Studies in nuclear macrophysics through fission and fission-like reactions

S S KAPOOR
Bhabha Atomic Research Centre, Trombay, Bombay 400085, India.

Abstract. Recent developments in the study of fission and fission-like reactions are briefly reviewed. After a brief introduction of some of the important features of the fission process, binary fission and fission-like processes in heavy ion-induced reactions are discussed. It is shown that studies of the fission fragment angular distributions which provide a way to determine relative contributions of compound nucleus fission and non-equilibrium fission-like events in heavy ion-induced fission have proved to be quite valuable in investigating the very short K-equilibration times of the order of $10^{-20}$ s involved in the nuclear dynamics of the dinuclear complex on its way to compound nucleus formation following nucleus-nucleus collision.

Keywords. Nuclear fission; fission-like reactions; fragment angular distributions; $K$-equilibration time.

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1. Introduction

While the discovery of the neutron in 1932 revealed the composition of the atomic nucleus, the discovery of fission in 1939 not only provided an experimental verification of one of the most fundamental equations ($E = MC^2$) of modern physics but also paved the way for large scale national funding for nuclear research due to realization of the energy potential of the tiny but mighty nucleus. Subsequent years had witnessed extensive studies of the atomic nucleus which showed apparent contradictions in its behaviour fitting into the general pattern of duality so characteristic of the twentieth century physics. While nuclear physics took a giant leap forward with the great success of the nuclear shell model based on independent particle motion, the bulk nuclear behaviour involving vibrations, rotations and stable nuclear deformation reflected the presence of co-operative phenomenon in nuclei.

A great deal about the nuclear structure was already learnt in the early years of nuclear physics research. The low and medium energy accelerators which were constructed in the fifties and sixties became powerful tools for the studies of nuclear spectroscopy through light and heavy ion reactions. In the early years, study of the fission process was the only means available to investigate large scale nuclear motion. However, in recent years, with the availability of heavy ion accelerators capable of accelerating a variety of heavy ions to energies up to about 10 MeV/A, it became possible to study the dynamics of nucleus-nucleus collisions in greater detail and in a more systematic way. As a result, over the years, the studies of the fission process and nucleus-nucleus collisions through the heavy ion reactions have led to the development of a new area of nuclear physics—the nuclear macrophysics—dealing with bulk nuclear behaviour. In this paper, some recent studies in this area dealing with the study of fission and fission-like reactions are briefly reviewed.
2. Fission phenomenon at a glance

Nuclear fission which involves massive motion and subsequent division of a nucleus, resulting in drastic rearrangement of one nucleus into two nuclei, is a highly complex nuclear reaction. The discovery of nuclear fission came as a big surprise, as it was very difficult to imagine that a uranium nucleus with a total binding energy of about 2000 MeV, could be split into two parts by the capture of a slow neutron. At first sight, it did appear that a giant rock has fallen apart by a feather touch, but the original ideas of Bohr and Wheeler (1939) putting forward the compound nucleus hypothesis and the liquid drop model in the classic paper of 1939, soon provided an acceptable explanation. All these years the liquid drop model (LDM) has provided a very useful theoretical framework to understand and explain many gross features of the fission process.

On the energetic consideration alone, spontaneous fission is possible for all nuclei with $A \geq 120$, as the splitting of these nuclei in two parts will result in a release of energy. But we now know that it is the presence of a fission barrier that keeps these nuclei stable. The fission process is basically governed by the characteristic features of the fission barrier present in the map of the potential energy versus deformation of a nucleus. Figure 1 taken from a recent review (Bjornholm and Lynn 1980) shows a schematic diagram of the potential energy contour of a nucleus as a function of the quadrupole and hexadecapole deformation parameters calculated from the simple liquid drop model. The fission barrier height, which is the maximum in the potential energy along the minimum energy trajectory for increasing nuclear elongation, depends on the fissionability parameter $X$, which is the ratio of Coulomb energy $E_{C}$ to twice the surface energy $E_{S}$ of the spherical nucleus. As shown schematically in figure 2, LDM predicts a rapid decrease of fission barrier heights with increasing

![Figure 1. Liquid drop model potential energy contour of a fissioning nucleus as a function of quadrupole and hexadecapole deformation parameters. Potential energy versus deformation along the fission direction is also shown (taken from Bjornholm and Lynn 1980).](image-url)
value of \( \frac{Z^2}{A} \). In the spontaneous fission process, an average behaviour showing an exponential decrease of the fission half-lives with increasing \( \frac{Z^2}{A} \) is indeed observed. Thus, to a first order, the general systematics of the observed spontaneous fission half-lives follow the LDM prediction.

But the liquid drop model is capable of providing only an approximate description of the process and fails to account for several of its detailed features. The most notable failure of the LDM has been its inability to explain the asymmetric mass distributions in fission. Additional new observations in fission which were discovered during the sixties and which could not be explained on the basis of liquid drop model were the existence of fission isomers and sub-barrier resonances. As is now well-known, incorporation of nuclear shell effects in the calculations of nuclear potential energies and, in particular, the concept of deformed nuclear shells introduced by Strutinsky (1967) has led to important advances in the fission theory. Strutinsky’s macroscopic-microscopic method of calculation of shell correction energies revealed that the fission barriers of actinide nuclei have double-humped shape as shown schematically in figure 3. The discovery of double-humped fission barrier stimulated extensive experimental work during the sixties and seventies to obtain experimental information on the systematics of the double-humped fission barriers through the studies of fission excitation functions and fission isomers. Inclusion of shell effects on fission barriers also made the exciting prediction of the possibility of the existence of an island of relatively stable superheavy nuclei around \( Z = 114, N = 184 \) nuclei as shown in figure 4 (Myers and Swiatecki 1966; also see the review by Flerov and Ter-Akopian 1984 and the references therein).

Figure 5 represents schematically the time scales involved during various stages of the fission process. It also summarizes the various stages at which different fission characteristics are determined. The scission point represents that stage of the process where the two nascent fragments are barely influenced by each other’s nuclear forces. Although the fission barrier height is much smaller than the total energy released in fission, the fission barrier crucially governs the fission characteristics. The spontaneous
Figure 3. Schematic diagram of double-humped fission barrier.

Figure 4. Predicted island of superheavy nuclei around the doubly closed shell $Z = 114$, $N = 184$.

Figure 5. Schematic illustration of different stages of the fission process and corresponding time scales.
fission probability, fission cross-sections and the fission fragment angular distributions are all determined by the potential energy landscape around the fission barrier. In addition to the fission barrier characteristics, the dynamics from saddle to scission is important in determining the fragment mass and charge distributions. The light charged particles emitted in fission seem to originate from around the scission configuration. Bulk of the prompt neutrons are emitted from the excited fission fragments after $10^{-20}$ s, when the fragments have acquired their full velocities under the mutual Coulomb repulsion. Some neutrons can also be emitted during the nuclear dynamics to the scission point configuration and before the fragments acquire their final velocities. The fraction of this type of neutrons, although small in low energy fission, is found to be large in heavy-ion induced fission.

There has also been considerable interest in the study of rare fission events such as light charged-particle accompanied fission, low energy symmetric fission, cold compact fission corresponding to a window on high fragment kinetic energies, and cold deformed fission with a window on low fragment kinetic energies. Study of light charged-particle accompanied fission provide information on the scission configuration as well as dynamics of the fission process at the instant of their emission (see, for example, Kapoor and Nadkarni 1985 and the references cited therein). Recent developments in the study of cold compact fission have been discussed by Hasse (1987) with regard to the new experimental data as well as the possible theoretical explanations. Considerable theoretical studies have also been carried out in recent years to investigate the dominant energy dissipation mechanism appropriate for the description of the heavy-ion collisions and the dynamical descent of a fissioning nucleus from saddle to scission. The energy dissipation from saddle-to-scission can arise due to two-body collision between individual nucleons or by one-body dynamics involving interaction between the nucleon and the mean field created by all the other nucleons, or by a combination of these two types of mechanisms. In the two-body dissipation mechanism, viscosity comes out to be small leading to extended scission configurations with a considerable pre-scission kinetic energy. On the other hand, the one-body dissipation mechanism leads to high dissipation, more compact shapes and very small pre-scission kinetic energy. Extensive theoretical studies on these dissipation mechanism have been reported in recent years (Sierk and Nix 1980; Nix and Sierk 1986). Comparison of experimental data on the observed average fission fragment kinetic energies with those calculated using wall-and-window one-body dissipation and two-body dissipation mechanism shows that the observed variation of the average fragment kinetic energies with $Z^2/A^{1/3}$ can be explained by both the mechanism, but their predictions for the pre-scission kinetic energies and for the saddle-to-scission time are different (Davies et al 1976; Sierk and Nix 1980; Carjan et al 1986).

Recent investigations have also shown that interesting information regarding energy dissipation in heavy-ion collisions and in the fission process can be deduced from the study of fragment-neutron angular correlations in fission. From these measurements, one deduces the average number of pre-fission neutrons and a comparison of this number with that expected on the basis of statistical calculations shows that the average number of pre-fission neutrons is much larger than that predicted from the statistical model calculations. It has been inferred that in heavy-ion induced fusion-fission reactions, excess neutrons are emitted during the dynamics from the formation phase of the di-nuclear system to the scission stage. Therefore information about energy dissipation can be deduced (Gavron et al 1987 and Hinde et al 1986)
from such studies (Kapoor 1988 and references therein). Studies of nuclear fission phenomenon which are aimed at probing the dynamics of the fissioning system are currently being actively pursued and these studies are expected to remain exciting and fruitful areas of further research.

Another important advance in the understanding of the fission process has come from the study of fission fragment angular distributions which provide information on fission barrier shapes. With the availability of heavy ion beams, the scope of studies of the fission phenomenon has considerably widened. In these reactions, one produces intermediate nuclei with large angular momenta and excitation energies. Studies of the fission and fission-like reactions in nucleus-nucleus collisions have unfolded several additional new features of the nuclear dynamics. In what follows we discuss some of these aspects of fission and fission-like processes in heavy-ion induced reactions.

3. Binary fission and fission-like process in heavy ion-induced reactions

The reactions resulting from nucleus-nucleus collision can be broadly classified into four types: elastic (and quasi-elastic) reactions, dinucleus reactions involving deep-inelastic collisions (DIC), non-compound nucleus fusion reactions (mono-nucleus intermediate configuration) and true compound nucleus reactions. Peripheral collisions with a small overlap of the matter densities of the target and the projectile result in elastic, quasi-elastic and transfer reactions. A somewhat larger overlap of the matter densities results in the deep-inelastic collision. These reactions are characterized by a rather short interaction time of a few times $10^{-21}$ s during which a large fraction of the kinetic energy of the colliding nuclei gets converted into intrinsic fragment excitation energies. In addition, a large number of nucleons are also exchanged between the target and the projectile nuclei as inferred from the observed width of the mass and charge distributions of the target-like and the projectile-like binary fragments emitted as the reaction products.

Fully damped deep inelastic binary collisions have also been called ‘quasi-fission’ reaction as the fragment kinetic energy distributions are similar to those in compound nucleus fission. In deep-inelastic collisions, the fragment angular distributions are characteristic of direct reactions and the average fragment masses are not much different from those in the entrance channel. In contrast, the “complete-fusion” reactions are those reactions in which the composite system disintegrates only after achieving complete equilibration in the mass degree of freedom and after losing the memory of the reacting masses. If a complete fusion reaction also achieves equilibration in all other degrees of freedom before disintegration, the reaction is identified as a compound nucleus reaction. In a number of reactions one observes a predominant symmetric peak of the fragment mass distributions with such a large cross-section that the number of $l$-values which are involved must include some $l$-values which cross the limiting spin $I_{f}^{RLDM}$ for zero fission barrier as calculated from the rotating liquid drop model (RLDM) (Cohen et al 1974). It is clear that for collisions corresponding to $l > I_{f}^{RLDM}$ formation of a true compound nucleus is not expected, as the system can undergo fission decay in a very short time due to the absence of a fission barrier and may not live long enough to achieve equilibration in all the degrees of freedom. The term “fast fission” has been suggested (Gregoire et al 1981) for this type of reaction.
In a simplified picture, one usually associates the different types of reactions with limiting \( l \)-values as shown in figure 6. \( I_c \) corresponds to the limiting \( l \)-value below which a "complete-fusion" reaction is possible. (As the value of \( l \) becomes larger than \( I_c \), one progressively enters the regions of deep inelastic reactions, quasi-elastic and elastic reactions). The other limiting value \( I_{RLDM} \) corresponds to that \( l \)-value at which the fission barrier, against instantaneous binary split of the system, becomes zero. The fast fission process has been thought to be confined to the region of \( l \)-band defined by \( I_{RLDM} < l < I_c \) and it is believed that for \( l < I_{RLDM} \) the system has sufficient time to equilibrate into a compound nucleus before undergoing subsequent decay. Experimentally, while the symmetric peak in the fission fragment mass distribution can be associated with the fission corresponding to the complete fusion events (consisting of both the fast fission and compound nucleus fission events) the relative contribution of the fast fission events can be determined only by examining deviations from the other predicted characteristics of fission process following compound nucleus formation. Study of the fragment angular distributions provides one such convenient way.

4. What is learnt from the study of fragment angular distributions?

Foundation of our present understanding of the fragment angular distribution was laid by Aage Bohr (Bohr 1955) with the application of the unified model to the highly deformed transition state nucleus passing over the saddle point on its way to fission. Bohr postulated that a fissioning nucleus spends a sufficiently long time at the fission transition state to define a spectrum of quasi-stationary states at the saddle point.
This model proved to be highly successful in explaining the fragment angular distributions at very low energies on the basis of available quantum states at the saddle point. Halpern and Strutinsky (1958) extended this model to the statistical regime of higher excitation energies with spectacular success. In this theoretical approach, fission fragments are assumed to be emitted along the direction of the nuclear symmetry axis at the fission saddle point (the transition state configuration). The transition state nuclei are assumed to be axially symmetric and are described by symmetric-top wave functions \( D_{MK}^I(\theta) \). The relevant quantum numbers are the total spin \( I \), its projection \( K \) on the nuclear symmetry axis, and its projection \( M \) on the space-fixed axis. The angular distribution for each state (specified by \( I, M \) and \( K \)) is given by:

\[
P_{MK}^I(\theta) = \frac{1}{2}(2I + 1)|D_{MK}^I(\theta)|^2,
\]

where \( \theta \) is the angle with respect to the space-fixed axis.

It has been observed that while for neutron-induced fission of lighter nuclei there are fluctuations in the fragment anisotropy versus neutron energy near the threshold, for heavier nuclei the anisotropy is found to vary smoothly. This feature can be qualitatively understood on the basis that the height of the outer barrier II of the double-humped fission barrier decreases as the nuclei become heavier and that the \( K \)-quantum number is not conserved in going over the well from the first barrier to the second barrier. This also shows that near the threshold, fragment angular distributions are determined by the quantum states of the barrier II.

For fusion-fission reactions with spin-0 target and projectile (i.e., \( M = 0 \)), one must sum the functions \( P_{0,K}^I(\theta) \) over \( K \) and \( I \) to obtain the expression for the angular distribution:

\[
W(\theta) = \sum \sigma(I) \sum_{K = -I}^{+I} \rho(I, K) P_{0,K}^I(\theta),
\]

where \( \rho(I, K) \) is the level density of the intrinsic states at the saddle point. The spin-dependent cross-section \( \sigma(I) \) is given by:

\[
\sigma(I) = \pi \hbar^2 (2I + 1) T(I),
\]

where,

\[
\hbar^2 = 2\mu E_{c.m.}/\hbar^2,
\]

where \( E_{c.m.} \) is the centre-of-mass bombarding energy and \( \mu \) is the reduced mass. The transmission coefficients \( T(I) \) are obtained from a reaction model that reproduces the measured compound nucleus fission cross-section. This equation is applicable when spin fractionation through competing decay modes is negligible.

The distribution of \( K \) is determined by assuming that \( K \) is conserved for the transition state shape from saddle to scission point and that the fission probability is proportional to the minimum number of open channels encountered along each fission path. The number of open channels is proportional to the density, \( \rho(I, K) \) of intrinsic states at the transition state:

\[
\rho(I, K) \propto \exp[(E - E_{\text{def}} - E_{\text{rot}})/T],
\]

\[
\propto \exp[-K^2/2\xi^2],
\]

where \( \xi \) is the width of the fission barrier.
where $E_{\text{def}}$ and $E_{\text{rot}}$ are the deformation and the rotational energies of the nucleus at the transition state shape, and

$$K_0^2 = J_{\text{eff}} T / h^2 = (J_0 / J_{\text{eff}})^{-1} (J_0 T / h^2). \quad (7)$$

The effective moment of inertia, $J_{\text{eff}}$, is given by

$$J_{\text{eff}}^{-1} = (J_{\parallel})^{-1} - (J_{\perp})^{-1} \quad (8)$$

Here $J_{\parallel}$ and $J_{\perp}$ are nuclear moments of inertia, parallel and perpendicular to the nuclear symmetry axis at the transition shape and $J_0$ is the moment of inertia for the spherical shape. So it is possible to deduce the values of $J_0 / J_{\text{eff}}$, which depends only on the transition state nuclear shape, from an analysis of the fragment angular distributions of the fission process induced by energetic projectiles. The transition state shapes deduced from an analysis of the fragment angular distributions (Reising et al 1966) of the 42.8 MeV $\alpha$-particle-induced fission of various nuclei are shown in figure 7 along with the curves calculated for a liquid drop barrier as well as for a double-humped barrier (Ramamurthy et al 1970). This figure shows that the transition state shapes of a heavy nucleus like uranium, excited to about 20–30 MeV correspond to the liquid drop model saddle shapes rather than that corresponding to the second barrier of the double-humped fission barrier. Using statistical thermodynamics, it was shown (Ramamurthy et al 1970) that as the excitation energy increases, the shell effects on the statistical quantities such as entropy begin to disappear. So the minimum number of open channels are encountered at the liquid drop barrier rather than at the second barrier. This is substantiated by the analysis of the observed dependence of $K_0^2$ on excitation energy (Ramamurthy et al 1970) as shown in figure 8. It shows that the transition state shape changes smoothly from barrier II shape to LDM shape with increase in excitation energy. So the transition state model has been quite successful in explaining the fragment angular distributions, which provides information on the transition state shapes of the fissioning nuclei. Since the angular distributions depend on both $K_0^2$ and $l_\text{max}$ (maximum $l$-values for which target projectile

![Figure 7](image.png)

Figure 7. Experimentally deduced values of $J_0/J_{\text{eff}}$ versus $Z^2/A$ compared with values for the saddle shapes corresponding to LDM prediction as well as barriers I and II of the double-humped barrier (taken from Ramamurthy et al 1970).
fusion is assumed to occur), one can also deduce the values of \( I_c \) and hence the heavy ion fusion cross-section

\[
\sigma_{\text{fus}} = \pi \lambda^2 (l_c + 1)^2,
\]

where \( \lambda \) is the projectile de Broglie wavelength, by using the LDM predictions of \( K_0^2 \) (or \( J_0 / J_{\text{eff}} \)) as shown in figure 9. Such determination of fusion cross-sections from the analysis of fragment angular distributions in the case of light heavy ion-induced reactions (Choudhury et al 1979) is shown in figure 9 along with the results of model predictions of \( \sigma_{\text{fus}} \).

In the above formulation, it is assumed that the shape of the transition-state nucleus is independent of \( K \) so that the value of \( E_{\text{rot}} \) can be calculated for a rigid rotor and the deformation energy \( E_{\text{def}} \) can be taken to be independent of \( K \). Under these assumptions, \( K \) has a Gaussian distribution with a variance \( K_0^2 = J_{\text{eff}} T / \hbar^2 \). However, these approximations are valid only for values of rotational energy much smaller than the relative changes in surface and Coulomb energy. For large values of \( I \) and/or \( Z^2 / A \), variations in the rotational energy with \( K \) become important. Then for each value of \( I \), one should calculate the properties of the saddle-point configuration as a function of \( K \). This leads to a \( K \) dependence both for the inertial parameters and for the fission-barrier heights and a non-Gaussian \( K \)-distribution. This modification of the transition-state theory (the flexible-rotor model) has been discussed by Prakash et al (1984). However, it is found that this generalization introduces only a small correction to the calculated fragment angular distributions even for very high spins.

In the case of heavy ion-induced fission reactions, when the composite system is formed with a large angular momentum, the fission barrier heights (as a function of the angular momentum) may become vanishingly small for several \( l \)-values for which normally compound nucleus formation is expected. How does one understand fragment angular distributions in such cases? The fragment angular distributions in heavy-ion induced fission reveal that, in general, the observed fission events consist of an admixture of events of two types, (i) the compound-nucleus fission (CNF) and (ii) the non-compound nucleus fission (NCNF) of a composite system which has equilibrated in all degrees of freedom except the \( K \) degree of freedom. Reaction
mechanism such as fast fission (Gregoire et al 1981), taking place for the composite systems with zero fission barriers, and quasifission (Swiatecki 1981), taking place for the composite systems with fission-barrier shapes more compact than the entrance-channel contact configuration, are events of the type (ii). Another class of NCNF events is the pre-equilibrium fission (Ramamurthy et al 1985) consisting of those fission events occurring in a time comparable to the characteristic relaxation time in the $K$-degree of freedom when the fission-barrier heights become comparable to the temperature of the composite system. From the analysis of the fragment angular distributions in a number of heavy-ion induced fission reactions a value of $8 \times 10^{-21}$ s was deduced for the characteristic time of the $K$-equilibration (Ramamurthy et al 1985).

The various reaction channels in heavy ion collisions corresponding to binary fission and fission-like reactions can be summarized schematically as shown in figure 10. The re-separation of the reaction products in the exit channel can take place at various stages of the reactions as shown on the right side of the figure. The stages at which the relative kinetic energy, mass, charge and $K$ equilibration take place are also shown along with their characteristic relaxation times.

To summarize, in this paper I have tried to bring out the fact that the study of fission and fission-like processes has considerably enriched nuclear physics by providing information on several aspects of nuclear behaviour which cannot be studied otherwise. Nuclear shell effects in highly deformed nuclear shapes such as those encountered during the fission process, the study of topology of nuclear potential energy surfaces, and the dynamics of fission and fission-like reactions in nucleus-nucleus collision can be cited as important examples of this type. There are, however, still many aspects of nuclear physics, particularly of the nuclear macrophysics which are yet to be explored through fission studies.
Figure 10. Schematic diagram depicting fission and fission-like reaction channels in heavy ion collisions.

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