Pramāṇa, Vol. 22, Nos 3 & 4, March & April 1984, pp. 275–282. © Printed in India.

Fusion-fission angular distributions: A new probe of fast fission fractionation in nucleus-nucleus collisions

S S KAPOOR, V S RAMAMURTHY and R RAMANNA
Bhabha Atomic Research Centre, Trombay, Bombay 400 085, India

Abstract. Fragment angular distributions in heavy ion-induced fission reactions have been analysed in terms of a two component model—fission following compound nucleus formation and fast fission events. It is seen that, contrary to the general assumption, fast fission competes with compound nucleus fission even when the composite system is formed with a spin less than the rotating liquid drop model limit for vanishing fission barrier.

Keywords. Fusion-fission angular distribution; fast fission; nucleus-nucleus collisions; fragment angular distribution.

1. Introduction

The availability of a number of heavy-ion accelerators in the last two decades has allowed studies of nuclear reactions induced by a variety of heavy ion projectiles spanning a wide range of collision energies. It is now well realized that nuclear reactions induced by heavy ions of energies up to 10–20 MeV per nucleon open up new possibilities for the study of the dynamical behaviour of bulk nuclear matter. In heavy-ion reactions, the reaction dynamics involves a large scale rearrangement of nuclear matter with the result that the whole domain of nuclear evolution during this rearrangement is open to studies. Nuclear reactions induced by heavy ions also involve transfer of large angular momenta. In some cases the angular momentum can be sufficiently large to cross the limiting value at which the intermediate composite nuclear system loses stability against fast binary split (Cohen *et al* 1974). This allows a study of the formation, further evolution and decay of the composite nuclear complexes formed with such large angular momenta. The time scales involved in heavy ion collisions are comparable to the characteristic times for equilibration in several degrees of freedom such as the mass-asymmetry, energy dissipation, angular momentum transfer etc which are known to evolve primarily through the nucleon exchange process. The study of nucleus-nucleus collisions thus provides a way to learn about the nuclear equilibration processes in several degrees of freedom of much physical significance.

The reactions resulting from nucleus-nucleus collisions at medium collision energies can be broadly classified into four types: elastic (and quasi-elastic) reactions, dinucleus reactions involving deep-inelastic collisions (DIC), non-compound nucleus fusion reactions (mono-nucleus intermediate configuration) and true compound nucleus reactions. Peripheral collisions with little overlap of the matter densities of the target and the projectile result in elastic, quasi-elastic and transfer reactions. A somewhat more penetrating collision results in the now well-known deep-inelastic collisions, where dissipation of hundreds of MeV of relative kinetic energy and transfer of angular momentum on a massive scale take place in a brief interaction time of the order of 10^{-21} sec through the mechanism of the nucleon exchange process (see for example; Schröder and Huizenga 1977; Schröder *et al* 1980; Kapoor and De 1982; De and Kapoor 1983). Fully damped deep inelastic binary collisions have also been called 'quasi-fission" reaction as the fragment kinetic distributions are similar to those in compound nucleus fission. In deep inelastic collisions fragment angular distributions are characteristic of direct reactions and the average fragment masses are not much different from those in the entrance channel. In contrast, the "complete-fusion" reactions are those reactions in which the composite system disintegrates only after achieving complete equilibration in the mass degree of freedom and after losing the memory of the reacting masses. Only if a complete fusion reaction also achieves equilibration in all other degrees of freedom before disintegrating, the reaction is identified as a compound nucleus reaction. In a number of reactions one observes a predominant symmetric peak of the fragment mass distributions with such a large cross-section that the number of l-values which are involved must include some l-values which cross the limiting spin $J_{B_F=0}$ for zero fission barrier as calculated from the rotating liquid drop model (RLDM). It is clear that in these cases formation of a true compound nucleus is not expected to be achieved, as the system will undergo fission decay in a very short time due to the absence of a fission barrier and will not live long enough to achieve equilibration in all the degrees of freedom. The term "fast fission" has been suggested for this type of reaction (Gregoire *et al* 1981).

In a simplified picture, one usually associates the different types of reactions with limiting l-values as shown in figure 1. l_{cr} is the limiting l-value only below which a "complete-fusion" reaction is possible. (As the value of l becomes larger than l_{cr}, one progressively enters the regions of deep inelastic reactions, quasi-elastic and elastic reactions). The other limiting value $J_{B_F=0}$ corresponds to that l-value for which there is no energy barrier against instantaneous binary split of the system. The fast fission process has been thought to be confined to the region of l-band defined by $l_{cr} < l < J_{B_F=0}$ and it is believed that for $l < J_{B_F=0}$ the system has sufficient time to equilibrate into a compound nucleus before undergoing subsequent decay. Experimentally, while

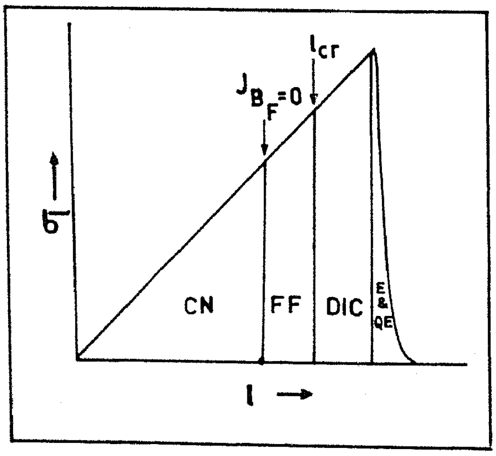

Figure 1. Decomposition of the total reaction cross section into cross sections for elastic (E) and quasielastic (QE) collisions, deep inelastic collisions (DIC), fast fission (FF) and compound nucleus formation (CN). l_{cr} is the critical angular momentum for fusion and $J_{B_F=0}$ is the rotating liquid drop model limit for vanishing fission barrier.

the symmetric peak in the fission fragment mass distribution can be identified with the complete fusion events (consisting of both the fast fission and compound nucleus fission events), identification of the fast fission events can only be achieved by examining deviations from the other predicted characteristics of fission process following compound nucleus formation. Study of the fragment angular distributions provides one such convenient way.

In the following sections, we demonstrate that important information regarding fast fission fractionation can be deduced from measured fusion-fission fragment angular distribution data. An important conclusion of the present analysis is that the fast-fission is not restricted to barrierless fission corresponding to partial waves with $l > J_{B_F=0}$ but also competes with compound nucleus fission in the presence of small barrier heights, that is, for some values of l which are smaller than $J_{B_F=0}$.

2. Fission-fragment angular distributions as signatures of fast fission phenomenon

It is well known from studies of fission induced by nucleons and light ions that if the compound nucleus resulting from the fusion of the target and the projectile carries some spin aligned with respect to the incident beam direction, the fragments arising from the fission of the compound nucleus exhibit an angular anisotropy with respect to the beam direction. The magnitude of the anisotropy depends both on the spin of the fissioning nucleus and on the distribution of quantum states of its fission transition state. Studies of fission fragment angular distributions in heavy ion-induced reactions therefore promise to be a sensitive probe of the rather large angular momentum transfer in these reactions if the fission transition state properties are known from nucleon and light ion-induced reactions. It has been indeed found (Choudhury *et al* 1980) that for a certain range of target-projectile combinations and bombarding energies, the angular momentum distributions deduced from fission fragment angular distributions and from total fusion cross-sections are mutually consistent. However, recently, anamolously large fragment anisotropies have been measured (Back *et al* 1983; Tsang *et al* 1983; Lesko *et al* 1983) for a number of other cases of target-projectile-high composite system spin combinations which cannot be understood in terms of the conventional theories of fission fragment angular distributions. This paper shows that from these interesting departures from theory, one can deduce the l-dependence of the fast fission fractionation taking place in competition with the dynamical evolution of the system towards a fully equilibrated compound nucleus. We first discuss below the angular distributions expected for fissions following compound nucleus formation and for fast fission events. It is then shown how quantitative information about fast-fission fractionation can be deduced from the analysis of the fragment angular distribution data.

2.1 *Fragment angular distributions in fission following compound nucleus formation*

The basic ingredients of the conventional theory of fragment angular distributions in fission following compound nucleus formation are the Bohr hypothesis (Bohr 1955) of well-defined quantum states of the fission transition state nucleus and Halpern-Strutinsky (Halpern and Strutinsky 1958) statistical theory of the distribution of these states. If J is the total angular momentum of the fissioning nucleus, M its projection on a space-fixed axis, usually taken as the beam direction and K its projection on the symmetry axis, the separation direction with respect to the space-fixed axis is given by the probability distribution function,

$$P^J_{MK}(\theta) = \frac{2J+1}{2}\left|d^J_{MK}(\theta)\right|^2, \tag{1}$$

where θ is the angle between the separation direction and the space-fixed axis and $d^J_{MK}(\theta)$ are the rotational wavefunctions. The final angular distributions of the fission fragments are obtained by summing over all possible values of the three quantum numbers. While the distributions of J and M are decided by the reaction mechanism, the distribution of K is obtained on the basis of statistical arguments. In the past, one always considered a fixed RLDM transition state shape for a given fissionability parameter x and spin J. But in a recent work (Prakash *et al* 1983) it was shown that for compound nuclei with large spin, it is necessary to include the dependence of transition state shape also on the quantum number K, and this model has been termed the flexible rotor model. Although the conclusions of the present work are not dependent on whether one uses flexible rotor model or the rigid rotor model in the analysis of the fragment angular distributions in fission following compound nucleus formation, in the present work we have employed the more rigorous flexible rotor model.

2.2 *Fragment angular distributions in fast fission*

There exists at present no rigorous theory of fragment angular distributions for fast fission events. However, in some recent experiments (Lesko *et al* 1983) involving very high spin and large values of Z^2/A of the composite system where the compound nucleus formation probability is very small, highly anisotropic angular distributions have been measured which are characteristic of a rather narrow distribution of K states around $K = 0$. Qualitatively, such a narrow distribution of K states is not unexpected since in the entrance channel, the dinuclear complex is formed with $K = 0$ (the angular momentum is aligned at right angles to the line joining the centres of mass of the target and the projectile nuclei). If, in the fast fission process, the exit channel maintains the same $K = 0$ configuration, a $1/\sin\theta$ type of angular distribution would be expected. However, misalignment in K can be introduced by such factors as nucleon exchanges between the target and the projectile in the entrance channel and between the separating nascent fragments in the exit channel, and the fragment anisotropy may get a little diluted from the highly anisotropic $1/\sin\theta$ distribution for fast-fission fragments. To take into account this effect, we characterize the effective K-distribution for fast fission events by a variance

$$\sigma_K^2 = \beta J^2, \tag{2}$$

where β is a parameter determined by the nucleon-exchange process and is a measure of the angular misalignment of the fission axis with respect to the line joining the centres of mass of the target and projectile nuclei.

3. Comparison with data

Based on the above discussions of fragment angular distributions in fission following compound nucleus formation and fast-fission events and the discussions presented in §1 regarding the angular momentum limits for compound nucleus formation and fast fission, it is possible to calculate the fragment angular distributions for any target-projectile combination at any bombarding energy. Figure 2 shows the measured angular distributions for the system $^{19}\mathrm{F}+{}^{208}\mathrm{Pb}$ at the three bombarding energies 110 MeV, 150 MeV and 190 MeV. The corresponding values as deduced from the measured fusion-fission cross-sections are $42\hbar$, $71\hbar$ and $93\hbar$ respectively. For this system, the rotating liquid drop model limit for zero fission barrier $J_{B_F=0}$ is $75\hbar$. Thus the experimental data span an interesting angular momentum range covering values of l both lower as well as higher than $J_{B_F=0}$. On the basis of the discussions presented above one would have expected that for the cases of bombarding energies of 110 MeV and 150 MeV, the measured angular distributions should be in agreement with the predictions of the statistical theory applicable to fission following compound nucleus formation. Only for the case of bombarding energy of 190 MeV, where some fast fission events are also involved, the measured anisotropy should be larger than that expected for the compound nucleus-fission. Although qualitatively such a behaviour is indeed apparent in figure 2, we find some deviations from the predictions of the statistical theory even at the lowest bombarding energy. Figure 3 shows a few other cases of the measured angular distributions where the bombarding energies are such that in all the cases, the critical angular momentum is much less than the RLDM limit for vanishing

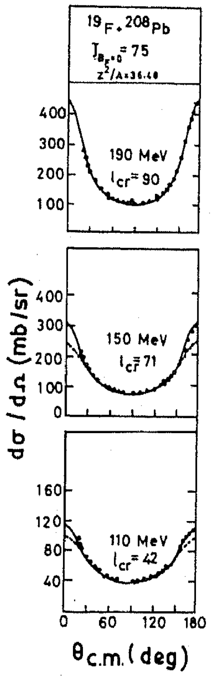

Figure 2. Differential fusion-fission fragment angular distributions for the $^{19}\mathrm{F}+{}^{208}\mathrm{Pb}$ reaction at bombarding energies 110 MeV, 150 MeV and 190 MeV. The experimental points are from Tsang *et al* (1983). The dashed line is the calculated fragment angular distribution based on the statistical theory for compound nucleus fission based on the flexible rotor model. The continuous curve is the result of the present calculations.

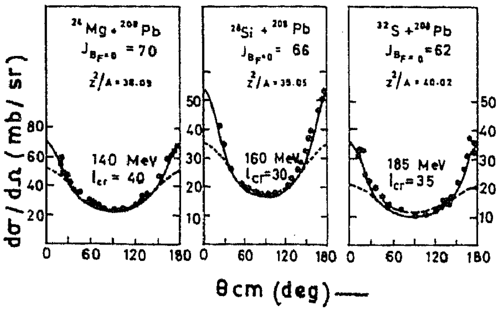

Figure 3. Differential fusion-fission fragment angular distributions for three systems ^{24}Mg $+ {}^{208}$Pb at 140 MeV, ^{28}Si $+ {}^{208}$Pb at 160 MeV and ^{32}S $+ {}^{208}$Pb at 185 MeV. The experimental points are from Tsang *et al* (1983) and Back *et al* (1983). The continuous and the dashed lines have the same meanings as in figure 2.

fission barrier, $J_{B_F=0}$. In these cases, the measured anisotropies are considerably larger than the calculated ones for compound nucleus fission. An interesting trend which can be noticed in figure 3 is that the deviations between the experimental results and the statistical theory predictions increase with increasing Z^2/A of the fissioning system. For the statistical theory calculations, we have used the liquid drop model (LDM) parameters of Pauli and Ledergerber (1971) which were arrived at by them to reproduce the systematics of the LDM fission barrier heights for actinide nuclei. Though the use of other sets of LDM parameters will result in slightly different angular distributions (5–10% change in the anisotropy values), the differences are not large enough to remove the aforesaid disagreement. We therefore conjecture that the assumption usually made that fast fission occurs only for partial waves $l > J_{B_F=0}$ is not valid; the fast fission competes with compound nucleus fission even for l-values less than $J_{B_F=0}$ and this competition is more for fissioning systems with higher values of the fissility parameter x (or Z^2/A).

To discuss the relative probabilities of fast fission and compound nucleus formation, we consider the collisions of target and projectile nuclei with angular momentum l in the entrance channel. Let l be less than both the critical angular momentum l_{cr} and the RLDM limit $J_{B_F=0}$ for vanishing fission barrier. Since $l < l_{\text{cr}}$, the system will fuse to form a composite system. This composite system at the time of formation is predominantly in a $K = 0$ configuration, with the spin J ($= l$ neglecting the spins of the target and the projectile) in a direction perpendicular to the line joining the centres of mass of the original target and projectile nuclei. Since also $l < J_{B_F=0}$, there exists a barrier for instantaneous binary split (fast fission) and the system is expected to relax in the shape and orientation degrees of freedom and a fully equilibrated compound nucleus is expected to be formed. In this paper, it is pointed out that this assumption is a valid one only if the barrier heights are large since for fission barrier heights comparable to the intrinsic temperature of the system, the fast fission process will compete with compound nucleus formation. Both an increasing spin and an increasing fissility (or Z^2/A) of the composite system tend to decrease the fission barriers and one expects to see increasing effects of this competition in the measured quantities such as the angular distribution data. In the absence of an exact dynamical treatment of the competition between fast fission and compound nucleus formation, we propose the following simple semiempirical expression for the probability for fast fission having the right asymptotic properties:

$$P_{FF}(J) = \exp[-\alpha B_F(J, K = 0)/T], \qquad (3)$$
$$P_{CN}(J) = 1 - P_{FF}(J)$$

where B_F is the fission barrier height for the composite system with spin J and $K = 0, T$ is the temperature and α is a parameter. It should be noted that $B_F(J, K = 0)$ is slightly different from the RLDM value of the fission barrier since in the latter case the barrier height is measured from the equilibrium ground state configuration with $K = J$. For small values of the fissility parameter (or Z^2/A) and low spin, $B_F(J, K = 0) \gg T$ and $P_{FF}(J) = 0$. On the other hand, for large values of Z^2/A and large spin such that $B_F(J, K = 0) \ll$ one has $P_{FF}(J) = 1$. In the limit $J > J_{B_F=0}$ when $B_F(J, K = 0) = 0$ one has $P_{FF}(J) = 1$. Thus the above simple expression for the probability of fast fission brings out the main features of the competition between fast fission and compound nucleus formation.

We have fitted the experimental data on the fragment angular distributions in terms of the two components—fission following compound nucleus formation and the fast fission—their relative probabilities being given by (3) and each having its own characteristic angular distributions as discussed earlier. These calculated angular distributions are also shown in figures 2 and 3 as solid lines. It was found that all the data of figures 2 and 3 can be fitted with a single set of the parameters corresponding to

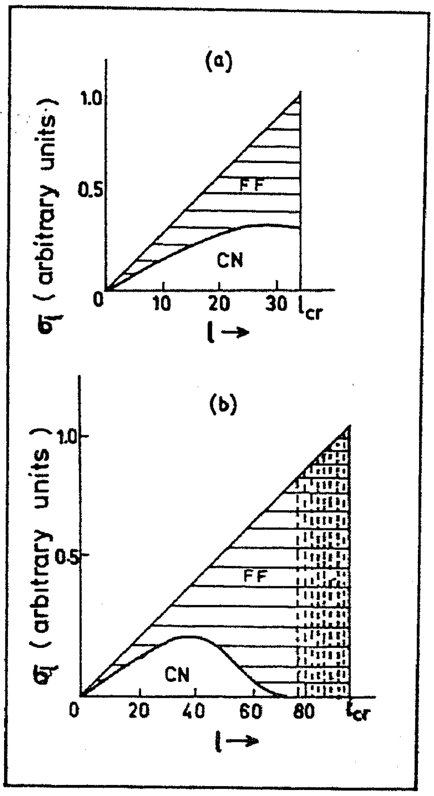

Figure 4. Decomposition of the total fusion cross-section into fast fission and compound nucleus fission according to the present analysis: **(a)** $^{32}S + ^{208}Pb$ at 185 MeV. **(b)** $^{19}F + ^{208}Pb$ at 190 MeV, the vertical lines define the area predicted to result in fast fission as discussed in §1. Horizontal lines in both the figures define the area which results in fast fission according to the present analysis.

$\beta = 0{\cdot}06$ and $\alpha = 0{\cdot}5$. The fractionation of fast fission and compound nucleus fission versus l as deduced from the above analysis for the typical cases of $^{32}S + ^{208}Pb$ at a bombarding energy of 185 MeV and $^{19}F + ^{208}Pb$ at 190 MeV are shown in figures 4a and 4b. It may be noted that on the basis of the usual assumptions made (as depicted in figure 1), no fast fission was expected in the case shown in figure 4a while fast fission was expected only in the region of vertical dashed lines in the case shown in figure 4b. Thus, contrary to the usual belief, the present analysis demonstrates that fast fission competes with compound nucleus formation over a significant range of l values, much below $J_{B_F=0}$. Taking into account this feature, the measured fragment angular distributions can be quantitatively accounted for in heavy ion reactions spanning a large range of Z^2/A values and bombarding energies. Although in this paper, we have not discussed the implications of the precise numerical values of the parameters α and β, it is nevertheless clear that a detailed examination of these values can bring out important dynamical features of the fusion process. Therefore an extension of this study would be to investigate the dynamics of equilibration leading to the formation of the compound nucleus, in particular, the time of equilibration in the K-degree of freedom in relation to characteristic times in the fission degrees of freedom.

4. Summary and conclusion

It is shown that information regarding the competition between fast fission and compound nucleus formation can be obtained from an analysis of fragment angular distribution data in heavy ion-induced fission reactions. With a simple parametrization of the fast fission fraction, data over a range of bombarding energies and for a number of projectiles are quantitatively fitted. The angular momentum dependence of the fast fission events has been deduced for some typical cases and is found to be different from the usual assumption of an l-window.

References

Back B B, Betts R R, Cassidy K, Glagola B G, Gindler J E, Glendenin L E and Wilkins B D 1983 *Phys. Rev. Lett.* **50** 818

Bohr A 1955 *Proc. Int. Conf. on the Peaceful uses of atomic energy, Geneva* (New York: United Nations) Vol. 2 p. 131

Choudhury R K, Govil R and Kapoor S S 1980 *Phys. Rev.* **C22** 1360

Cohen S, Plasil F and Swiatecki W J 1974 *Ann. Phys. (New York)* **82** 557

De J N and Kapoor S S 1983 *Phys. Rev.* **C27** 1928

Gregoire C, Ngo C and Remand B 1981 *Phys. Lett.* **B99** 17

Halpern I and Strutinski V M 1958 *Proc. Int. Conf. on the Peaceful uses of atomic energy, Geneva* (New York: United Nations) Vol. 15 p. 408

Kapoor S S and De J N 1982 *Phys. Rev.* **C26** 172

Lesko K T, Gil S, Laggarini A, Metag V, Seamster A G and Vandenbosch R 1983 *Phys. Rev.* **C27** 2999

Prakash M, Ramamurthy V S, Kapoor S S and Alexander J M 1983 (Preprint)

Pauli H C and Ledergerber T 1971 *Phys. Rev.* **A175** 545

Schröder W U, Birkelund J R, Huizenga J R, Wilcke W W and Randrup J 1980 *Phys. Rev. Lett.* **44** 308

Schröder W U and Huizenga J R 1977 *Annu. Rev. Nucl. Sci.* **27** 465

Tsang M B *et al* 1983 *Phys. Lett.* **129** 18