IC/69/124
INTERNAL REPORT
(Limited distribution)

INTERNATIONAL ATOMIC ENERGY AGENCY

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

ALGEBRA OF PION SOURCES *

T. PRADHAN

International Centre for Theoretical Physics, Trieste, Italy,

and

Saha Institute of Nuclear Physics, Calcutta, India,

and

A.V. KHARE

Saha Institute of Nuclear Physics, Calcutta, India.

ABSTRACT

Assuming that the space of integrals of source terms (sources) in the Klein-Gordon equation for the pion fields together with isospin generators form an SU(2) $\otimes$ SU(2) algebra, which is a good symmetry of the strong interactions for hadrons at rest, we calculate odd isospin s-wave scattering lengths for the collision of massive pions with hadron targets as well as pion-nucleon coupling constant. The results are in good agreement with experiment.

MIRAMARE - TRIESTE

September 1969

* To be submitted for publication.

Many successful calculations [1] of pure strong interaction processes have been carried out during recent years with the aid of assumed $SU(2) \otimes SU(2)$ structure of vector and axial vector currents of hadrons. In all these calculations a vital role is played by the PCAC hypothesis which provides the necessary link between the axial vector current and strong interactions. This hypothesis, however, is not precisely defined [2] and may be the source of some of the failures [3] of this brand of algebraic approach to strong interaction physics. It would therefore be worthwhile to look for alternative approaches where the PCAC hypothesis would not be necessary at all for calculation of strong interaction parameters. It may be noted in this context that the scattering matrix for meson hadron collision is usually written in terms of retarded commutator of meson source densities. It is therefore natural to ask whether these meson sources, i.e., the space integrals of these densities, together with isospins, which are known to be constants of motion for strong interactions, do form a closed algebra. In this communication we discuss the consequences of an $SU(2) \otimes SU(2)$ algebraic structure of the isospin generators and pion sources. We show that predictions for scattering lengths and coupling constants can be made without any necessity of going to zero pion mass limit, provided we assume that for hadrons at rest this $SU(2) \otimes SU(2)$ is a good symmetry of the Hamiltonian for strong interactions. The results agree well with experiment for pion-nucleon scattering.

The invariant T-matrix for forward scattering of charged pions on a hadron target at rest can be written as

$$T^{\mp}(k) = -2iM \int d^4x\, e^{-ikx}\, \theta(x_0) \langle t | [j^{\pm}(x), j^{\mp}(0)] | t \rangle$$

$$-2iM \int d^4x\, e^{-ikx}\, \delta(x_0) \langle t | \left\{ [j^{\pm}(x), \dot{\phi}^{\mp}(0)] + ik_0 [j^{\pm}(x), \phi^{\mp}(0)] \right\} | t \rangle , \quad (1)$$

where $|t\rangle \equiv$ hadron state, $j^{\pm}(x) = (\Box - m^2)\,\phi^{\pm}(x)$, $\phi^{\mp}(x)$ being pion field operators, M is the target mass and k is the pion four momentum. The superscripts $\pm$ represent the charge of the pion. At threshold, i.e., for $\underset{\sim}{k} = 0$, this equation can be cast into the form

$$T^{\mp}(m) = -2iM \int_{-\infty}^{+\infty} dx_0\, e^{imx_0}\, \theta(x_0) \langle t | [P^{\pm}(x_0), P^{\mp}(0)] | t \rangle$$

$$-2iM \langle t | [P^{\pm}(0), \dot{\Phi}^{\mp}(0)] | t \rangle + 2Mm \langle t | [P^{\pm}(0), \Phi^{\mp}(0)] | t \rangle , \quad (2)$$

where $P^{\pm}(x_0) = \int d^3x\, j^{\pm}(x)$ is the pion source operator and $\Phi^{\pm}(x_0) = \int d^3x\, \phi^{\pm}(x)$. On writing

$$e^{imx_0} = \frac{1}{im} \frac{\partial}{\partial x_0} \left(e^{imx_0} \right) ,$$

and then performing integration by parts, eq. (2) becomes

$$T^{\mp}(m) = \pm \frac{2M}{m} \langle t | [P^{\pm}(0), P^{\mp}(0)] | t \rangle +$$

$$+ \frac{2M}{m} \int_{-\infty}^{+\infty} dx_0\, e^{imx_0}\, \theta(x_0) \langle t | [\dot{P}^{\pm}(x_0), P^{\mp}(0)] | t \rangle$$

$$-2iM \langle t | [P^{\pm}(0), \dot{\Phi}^{\mp}(0)] | t \rangle + 2Mm \langle t | [P^{\pm}(0), \Phi^{\mp}(0)] | t \rangle . \quad (3)$$

Further simplification of this result can be made if we assume that:

i) The operators $P^{\pm}$ and P^0 together with the isospin generators form an $SU(2) \otimes SU(2)$ group, i.e.,

$$[I^{\alpha}, I^{\beta}] = i\varepsilon_{\alpha\beta\gamma} I^{\gamma}; [I^{\alpha}, P^{\beta}(0)] = i\varepsilon_{\alpha\beta\gamma} P^{\gamma}(0); [P^{\alpha}(0), P^{\beta}(0)] = i\varepsilon_{\alpha\beta\gamma} P^{\gamma}(0). \quad (4)$$

ii) The pion field transformation law is

$$[P^{\alpha}(0), \Phi^{\beta}(0)] = \delta_{\alpha\beta} S \quad , \tag{5}$$

where S is an isosinglet operator which is a non-linear functional of the square of the pion field operator.

iii) The SU(2) $\otimes$ SU(2) defined in eq. (4) is a good symmetry of the strong interaction Hamiltonian for hadrons at rest, in which case $\dot{P}^{\pm}(x_0) = 0$.

The validity of these assumptions can be tested by comparing the consequences that follow from them with experiment. With these assumptions eq. (3) reduces to

$$T^{\mp}(m) = \pm \frac{2iM}{m} \langle t|2I_3|t\rangle + 2Mm \langle t|S|t\rangle \quad . \tag{6}$$

In obtaining this from eq. (3), Jacobi identity has been used to convert $\langle t|[P^{\pm}(0), \dot{\Phi}^{\mp}(0)]|t\rangle$ into $\langle t|[\dot{P}^{\pm}(0), \Phi^{\mp}(0)]|t\rangle$, which vanishes by virtue of assumption (iii). From eq. (6) we have

$$T_{odd}(m) = -\frac{2M}{m} \qquad T_{even}(m) = 2Mm \langle t|S|t\rangle \tag{7}$$

where T_{odd} and T_{even} are defined by the equation:

$$T^{\mp}(\omega) = T_{even}(\omega) \mp \langle t|2I_3|t\rangle \; T_{odd}(\omega) \tag{8}$$

On using the definition,

$$T(m) = 8\pi \left(1 + \frac{M}{m}\right) a \quad ,$$

for the dimensionless s-wave scattering length "a", we obtain from eq. (7),

$$a_{odd} = \frac{-1}{4\pi} \left(\frac{\mu}{m}\right) \quad , \quad a_{even} = \frac{1}{4\pi} \left(\frac{\mu}{m}\right) m^2 \langle t|S|t\rangle \quad , \tag{9}$$

where μ is the reduced mass of the pion and the target. Since $\langle t|S|t\rangle$

cannot be calculated without further dynamical input, we obtain from our $SU(2) \otimes SU(2)$ symmetry only the odd isospin scattering length. The results for nucleon, pion and kaon targets are given in Table I along with the corresponding results obtained from vector-axial vector algebra and the experimental results. It will be noticed that our scattering lengths are different from those of the V-A current algebra which are expressed in terms of weak interaction parameters. Our results do not contain such parameters because we work in terms of strong interaction operators alone. For πN scattering our value for the odd scattering length is about 20% below the experimental result whereas the V-A current algebra value is about 20% above. Although we cannot obtain any definite result for the even isospin scattering lengths, it is possible to make an estimate based on eq. (9). From dimensional considerations, $\langle t|S|t\rangle$ must be proportional to the inverse of the square of some mass. It is not unreasonable to take this mass to be the target mass. In that case

$$a_{\text{even}} \sim \left(\frac{m}{M}\right)^2 , \tag{10}$$

which has the same order of magnitude as the experimental s-wave pion-nucleon even scattering length. It should be noted here that V-A current algebra gives $a_{\text{even}} = 0$. It may be noted in this context that one goes to the soft pion limit in the V-A current algebra method. If we take such a zero pion mass limit of our result, we should also get vanishing value for the even scattering length, except in the pion-pion scattering case where the vanishing of this scattering length would follow only if the target pion mass is kept finite while letting the scattered pion mass go to zero.

A further application of our scheme can be made in the determination of the pion-nucleon coupling constant. This is done by combining eq. (7) with the unsubtracted dispersion relation

$$\mathrm{Re}\, T_{odd}(\omega) = - \frac{4M\omega f^2\left(1 - \frac{m^2}{4M^2}\right)}{\omega^2 - \left(\frac{m^2}{2M}\right)^2} +$$

$$\frac{2M\omega}{\pi} \int_m^\infty d\omega' \frac{\sqrt{\omega'^2 - m^2}}{\omega'^2 - \omega^2} \left(\sigma_{tot}^{p\pi^+}(\omega') - \sigma_{tot}^{p\pi^-}(\omega')\right) . \qquad (11)$$

This leads to the sum rule

$$1 = 2f^2 + \frac{m^2}{\pi} \int_m^\infty \frac{d\omega}{\sqrt{\omega^2 - m^2}} \left(\sigma_{tot}^{p\pi^-}(\omega) - \sigma^{p\pi^+}(\omega)\right) , \qquad (12)$$

from which a numerical evaluation of f^2 can be made. Using the results of Adler and Weisberger [4] for the integral over pion-nucleon total cross-sections we get

$$\frac{f^2}{4\pi} = 0.078 ,$$

which agrees fairly well with the result $\frac{f^2}{4\pi} = 0.081 \pm 0.002$ obtained by comparing forward dispersion relations with experiment [5].

The good agreement of our results with experiment implies that the SU(2) $\otimes$ SU(2) algebra of isospin and source operators of pions is a good symmetry of the strong interactions, at least for hadrons at rest. The source operators $P^{\pm}$ connect nucleons at rest with negative parity pion-nucleon resonant states in addition to the pion-nucleon continuum. Our assumption that the sources are time independent amounts to neglecting the contribution of the negative parity resonant states to threshold scattering. This sounds fairly reasonable in view of the high excitation energy of these resonant states.

Table I

TARGET	SCATTERING LENGTHS FROM ALGEBRA OF MESON SOURCES	SCATTERING LENGTHS FROM ALGEBRA OF V & A CURRENTS	EXPERIMENTAL VALUES
Nucleon	$a_{1/2} - a_{3/2} = \frac{3}{4\pi}\left(\frac{M_N}{M_N + m}\right) = 0.21$	$a_{1/2} - a_{3/2} = \frac{3g_V m^2}{2\pi f_\pi^2}\left(\frac{M_N}{M_N + m}\right) = 0.30$	$a_{1/2} - a_{3/2} = 0.26$
Kaon	$a_{1/2} - a_{3/2} = \frac{3}{4\pi}\left(\frac{M_N}{M_K + m}\right) = 0.18$	$a_{1/2} - a_{3/2} = \frac{3}{2\pi}\frac{g_V m^2}{f_\pi^2}\left(\frac{M_K}{M_K + m}\right) = 0.27$	—
Pion	$2a_0 - 5a_2 = \frac{3}{2\pi} = 0.49$	$2a_0 - 5a_2 = 0.70$	—

s-wave scattering lengths for collision of pions with N, K and π.

ACKNOWLEDGMENTS

One of us (TP) would like to thank Professor F.E. Low and other participants of the Summer School in Theoretical Physics held at Nainital, India, in June-July 1969, for their comments during the presentation of this topic in one of the seminars held there. He would also like to thank Professor G. Furlan of the University of Trieste for very helpful discussions on the subject during preparation of this manuscript. Finally he would like to take this opportunity to thank Professors Abdus Salam and P. Budini as well as the International Atomic Energy Agency for hospitality at the International Centre for Theoretical Physics, Trieste, where this work was completed. Thanks are also due to the Ford Foundation for making possible his Associateship at the ICTP.

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2) No precise meaning can be given to the smooth variation hypothesis of matrix elements of the pion source density with the square of momentum transfer required for the application of the PCAC equation. Further, there are ambiguities when matrix elements between multiparticle states are considered. See, for instance, S.L. Adler and R.F. Dashen, "Current Algebras and Applications to Particle Physics" (W.A. Benjamin, Inc., New York and Amsterdam 1968), Chapter 1. Besides, in the presence of electromagnetic and weak interactions, the PCAC equation itself gets altered. See, for instance, S.L. Adler, Phys. Rev. 177, 2426 (1969); S.R. Choudhury, Y. Tomozawa and Y.P. Yao, Michigan preprint (1969).

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