Eqn. (3.5) appears as the Stückelberg field with correct compensating transformation in Eqn. (3.7). Note that in the last term in (3.8), the contribution from the Stückelberg field vanishes. Thus the model described by the Lagrangian (3.8) is invariant under the $U(1)$ transformation of P_μ when the scalar field also undergoes a compensating transformation. Thus we see here that two theories which are related by duality have the same symmetry. The same feature was observed in the case of the duality equivalence of topologically massive $B\wedge F$ theory in $3+1$ dimensions to two different massive gauge theories [20]. In the absence of the sources terms (*i.e.*, $J_\mu = 0$), by setting the gauge condition $\Theta = 0$ and with the identification $\frac{mP_\mu}{2} = f_\mu$, the above Lagrangian reduces to that BISD model (1.3). Thus using Buscher's procedure we have shown that the BICS theory is equivalent to the gauge invariant form of the BISD model. We see here that the

BISD model described by (1.3) is the gauge fixed version of the dual model obtained by Buscher's procedure from BICS theory.

¿From the partition functions of Eqn. (3.1 and 3.7), by functional differentiation with respect to corresponding source terms and then setting sources to be zero, we get the following mapping between the correlators

$$\Big\langle \ F^*_\mu \ F^*_\nu \ \Big\rangle_{BICS} \ \leftrightarrow \ \frac{1}{4} \ \langle \ (mP_\mu + 2\partial_\mu \Theta) \ (mP_\nu + 2\partial_\nu \Theta) \ \rangle_{BISD} \\ - \ ig_{\mu\nu}\delta(x-y) \ , \tag{3.10}$$

where F^*_μ is the dual of $F^{\nu\lambda}$ and the last term in the above is a non-propagating contact term. It is interesting to note that the (dual) field strength of the BICS theory is equivalent to a gauge invariant combination of fields in the gauge invariant form of BISD model (This result is also true in the case of the MCS theory and the SD model). In particular, remarkably there is no β dependence in the operator equivalence.

IV. BOSONIZATION

As discussed in the introduction, the MCS gauge theory has been shown to be equivalent to the massive Thirring model. The operator correspondence at the level of correlators between the fermionic current and the dual gauge field strength of MCS has been shown [6,15,16]. Also the commutators between the components of the fermionic current has been evaluated using this correspondence and the Dirac brackets of MCS theory. Further, non-zero Schwinger term is found to result in the $[j_0(x), j_i(y)]$ commutators, where j_μ is the fermionic current. It is then clearly of considerable interest to extend the previous study to the BICS case and investigate the role of non-linearity in the bosonization of the Born-Infeld theories.

In this section, we first start from the the BIT model (1.6) and show that to leading order in the fermion mass μ, its partition function is equivalent to that of BICS theory. This is also re-derived starting with an interpolating master Lagrangian. We also provide the bosonization rules relating the fermion current correlators of BIT to (dual) field strength correlators of the BICS theory. We find that this mapping is β independent. We also use this correspondence to obtain the current algebra for the fermionic theory. Since this requires the Dirac brackets for the BICS theory which to the best of our knowledge have not been evaluated so far, hence we have calculated them in the Appendix. Using these Dirac brackets, the current algebra relations are evaluated and are found to be β dependent. As expected, in the limit $\beta \to \infty$, the MCS-Thirring model results are reproduced.

A. Non-Polynomial Thirring model

We start with the partition function of a massive Dirac fermion with a non-polynomial Thirring type current-current coupling whose partition function is

$$Z_T = \int D\bar{\Psi} D\Psi exp \left(i \int d^3x \mathcal{L} \right) \ , \tag{4.1}$$

where $\mathcal{L}$ is given by Eqn. (1.6).

The non-polynomial interaction term in this Lagrangian is linearized by introducing an auxiliary field χ to express (1.6) as

$$L = \bar{\Psi}(i\gamma_\mu \partial^\mu - \mu)\Psi + \chi + \frac{\beta^4}{4\chi} - \frac{\chi\lambda}{\beta^2} j_\mu j^\mu \ . \tag{4.2}$$

Next we linearize the quadratic term $j_\mu j^\mu$ by an auxiliary vector field f_μ and write the Lagrangian as

$$\mathcal{L} = \bar{\Psi}(i\gamma_\mu \partial^\mu - \mu)\Psi + \chi + \frac{\beta^4}{4\chi} + \frac{\lambda\chi}{\beta^2}(\frac{1}{4} f_\mu f^\mu - f_\mu j^\mu) \ . \tag{4.3}$$

Using this Lagrangian in (4.1), we integrate out the Dirac field and use the well-known result for the evaluation of the determinant of the Dirac operator in a gauge invariant regularization [6]:

$$ln \ det \ (i\gamma_\mu \partial^\mu - \mu - g f_\mu \gamma^\mu) = \pm \frac{ig^2}{8\pi} \int d^3x \epsilon_{\mu\nu\lambda} f^\mu \partial^\nu f^\lambda + O(\frac{1}{\mu}) + ... \ . \tag{4.4}$$

The leading order term is the odd parity Chern-Simons term and as is well known, in the limit $\mu \to \infty$, only this term survives. This applied to the present case gives

$$
\begin{aligned}
ln\ det\ (i\gamma_\mu \partial^\mu - \mu - \frac{\chi\lambda}{\beta^2} f_\mu \gamma^\mu) &= \pm \frac{i\lambda^2}{8\beta^4\pi} \int d^3x \chi \epsilon_{\mu\nu\lambda} f^\mu \partial^\nu (\chi f^\lambda) + O(\frac{1}{\mu}) + ... \\
&= \pm \frac{i\lambda^2}{8\beta^4\pi} \int d^3x \chi^2 \epsilon_{\mu\nu\lambda} f^\mu \partial^\nu f^\lambda + O(\frac{1}{\mu}) + ... \ .
\end{aligned} \tag{4.5}
$$

As a result, in the limit $\mu \to \infty$, the partition function becomes

$$
Z = \int D\chi D f_\mu exp\ i \Big(\int d^3x \mathcal{L}_{eff} \Big) \ , \tag{4.6}
$$

where

$$
\mathcal{L}_{eff} = \chi + \frac{\beta^4}{4\chi} + \frac{\beta^2}{4\chi} g_\mu g^\mu \pm \frac{\lambda}{8\pi} \epsilon^{\mu\nu\lambda} g_\mu \partial_\nu g_\lambda \ , \tag{4.7}
$$

(we have redefined $\frac{\sqrt{\lambda}\chi}{\beta^2} f_\mu = g_\mu$). We identify the above Lagrangian (4.7) with that given in (2.7) describing BISD model (in the same spirit as [6–8]). Now we integrate out the auxiliary field χ and resulting Lagrangian is

$$
\mathcal{L}_{BISD} = \beta^2 \sqrt{1 + \frac{1}{\beta^2} g_\mu g^\mu} \pm \frac{\lambda}{8\pi} e^{\mu\nu\lambda} g_\mu \partial_\nu g_\lambda \ , \tag{4.8}
$$

which is the BISD action given in (1.3) with the identifications $m = \mp \frac{4\pi}{\lambda}$. Thus we have shown that to order $\frac{1}{\mu}$, the Thirring model partition function is equivalent to that of BISD model. Now using the equivalence discussed in the previous section, we can identify this Lagrangian, with the BICS Lagrangian (1.4) and thus we conclude that

$$
Z_T \equiv Z_{BICS} \ . \tag{4.9}
$$

B. Master Lagrangian

Next we derive the same result from a master Lagrangian, as has been done in the usual MCS to Thirring model equivalence.

The master Lagrangian we start with is

$$L = \bar{\Psi}(i\gamma_\mu\partial^\mu - \mu)\Psi - \alpha f_\mu j^\mu - \beta^2\sqrt{1 - \frac{1}{2\beta^2}F_{\mu\nu}F^{\mu\nu}}$$
$$+\frac{1}{4}\epsilon_{\mu\nu\lambda}f^\mu F^{\mu\lambda} + \frac{1}{4}\epsilon_{\mu\nu\lambda}F^{\mu\nu}J^\lambda \ . \qquad (4.10)$$

In the above J_μ is the source for the dual of the field strength $F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)$, α is a dimensionful constant, j_μ is the conserved fermionic current, $\bar{\Psi}\gamma_\mu\Psi$. The Lagrangian (4.10) is invariant under (i) the local $U(1)$ transformation of gauge potential A_μ (ii) under another independent local $U(1)$ gauge transformation where Ψ and f_μ are minimally coupled and (iii) global shift symmetry of A_μ.

Next we implement the Buscher's procedure to re-express this theory into an equivalent one. Note that in contrast to section III, here we have a mixed Chern-Simons term involving 1-forms f and A. To apply the Buscher's procedure we make use of the shift symmetry ($\delta A_\mu = \epsilon_\mu$, where ϵ_μ is constant) of the Lagrangian (4.10). Note that in (4.10) it was necessary to couple the source to the dual field strength of A_μ so as to have the shift symmetry.

As in the previous section, we first linearize the Born-Infeld term using an auxiliary Φ field. The Lagrangian which is invariant under the local shift of A_μ is given by

$$L = \bar{\Psi}(i\gamma_\mu\partial^\mu - \mu)\Psi - \alpha f_\mu j^\mu + \Phi + \frac{\beta^4}{4\Phi} - \frac{\Phi}{2\beta^2}(F_{\mu\nu} - G_{\mu\nu})(F^{\mu\nu} - G^{\mu\nu})$$
$$+ \frac{1}{4}\epsilon_{\mu\nu\lambda}(F^{\mu\nu} - G^{\mu\nu})(f^\lambda \ + \ J^\lambda) - \frac{1}{4}\epsilon_{\mu\nu\lambda}G^{\mu\nu}\partial^\lambda\Theta \ , \qquad (4.11)$$

where $G_{\mu\nu}$ is a 2-form gauge potential and Θ is the multiplier field. Note that as in the previous section, we have constrained the dual field strength of $G_{\mu\nu}$ to be flat. We integrate out $G_{\mu\nu}$ and A_μ fields from the partition function corresponding to (4.11). Thus we get

$$Z = \int D\bar{\Psi}D\Psi Df_\mu D\Theta D\Phi \ exp \ \left(i\int d^3x \ L_{eff}\right) \ , \qquad (4.12)$$

where

$$L_{eff} = \bar{\Psi}(i\gamma_\mu\partial^\mu - \mu)\Psi - \alpha f_\mu j^\mu + \Phi + \frac{\beta^4}{4\Phi} + \frac{\beta^2}{16\Phi}(f_\mu + \partial_\mu\Theta)(f^\mu + \partial^\mu\Theta)$$
$$+ \frac{\beta^2}{8\Phi}(f_\mu + \partial_\mu\Theta)J^\mu + \frac{\beta^2}{16\Phi}J_\mu J^\mu \ . \qquad (4.13)$$

Next, the integrations over f_μ, and Θ which are Gaussian are carried out with the gauge condition $\Theta = 0$ [1] and then we integrate out Φ also to get the partition function

$$Z[J_\mu] = \int D\bar{\Psi} D\Psi \ exp \ \left(i \int d^3x \ \mathcal{L} \right) \ , \tag{4.14}$$

where

$$\mathcal{L} = \bar{\Psi}(i\gamma_\mu \partial^\mu - \mu)\Psi - \alpha J_\mu j^\mu + \beta^2 \sqrt{1 - \frac{\alpha^2}{\beta^2} j_\mu j^\mu} \ . \tag{4.15}$$

In the absence of source J_μ, the Lagrangian (4.15) reduces to that of the BIT model (1.6) with the identification $\alpha^2 = \lambda$.

Instead of integrating the bosonic variables from the partition function of the master Lagrangian (4.10), one can integrate $\bar{\Psi}$ and Ψ. Using Eqn. (4.4), we get

$$Z[J_\mu] = \int DA_\mu Df_\mu exp \ \left(i \int d^3x \mathcal{L} \right) \ , \tag{4.16}$$

where

$$\begin{aligned} \mathcal{L} = \beta^2 \sqrt{1 - \frac{1}{2\beta^2} F_{\mu\nu} F^{\mu\nu}} + \frac{1}{4} \epsilon_{\mu\nu\lambda} f^\mu F^{\nu\lambda} + + \frac{1}{2} \epsilon_{\mu\nu\lambda} F^{\mu\nu} J^\lambda \\ - \frac{\alpha^2}{8\pi} \epsilon_{\mu\nu\lambda} f^\mu \partial^\nu f^\lambda + O(\frac{1}{\mu}) + \ldots \ . \end{aligned} \tag{4.17}$$

Since the partition functions in Eqns. (4.14) and (4.16) are obtained from the same master Lagrangian, this shows their equivalence.

Integrating out f_μ from (4.16) with the gauge condition $\partial_\mu f^\mu = 0$, we get the partition function to be

$$Z = \int DA_\mu exp \ i \ \left(\int d^3x \mathcal{L} \right) \ , \tag{4.18}$$

where

$$\mathcal{L} = \beta^2 \sqrt{1 - \frac{1}{2\beta^2} F_{\mu\nu} F^{\mu\nu}} + \frac{m}{2} \epsilon_{\mu\nu\lambda} A^\mu \partial^\nu A^\lambda + \frac{1}{2} \epsilon_{\mu\nu\lambda} F^{\mu\nu} J^\lambda \ , \tag{4.19}$$

---

[1] We can as well fix the gauge condition to be $\partial_\mu f^\mu = 0$ and then integrate over f_μ and Θ.

with the identification $\frac{4\pi}{\alpha^2} = m$. For $J_\mu = 0$, this gives

$$Z_T \equiv Z_{BICS} \ . \tag{4.20}$$

Using the fact that (4.14) and (4.18) are equivalent, one can easily derive the mapping between the correlators of these two models by taking the functional derivatives with respect to the source J_μ. Thus we get

$$\langle j_{\mu_1}(x_1)..\ j_{\mu_n}(x_n)\rangle_{BIT} = \frac{1}{\alpha^{2n}} \left\langle F^*_{\mu_1}(x_1)..\ \ F^*_{\mu_n}(x_n)\right\rangle_{BICS} \ , \tag{4.21}$$

where F^*_μ is the dual field strength corresponding to A_μ. This implies the following operator correspondence between the fermionic current and the dual field strength:

$$j_\mu = \frac{1}{2\alpha}\epsilon_{\mu\nu\lambda}F^{\nu\lambda} \ . \tag{4.22}$$

Here we note that the mapping (4.21) is independent of the β parameter and thus same as that obtained in the case of MCS - massive Thirring model.

C. Current Algebra

In this subsection we extend the analysis of bosonization by studying the algebra of fermionic currents by using the operator equivalence (4.22).

We see that with the identification (4.22), the term $\sqrt{1-\frac{1}{2\beta^2}F_{\mu\nu}F^{\mu\nu}}$ in BICS goes to $\sqrt{1-\frac{\alpha^2}{2\beta^2}j_\mu j^\mu}$ and if further $\alpha^2 = \lambda$ then this is precisely the non-linear current-current term in Eqn. (1.6). From the relation (4.22) between the fermionic current and the dual field strength we can obtain the potential A_i in terms of the components of the current j_μ. Then using the Dirac brackets evaluated in the Appendix A we get the various commutators between the currents. From Eqn. (4.22) we have for the components

$$j_0 = \frac{1}{\alpha}\epsilon_{0ij}\partial^i A^j \ , \tag{4.23}$$

$$j_i = -\frac{1}{2\alpha}\epsilon_{0ij}F^{0j} \ . \tag{4.24}$$

Inverting these relations we get

$$A_i = \frac{\alpha}{\nabla^2}\epsilon_{0ij}\partial^j j^0 \ , \qquad (4.25)$$

$$\Pi_i = -\frac{\alpha}{R}\epsilon_{0ij} j^j - \frac{m\alpha}{2\nabla^2}\partial_i j^0 \ , \qquad (4.26)$$

where Π_i is the conjugate momentum corresponding to A_i and $R(x) = \sqrt{1 - \frac{\alpha^2}{2\beta^2} j_\mu j^\mu}$. In obtaining (4.25) we have used the condition $\partial_i A^i = 0$. Here we see that the conjugate momentum Π_i depends on R and hence on β^2. For the second Eqn. (4.26) we have used the defining relation of the conjugate momenta Π_i (viz: $\Pi_i = -\frac{F_{0i}}{R} + \frac{m}{2}\epsilon_{0ij}A^j$).

Now using the non-vanishing Dirac brackets of BICS theory (see appendix, Eqns. A14, A15), we evaluate the commutators between the different components of the fermionic current. Thus we get

$$[\ j_0(x),\ j_0(y)\ ] = \left[\frac{1}{\alpha}\epsilon_{0ij}\partial^i A^j(x),\ \frac{1}{\alpha}\epsilon_{0lm}\partial^l A^m(y)\right] = 0 \ . \qquad (4.27)$$

Using the above commutator (4.27) we get

$$\{A_i(x), \Pi_j(y)\}^* = \{\frac{\alpha}{\nabla^2}\epsilon_{0ij}\partial^j_{(x)} j^0(x),\ \ -\frac{\alpha}{R(x)}\epsilon_{0ij} j^j(y)\}^* \ . \qquad (4.28)$$

Using the Dirac bracket (A14) and with some algebra, from the above Eqn. (4.28) we get

$$\left[\ j_0(x),\ \frac{1}{R(y)} j_i(y)\ \right] = i\frac{1}{\alpha^2}\partial_i^{(x)}\delta(x-y) \ . \qquad (4.29)$$

Similarly from

$$\{\Pi_i(x), \Pi_j(y)\}^* = \{-\frac{\alpha}{R(x)}\epsilon_{0il} j^l(x) - \frac{m\alpha}{2\nabla^2}\partial_i^{(x)} j^0(x),\ \ -\frac{\alpha}{R(x)}\epsilon_{0jm} j^m(y) - \frac{m\alpha}{2\nabla^2}\partial_j^{(y)} j^0(y)\}^* \ , \qquad (4.30)$$

using the above commutators (4.27, 4.29), and the Dirac bracket (A15) we get

$$\left[\ \frac{1}{R(x)} j_i(x),\ \frac{1}{R(y)} j_j(y)\ \right] = -i\frac{m}{\alpha^2}\epsilon_{oij}\delta(x-y) \ . \qquad (4.31)$$

Here we see that the commutators (4.29, 4.31) are β dependent (note R is β-dependent) even though the correlator mapping (4.21) is β-independent. In the limit $\beta^2 \to \infty$, from (4.29, 4.31) we get

$$[\ j_0(x),\ j_i(y)\ ] = i\frac{1}{\alpha^2}\partial_i^{(x)}\delta(x-y) - \frac{\alpha^2}{4\beta^2}[j_0(x), j_\mu j^\mu j_i(y)] + O(\frac{1}{\beta^4})\ + ..\ , \tag{4.32}$$

$$[j_i(x),\ j_j(y)\ ] = -i\frac{m}{\alpha^2}\epsilon_{oij}\delta(x-y) - \frac{\alpha^2}{4m}[j_i(x)j_\mu j^\mu,\ j_j(y)] - \frac{\alpha^2}{4\beta^2}[j_i(x), j_\mu j^\mu j_j(y)]\ +\ O(\frac{1}{\beta^4})\ + ..\ . \tag{4.33}$$

Note that the operator correspondence is β independent, since it is a general feature of bosonization. However, the current algebra, which is an observable of the theory, depends on the details of the theory and hence on β.

It is easy to see from the above Eqns. (4.32, 4.33) that as $\beta^2 \rightarrow \infty$, the above commutators (4.27,4.29,4.31) reduce to the corresponding ones of the massive Thirring model evaluated using the Dirac brackets of MCS theory [8].

In order to understand the free theory limit, it is convenient to scale the fields $A_\mu \rightarrow \alpha A_\mu$ so that (4.22) is α independent. Then as $\alpha \rightarrow 0$, only Chern-Simons term survives in the action while the Fermionic sector becomes free theory and gives the same prescription of the current bosonization.

V. CONCLUSION

In this paper we have made a comprehensive study of the BICS theory and the SDBI model and shown their equivalence in a variety of ways. We have extended the earlier studies on the MCS theory and the SD model to the Born-Infeld case. One of the motivation is to investigate the application of the known techniques to non-linear theories.

We started with the SDBI theory, which has only second-class constraints, and following the improved BFT scheme we obtained the gauge invariant Hamiltonian and also converted the constraints to first class. The resulting theory was shown to be equivalent to the BICS theory. This demonstrates that the improved BFT procedure is also applicable to highly non-linear theories. Previously this method has been successfully applied to other non-linear theories like massive Yang-Mills theory [21]. Apart from its intrinsic interest this study also provides yet another example of the application of this scheme to a different type of nonlinear theory.

Next we showed that the equivalence between these theories, viz; BICS and BISD , is actually a duality equivalence thereby extending the approach of Buscher's procedure to relate two *non-linear dual theories* .

We have also provided the operator correspondence between the fermionic current of the BIT and the dual gauge field of the BICS at the level of correlators. This has been used to calculate the commutators between different components of the fermionic current. For this purpose, we have calculated the Dirac brackets of the BICS theory using symplectic quantization scheme in Appendix A and this by itself is an interesting new result. The current algebra is found to be β dependent and the leading order correction term in $\frac{1}{\beta^2}$ has been computed. As expected, in the limit $\beta^2 \to \infty$, the results corresponding to massive Thirring model are recovered. It is interesting to note that the coupling constant α^2 that appears linearly in BIT, appears inversely as Chern-Simons mass of BICS. This can be of use in studing the non-perturbative aspects of BICS.

Acknowledgments

One of us (EH) would like to thank Institute of Physics, Bhubaneswar for the warm hospitality extended to him during his visit to the institute where a part of this work was done.

APPENDIX A: SYMPLECTIC QUANTIZATION

In this appendix we compute the Dirac brackets of the BICS theory using the symplectic quantization scheme.

The symplectic quantization scheme of Faddeev and Jackiw [19,22,23] has recently been studied and applied to different models. Primary constraints of the Dirac procedure do not appear in this Lagrangian scheme. Here one starts with the Lagrangian which is first order in time derivatives. One then has to invert the symplectic matrix to obtain the Dirac brackets. If the system has true constraints, then the symplectic matrix become singular. In this case the configuration space is enlarged by introducing multiplier fields corresponding to each of the constraints and the constraints are introduced back in to the Lagrangian using them. After incorporating all the constraints, the symplectic matrix can still be singular signaling the gauge invariance of the theory. At this stage the gauge fixing conditions are introduced so as to make the symplectic matrix non singular and from its inverse Dirac brackets are obtained.

Here we apply this scheme to the BICS theory described by the Lagrangian (1.4) and obtain the Dirac brackets. We start with a first order form of (1.4) as given by

$$\mathcal{L} = \Pi_i \partial^0 A^i - R\Pi_i \Pi^i + \frac{m^2}{4} R A_i A^i + \beta^2 R + (\partial^i \Pi_i + \frac{m}{4}\epsilon_{0ij} F^{ij}) A^0 \ . \tag{A1}$$

Here as is common in the symplectic scheme, Π_i is not identified with the conjugate momentum corresponding to the field A_i and $R = \sqrt{1 - \frac{1}{2\beta^2} F_{\mu\nu} F^{\mu\nu}}$. The above Lagrangian (A1) is of the form

$$L = a_i \partial^0 \zeta^i - V(\zeta) \ , \tag{A2}$$

and in that case the symplectic matrix is defined as

$$f_{ij} = \partial_i a_j - \partial_j a_i \ , \tag{A3}$$

where the derivatives are taken with respect to ζ. In our case, using (A1), we get the a_i to be

$$a_0^A = 0 \qquad a_i^A = \Pi_i, \quad a_i^\Pi = 0 \ , \tag{A4}$$

where a_0^A stand for the coefficient of $\partial^0 A_0$. Then the only non-vanishing elements of the symplectic matrix are

$$f_{ij}^{A\Pi}(x,y) = \frac{\partial a_j^\Pi}{\partial A_i} - \frac{\partial a_i^A}{\partial \Pi_j} = -\delta_{ij}\delta(x-y) = -f_{ij}^{\Pi A}(x,y) \ . \tag{A5}$$

Thus we get the symplectic matrix to be

$$f_{ij}(x,y) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\delta_{ij} \\ 0 & \delta_{ij} & 0 \end{pmatrix} \delta(x-y) \ , \tag{A6}$$

which is singular, showing the constrained nature of the theory. These constraints are obtained by solving for the zero modes of f_{ij} from[2]

$$\int dxdy \left[a\frac{\delta}{\delta A_0} + b_i \frac{\delta}{\delta A_i} + c_i \frac{\delta}{\delta \Pi_i} \right] V = 0 \ , \tag{A7}$$

where $V = R\Pi_i\Pi^i - \frac{m^2}{4}RA_iA^i - \beta^2 R - (\partial^i\Pi_i + \frac{m}{4}\epsilon_{0ij}F^{ij})A^0$. From this we get the constraint

$$\partial_i\Pi^i + \frac{m}{2}\epsilon_{0ij}F^{ij} = 0 \ . \tag{A8}$$

Now we introduce this constraint into the kinetic part of the Lagrangian using a multiplier field η. In this way, the modified Lagrangian takes the form

$$\mathcal{L}^1 = \Pi_i\partial^0 A^i - \partial^i\eta[\partial^0\Pi_i + \frac{m}{2}\epsilon_{0ij}\partial^0 A^j] - R\Pi_i\Pi^i + \frac{m^2}{4}RA_iA^i + \beta^2 R \ , \tag{A9}$$

where we have absorbed $A_0(\partial_i\Pi^i + \frac{m}{2}\epsilon_{0ij}F^{ij})$ into the second term of the above Lagrangian (A9). From (A9) we get

$$a_i^A = \Pi_i + \frac{m}{2}\epsilon_{0ij}\partial^j\eta, \quad a_i^\Pi = -\partial_i\eta \ , \tag{A10}$$

---

[2]Since the Euler-Lagrange equation following from (A2) is $f_{ij}\partial^0\zeta^j = \frac{\partial\ V(\zeta)}{\partial\zeta_i}$, the zero mode of $V(\zeta)$ is related to that of f_{ij}, where f_{ij} is defined in (A3).

and hence the symplectic matrix is

$$f_{ij}(x,y) = \begin{pmatrix} 0 & -\,\delta_{ij} & -\,\frac{m}{2}\epsilon_{0ij} \\ \delta_{ij} & 0 & \partial_i \\ \frac{m}{2}\epsilon_{0ij}\partial^j & -\,\partial_i & 0 \end{pmatrix} \delta(x-y)\ , \tag{A11}$$

which is also singular. But this does not give any new constraint showing the gauge symmetry of the theory. So we introduce the gauge fixing condition $\partial_i A^i$ using a multiplier field λ into the kinetic part of the Lagrangian (A9) to get the modified Lagrangian

$$\mathcal{L}^{(2)} = (\Pi_i + \frac{m}{2}\epsilon_{0ij}\partial^j\eta - \partial_i\lambda)\partial^0 A^i - \partial_i\eta\partial^0\Pi^i - R\Pi_i\Pi^i + \frac{m^2}{4}RA_iA^i + \beta^2 R\ . \tag{A12}$$

The non-singular symplectic matrix following from the above Lagrangian (A12) is

$$f^{(2)}_{ij}(x,y) = \begin{pmatrix} 0 & -\,\delta_{ij} & -\,\frac{m}{2}\epsilon_{0ij}\partial^j & \partial_i \\ \delta_{ij} & 0 & \partial_i & 0 \\ \frac{m}{2}\epsilon_{0ij}\partial^j & -\,\partial_j & 0 & 0 \\ -\partial_i & 0 & 0 & 0 \end{pmatrix} \delta(x-y)\ , \tag{A13}$$

from the inverse of which we get the following non-vanishing Dirac brackets

$$\{A_i(x),\ \Pi_j(y)\}^* = (\delta_{ij} + \frac{\partial_i\partial_j}{\nabla^2})\delta(x-y), \tag{A14}$$

$$\{\Pi_i(x),\ \Pi_j(y)\}^* = -\frac{m}{2\nabla^2}(\epsilon_{0mi}\partial^m\partial_j - \epsilon_{0mj}\partial^m\partial_i)\delta(x-y). \tag{A15}$$

In the above (A14, A15), all the derivatives are with respect to x. Here we note that the Dirac brackets are independent of β^2 and hence are the same as in the MCS theory. This is a reflection of the fact that the gauge symmetry in the BICS theory and MCS theory is the same. The nonlinearity of the BICS Lagrangian shows up only in the definition of Π_i .

REFERENCES

[1] For a review see A. A. Tseytlin; hep-th/9908105; Yuri Golfand memorial volume, ed. M. Shifman, World Scientific, 2000.

[2] S. Deser and R. Jackiw, Phys. Lett. **B139**, 371 (1984) .

[3] P. K. Townsend, K. Pilch and P. van Nieuwenhuizen, Phys. Lett. **B136**, 38 (1984) .

[4] R. Banerjee H. J. Rothe and K. D. Rothe, Phys. Rev. **D55**, 6369 (1997).

[5] Y-W. Kim and K. D. Rothe, Int. J. Mod. Phys. **A13**, 4183 (1998) and references therein.

[6] A. N. Redlich, Phys. Rev. **D29**, 2366 (1984).

[7] E. Fradkin and F. A. Schaposnik, Phys. Lett. **B338**, 253 (1994).

[8] R. Banerjee, Phys. Lett. **B358**, 297 (1995).

[9] P. K. Tripathy and A. Khare, Phys. Lett. **B504**, 152 (2001).

[10] I. A. Batalin and E. S. Fradkin, Nucl. Phys. **B279**, 514 (1987).

[11] I. A. Batalin and I. V. Tyutin, Int. J. Mod. Phys. **A6**, 3255 (1991).

[12] T. H. Buscher, Phys. Lett. **B194**, 59 (1987); Phys. Lett. **B201**, 466, (1988).

[13] M. Rocek and E. Verlinde, Nucl. Phys. **B373**, 630 (1992).

[14] E. Harikumar, Ph.D Thesis, Uni. Hyderabad (unpublished).

[15] N. Shaji, R. Shankar and M. Sivakumar, Mod. Phys. Lett. **A5**, 593 (1990).

[16] R. Shankar and M. Sivakumar, Mod. Phys. Lett. **A6**, 2379 (1991).

[17] D. G. Barci, L. E. Oxman and S. P. Sorella, Phys. Rev. **D59**, 105012 (1999).

[18] Y-W. Kim, M-I. Park, Y-J. Park and S. J. Yoon, Int. J. Phys. **A12**, 4217 (1997).

[19] L. Faddeev and R. Jackiw, Phys. Rev. Lett. **60**, 1692 (1988).

[20] E. Harikumar and M. Sivakumar, Phys. Rev. **D57**, 3794 (1998).

[21] N. Banerjee, R. Banerjee and S. Ghosh, Ann. Phys. **241**, 237 (1995).

[22] J. Barcelos-Neto and C. Wotzasek, Mod. Phys. Lett. **A7**, 1737 (1992).

[23] J. Barcelos-Neto and C. Wotzasek, Int. J. Phys. **A7** 4981 (1992).