Segregation of granular materials in rotating cylinders

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Abstract

Mixtures of flowing granular materials containing particles of different sizes or different densities have a tendency to segregate. We focus here on the segregation that occurs in the cross-section of the cylinder using short cylinders (axial length is much smaller than the diameter) ensuring small variation along the cylinder axis. The typical structure formed in these systems is a radial core of the smaller or more dense particles. Coupling of composition with flow can lead to instability causing a pattern of *radial streaks*. The predictions of hard sphere mixture theories are briefly discussed first. Results of experimental and modelling studies of segregation are reviewed for density-induced radial segregation of equal sized particles, and size-induced radial streak formation. Experimental results for size-induced radial segregation are presented.

Keywords: Granular flow; Segregation; Rotating cylinder; Surface flow

1. Introduction

Granular segregation in rotating cylinders is undoubtedly of considerable practical significance. Rotary cylinders are used as kilns, mixers, dryers and granulators and segregation is often the cause of process and product defects. The rotary cylinder system is attractive for studying the basics of granular segregation in surface flows which can display remarkable complexity. For example, mixtures can spontaneously form regular layered patterns when poured to form a heap. Differences in particle shape (leading to differences in angle of repose) may be sufficient to produce these layered patterns [1].

The flow in rotating cylinders comprises a thin surface flowing layer with the remaining particles rotating as a fixed bed. We focus here on the segregation that occurs in the cross-section of the cylinder using short cylinders (axial length is much smaller than the diameter) ensuring small variation along the cylinder axis. Even in this case several outcomes are possible. The objective here is to review observed phenomena and models based on a simple continuum approach.

Rotation of a mixture of particles in a cylinder usually results in rapid *radial segregation* with the small (or high density) particles concentrating in a central core and the large (or low density) particles in the periphery [2–12]. However, segregation is never complete because of the diffusional mixing produced by interparticle collisions in the flowing layer. Nityanand et al. [4] showed that the pattern of radial segregation could reverse (small particles at the periphery and large particles in the core) at high rotational speeds in mixtures with different sized particles. More recently, Thomas [12] showed for mixtures with a few large particles that the large particles could concentrate at any radial position in the bed depending on the size ratio. Such double segregation was also reported by Dolgunin and Ukolov [13] and Thomas [12] in chute flow of mixtures.

The basic explanation for regular segregation was provided by Savage and Lun [14] and is geometric in origin. In densely flowing layers, small void spaces are more likely than large ones so that the small particles can preferentially drop into them. The upward flux to ensure a constant bulk density should be uniform for both types of particles resulting in a net downward flux of the small particles and a net upward flux of the large particles. Thomas [12] proposed that reverse segregation occurs when the large particles are massive enough to move down by pushing aside the small particles. The theory of hard sphere mixtures provides an alternate framework for analysis, but the theory is valid only for slightly inelastic spheres [15,16]. We review the predictions of this theory for the cases of mixtures of different density particles and of different size particles, with comparisons to computational results. Application of the hard sphere results for density radial segregation in a rotating cylinder are given. The conditions for an instability causing the formation of a pattern of *radial streaks* are discussed next along with a model. Finally, new experimental results for radial size segregation are presented.

2. Hard sphere theory

Jenkins and Mancini [15] have extended the theory of hard sphere mixtures [17] to the case of slightly inelastic spheres. The theory gives the complete set of balance equations and constitutive equations for flow and species transport for binary mixtures. The diffusion fluxes are of interest here, and may arise due to gradients in number fraction, pressure and temperature. The equations for the fluxes simplify considerably in the limit of an ideal gas, and are given below [18,19]. The pressure diffusion

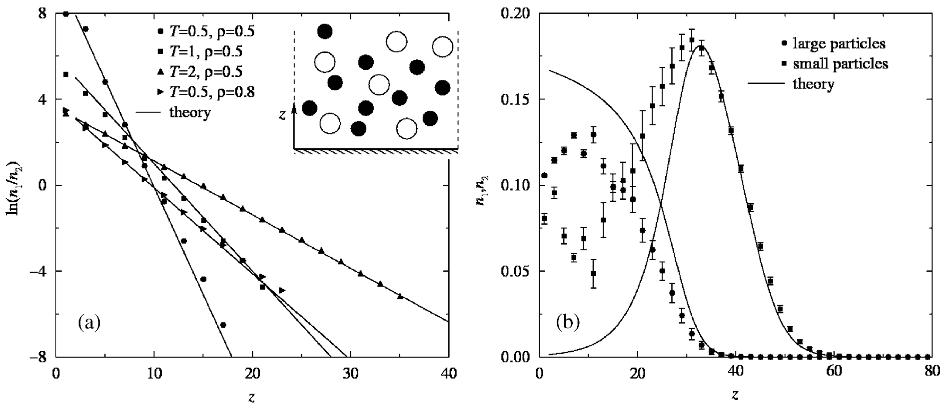

Fig. 1. Inset: Schematic view of the system: elastic hard spheres in a gravitational field at constant temperature. (a) Equilibrium number ratio profiles for equal size particles for different temperatures (T) and density ratios ($\bar{\rho}$). (b) Equilibrium dimensionless number density profiles for equal density particles. Points are the results of Monte Carlo simulations and solid lines are the predictions of kinetic theory [18].

flux is

$$\boldsymbol{j}_1^p = \frac{D_{12}\rho_1\rho_2}{\rho^2 T}(m_1 - m_2)\nabla p \,, \tag{1}$$

the temperature diffusion flux is

$$\boldsymbol{j}_1^T = -\frac{D_{12}\rho_1\rho_2 n^2}{\rho^2} K_T(m_1 - m_2)\nabla \ln T \,, \tag{2}$$

and the ordinary diffusion flux is

$$\boldsymbol{j}_1^f = -\frac{D_{12}m_1 m_2 n^2}{\rho}\nabla f \,, \tag{3}$$

where p, T and $f = n_{1/n}$ are pressure, temperature and number fraction, respectively. D_{12} is the binary diffusion coefficient while K_T is the thermal diffusion ratio. Particle mass is denoted by m, the mass density by ρ while n corresponds to the number density. Subscripts 1 and 2 denote the two different particle species. Thus, ordinary diffusion results in mixing, while pressure and temperature diffusion result in segregation of the species. From the above equations, it is clear that the heavier particles go to the higher pressure and lower temperature regions. At higher number densities, the constitutive equations obtained are similar in form but algebraically more complex [18].

Consider the case of an isothermal system comprising elastic particles in a gravitational field at equilibrium (Fig. 1, inset). In this case, the pressure gradient is $\partial p/\partial z = -\rho g$. Suppose that bulk density is low enough for the mixture to behave as an ideal gas. The balance between the pressure and ordinary diffusion fluxes then gives

$$\ln\left(\frac{n_1}{n_2}\right) = \ln\left(\frac{n_{10}}{n_{20}}\right) - \frac{(m_1 - m_2)g(z - z_0)}{T} \,. \tag{4}$$

It can be shown that a similar equation is obtained for particles of equal size which is valid for all bulk densities [18].

Fig. 1a shows the comparison between predictions of Eq. (4) to computational results from Monte Carlo simulations for equal size particles of different density. There is good agreement between the two for a wide range of temperatures. Similar agreement is obtained for frictional and elastic particles in chute flow but only when a fitted "effective" temperature is used [18]. These results suggest that the form of the diffusion flux given above may be reasonable for real systems with equal size particles but the temperature should be taken to be a fitted parameter. The case of mixtures of different sized particles is more complex. Fig. 1b shows a comparison between the predictions of the full theory to Monte Carlo simulation results for elastic particles in a gravitational field. Although the predictions in the low-density regions are reasonable, at higher densities there are significant deviations. Most notably, the theory does not show the reversal in the number density profiles seen in the Monte Carlo results. The Monte Carlo results indicate that there is a small region of regular segregation (small particles moving down) and a larger region of reverse segregation (small particles moving up). In real systems, the region of regular segregation is much larger because of the high bulk density of the flowing layer.

3. Density segregation

The hard sphere results are applied to analyse the segregation of equal size but different density particles in a rotating cylinder. The convective diffusion equation describing the variation of concentration of the high-density particles in the flowing layer is

$$\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + v_y \frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(D \frac{\partial f}{\partial y} - \gamma_s \frac{D}{R} f(1-f) \right) , \tag{5}$$

where D is the diffusion coefficient, R is the cylinder radius and γ_s is the characteristic segregation velocity. The flow is nearly unaffected by particle density, hence the velocity field (v_x, v_y) for the pure component system [20] can be used. A correlation for D is used and γ_s is treated as a fitting parameter [8].

Fig. 2a shows a comparison of the model predictions to experimental results for mixtures of equal sized steel balls and glass beads. There is good agreement between theory and experiment. The dynamics of segregation are also reasonably well predicted by the model. Fig. 2b shows the variation of the intensity of segregation (I_s, standard deviation of the concentration from the mean over the cross-section of the bed) with cylinder revolutions, for an initially segregated system. When the cylinder is less than $\frac{1}{2}$ filled ($\bar{H} = 0.45$) there is an optimal time of mixing at which the best mixing is achieved. This is because of fast initial mixing followed by segregation, and the behaviour is described by the model.

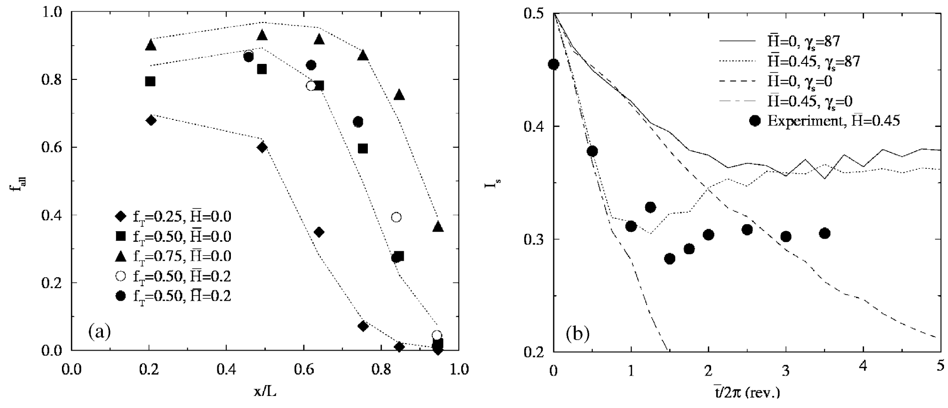

Fig. 2. (a) Equilibrium number fraction of steel balls in the bed (f_{all}) versus dimensionless distance along the free surface (x/L) obtained for mixtures of steel balls and glass of different compositions (f_T). $\bar{H}$ denotes the distance of the free surface from the cylinder axis, and lines show the corresponding model predictions. (b) Variation of the intensity of segregation (I_s) with cylinder revolutions of an initially segregated mixture for different extents of filling of the cylinder. Symbols are experimental measurements and lines are predictions [8].

4. Size segregation—streak formation

The most notable feature of the flow of mixtures of different sized particles is that the flow is significantly dependent on composition. Increase in the concentration of small particles results in faster flow. We highlight this here by means of an example—the radial streak formation process in a size segregating system. Radial streaks are found to form for slightly more than half-filled cylinders at low rotational speeds [21]. The instability is facilitated by large size differences between the particles. Starting with an initial configuration that is radially segregated, upon rotation the boundary between the small and large particles becomes wavy, the bumps grow and eventually the fingers reach the periphery of the cylinder (Fig. 3).

A model that describes the above behaviour has several elements and assumptions [22]. For the purpose of calculating the mean velocity in the layer, the particles are assumed to be completely segregated with the small particles at the bottom. The velocity profile is assumed to be piecewise linear with the velocity gradient inversely proportional to the square of the particle diameter. Further, to take into account the variation of the surface angle with the composition of the flowing layer in a simple way, each half of the free surface is allowed to pivot about a point. The depth averaged, unsteady mass, momentum and species balance equations for the layer are solved simultaneously to obtain the dynamic variation of the flow and concentration in the system. The variation of the bed composition as a result of inflow and outflow from the layer is also computed. Streak formation starts with a slight excess of small particles in the layer, due to a small bump entering the layer from the bed. This speeds up the flow in the layer causing the bump to become longer.

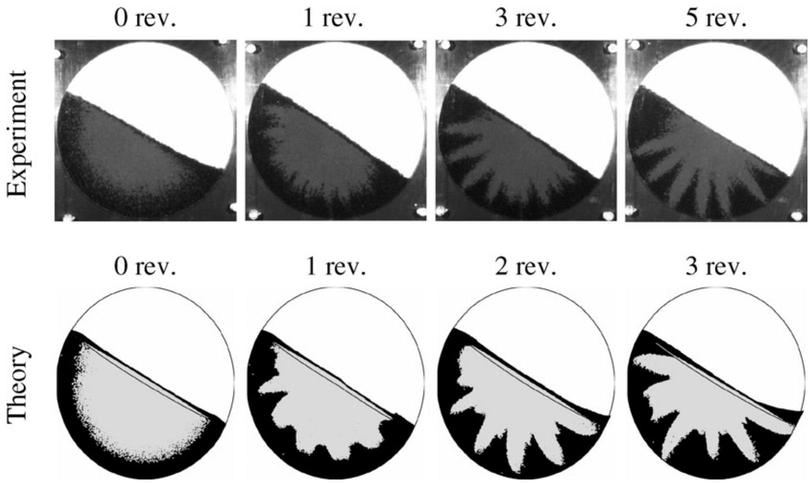

Fig. 3. Experimental and computational results showing the dynamics of streak formation at a rotational speed of 0.75 rpm. A 50% vol/vol mixture of 3 mm (dark) and 1 mm (light) glass beads is used for the experiments [22].

Since the cylinder is nearly half-full a streak once formed enters the flowing layer all at once reinforcing the instability. Further, because the surface angle is smaller for the small particles, the surface angle reduces as small particles enter the layer and this increases the rate of inflow of particles from the bed into the layer. Thus, the rate at which the streak enters the layer is increased by this pumping action. We found that streaks are not formed if pumping is switched off. The variation of the concentration in the layer results in oscillations of the free surface, each oscillation corresponding to the passage of a streak through the layer. The dynamics of the streak formation is qualitatively predicted by the model (Fig. 3). The suppression of streaks at high rotational speeds and at lower fillings is also predicted by the model [22].

5. Size radial segregation

Experiments to obtain the radial concentration profile were carried out in a quasi-2d cylinder of radius 16 cm and length 1 cm, driven by a computer-controlled stepper motor. Mixtures of monodisperse and highly spherical steel balls of different sizes were used. In all the experiments, the cylinder was filled 50% by volume with mixtures of specified compositions and rotated at different speeds for 150 revolutions to achieve a steady state. To measure the bed profile, the cylinder was removed from the drive, placed on an inclined surface and the top face plate was replaced by a sampling plate with twenty-three 10 mm holes at 13 different radial positions. The volume of material sampled was precisely controlled and relatively small, and the sample was sieved to

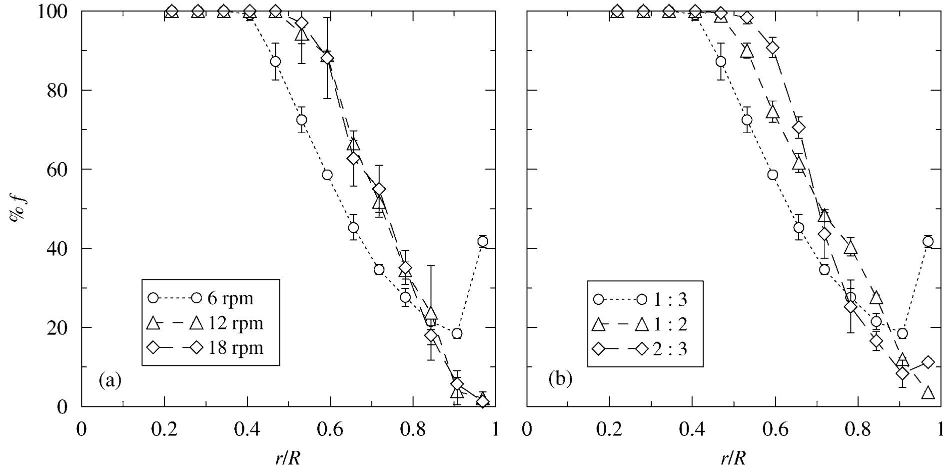

Fig. 4. Variation of the percentage weight fraction of small particles (f) with scaled radial distance (r/R) for (a) mixture of 1 and 3 mm particles at different rotational speeds and (b) mixture of particles for three different size ratios at a rotational speed of 6 rpm.

obtain the weight fraction of the small particles (f). Each experiment was repeated at least 3 times.

Fig. 4a shows the variation of the weight fraction of the small particles with radial distance in the bed for different rotational speeds. At the lowest rotational speed, significant reverse segregation is observed. At higher rotational speeds, reverse segregation is suppressed and there is little effect of rotational speed on the profile. This trend is the reverse of that observed by Nityanand et al. [4] who found reverse segregation at high rotational speeds. Fig. 4b shows the effect of size ratio on the profiles. At size ratios closer to unity, reverse segregation is reduced. Surprisingly, the slope of the graph for a mixture of 2 and 3 mm particles is higher, and thus the extent of segregation is the highest in this case. The effect of size ratio observed is very different from the predictions of the model of Prigozhin and Kalman [10]. The observed behaviour is not predicted by hard sphere theory either.

6. Conclusions

We reviewed the range of phenomena observed in the segregation of particle mixtures in the cross-section of a rotating cylinder. Both density and size differences are considered. A review of the predictions of the hard sphere theory for diffusion fluxes shows that density segregation is well predicted by hard sphere theory provided that the temperature is taken to be fitting parameter. Hard sphere theory, however, gives qualitatively different results for mixtures of different size particles. The segregation flux expression obtained from hard sphere theory is shown to give reasonable predictions for density radial segregation in a rotating cylinder. In the case of different size particles at low rotation speeds, streak formation occurs. A model based on dynamic variation of flow, composition and surface angle is shown to give a qualitative description of the process. The radially segregated profiles for mixtures of different size particles reveal double segregation and a complex dependence on size ratio. Hard sphere theory does not predict the observed behaviour in this case and new models are needed.

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