RIGIDITY SPECTRUM OF COSMIC RAY HELIUM NUCLEI

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Abstract

An experiment has been carried out using an oriented stack of nuclear emulsions to determine the rigidity spectrum of cosmic ray helium nuclei between 12 and 40 GV, by taking advantage of the variation of the geomagnetic cut-off rigidity in the east-west plane over Hyderabad, India. A total of 2433 identified helium nuclei recorded in the stack, has been divided into 8 angular intervals in the east-west plane corresponding to 8 different cut-off rigidities. From this the integral fluxes of helium nuclei at the top of the atmosphere have been obtained for all the 8 rigidity intervals. The vertical flux above an effective threshold rigidity of 16·73 GV has been determined with high statistical accuracy and has a value of $15 \cdot 0 \pm 0 \cdot 5$ helium nuclei $(m^2.sr.sec.)^{-1}$. The rigidity spectrum of these nuclei between 12 and 40 GV can be well represented by a power law of the type $N\ (>R) = 1990\ R^{-1 \cdot 74 \pm 0 \cdot 11}\ (m^2.sr.sec.)^{-1}$ and is the first direct determination so far made in this rigidity region.

The differential rigidity spectra of protons, helium nuclei and S-nuclei of the cosmic radiation in the vicinity of the earth at solar minimum (1965) have been constructed with the existing world data and it is found that for rigidities $\gtrsim 10$ GV, the three spectra have, within experimental errors, the same slope of 2·6. The ratios P/He and He/S of the differential fluxes have been studied as a function of rigidity. It is found that for $R > 2$ GV, the ratio P/He has, within experimental errors, a constant value of 6·3; as for the ratio He/S, it seems that the experimental data above a GV is not inconsistent with a constant value of 14 over the entire rigidity interval considered here.

1. INTRODUCTION

THE integral and differential rigidity spectra of cosmic ray helium nuclei have been extensively studied in the past between 500 MV and 5 BV using satellite and balloon-borne detectors. It is now well known that the intensity, as also the shape of the spectrum in this rigidity region, changes appreciably with solar activity; there is also evidence that even at the time of minimum solar activity in the 11 year cycle, the spectrum observed near the earth does not represent the one in the near interstellar space because of residual solar modulation. The effect of solar modulation, however, would decrease with increasing rigidities and it seems reasonable to suppose from our present knowledge that the shape of the spectrum above 10 GV would represent not only the spectrum in the near interstellar space but also the injection spectrum of these particles. Hence such information is essential to understand the origin of cosmic rays, their propagation in interstellar space and the solar modulation they undergo.

Since direct determinations of rigidities of individual cosmic ray particles with $R > 10$ GV are difficult to make, existing data so far have been obtained mainly from measurements of the vertical intensities at different geomagnetic latitudes. The maximum value of the rigidity upto which this method can be employed is about 17 GV, this being the vertical geomagnetic cut-off rigidity over equatorial latitudes. However, if one considers all zenith angles in the east–west plane over equatorial stations, it is found that the cut-off rigidity varies from about 11 GV to about 70 GV, and in practice one can usefully cover the range of 12 GV to about 40 GV. Furthermore, the fact that (*a*) recent calculations using sixth degree simulation of the earth's magnetic field permits reliable estimate of the geomagnetic threshold rigidity for any given arrival direction and (*b*) penumbral bands are almost absent over low latitudes, permits one to extend the rigidity spectrum upto about 40 GV. The present experiment is the first attempt to exploit this method in the case of He-nuclei and obtain results of a high statistical accuracy. It was carried out using a hypersensitised nuclear emulsion stack which was flown oriented in the east–west plane over Hyderabad, India, under 10·2 g./cm.^{2} of residual atmosphere. Results from an early analysis based on 528 primary helium nuclei recorded in this stack were presented at the International Conference on Cosmic Rays, London and the results from a similar analysis with 1102 helium nuclei were reported in the Cosmic Ray Symposium,[2] Bombay. The present paper is based on the final analysis of a total of 2433 tracks and was presented at the International Conference on Cosmic Rays, Calgary (1967).

2. EXPERIMENTAL DETAILS

An emulsion stack consisting of 265 hypersensitised Ilford G–5 pellicles each of size 20 cm. × 15 cm. and nominal thickness 0·06 cm. was flown on 6th April 1963 under 10·2 g./cm.^{2} of residual atmosphere over Hyderabad, India, where the vertical geomagnetic cut-off rigidity for a positively charged particle is 16·92 GV. The stack was kept oriented in the east-west plane and was flipped through 180° when it just reached the ceiling altitude; thereafter it was kept oriented to an accuracy of $\pm 2°$ during the entire period of 398 minutes at the ceiling altitude. The thicknesses of individual emulsions were measured before processing.

Microscope scanning was carried out in each emulsion along a line parallel to the 20 cm. top edge, at a depth of 7 mm., for tracks making projected zenith angles $\leq 65°$ and dip angles $\leq 11·7°$ with ionisation corresponding to 3–6 times that of a singly charged relativistic particle. After confirming that each track entered the stack from the top edge, it was traced down till it either left the stack or led to an interaction. Tracks were unambiguously identified as due to helium nuclei if they either showed no change of ionisation till they left the stack or by the typical high energy interactions they produced. In this manner scanning was carried out on a total length of 2536 cm. from which 2650 helium nuclei were identified.

In order to eliminate any scanning bias in locating tracks close to the air and glass surfaces of the emulsions, a study of the number of tracks observed as a function of depth in the emulsions was carried out. This showed that, though a small effect did exist, it can be completely eliminated by accepting only tracks located beyond 30 μm. from either emulsion surface at the scan line. Thus 2433 tracks were left for further analysis.

Mention may be made here that the use of hypersensitised emulsions resulted in a grain density of about 36 grains per 100 μm. for relativistic singly charged particles (a value twice as high as in normal emulsions) thus permitting high scanning efficiency for tracks of He-nuclei. In spite of this it is essential to determine the scanning efficiency as a function of zenith angle with an accuracy better than the statistical accuracy involved in this experiment. For this purpose a scan length corresponding to 493 tracks was rescanned by different observers. These tracks, when divided according to zenith angle intervals of 0–15°, 15–30°, 30–45° and 45–65° in the east-west plane, showed that the scanning efficiency was a little smaller for larger zenith angles; the estimated values are shown in Table I.

TABLE I

Integral flux of primary helium nuclei at different zenith angle interval

Angular interval	Effective cut-off rigidity Re in BV	No. of helium nuclei observed	Ascent correction (No. of tracks)	Scanning effi- ciency %	J(> R) particles per ($m.^2$ sr.sec.) at top of atmosphere
65–45° W.	12·5	353	17·4±1·8	84·5±4·2	23·20±2·03
45–30° W.	13·3	401	19·3±1·9	90·4±2·8	21·10±1·48
30–15° W.	14·4	485	22·1±2·2	94·4±1·4	19·30±1·12
15– 0° W.	15·9	435	20·4±2·1	94·4±1·4	16·50±0·95
0–15° E.	18·0	348	16·4±1·7	94·4±1·4	13·18±0·85
15–30° E.	21·2	239	11·4±1·2	94·4±1·4	9·89±0·65
30–45° E.	25·7	125	6·0±0·6	90·4±2·8	6·54±0·68
45–65° E.	33·3	67	3·1±0·3	84·5±4·2	4·30±0·61

3. FLUX AND INTEGRAL RIGIDITY SPECTRUM OF HE-NUCLEI

The 2433 selected helium nuclei were then divided according to their arrival directions into eight zenith angle intervals in the east–west plane as shown in Table I. In order to estimate the number of particles entering the top of the stack at the flight altitude, it is necessary to correct not only for interactions of He-nuclei above the scan line but also for the small number that would have entered through the other side of the stack during the time of ascent, *i.e.*, before the stack was flipped through 180°. The number thus recorded during ascent time at the scan line for a given zenith angle interval is given by $N(\theta)\,d\theta = C(\theta)\,F(\theta)\,d\theta$, where $F(\theta)$ is the flux of particles at the top of the atmosphere at a given zenith angle θ, and $C(\theta)\,d\theta$ is the collecting power of the stack at that angle. The collecting power as a function of zenith angle has been computed and the method of computation is described in Appendix I. Since the ascent contribution is only about 5%, one can calculate it by first obtaining F (θ) without this correction and then by successive approximation estimate the true number of particles recorded during ascent. The corrections thus obtained are included in Table I. The fluxes at the top of the atmosphere corresponding to the eight angular intervals were then calculated using an effective mean free path of 52 ± 4 g./cm.^{2} in air,[3] which takes into account the attenuation and production of helium nuclei due to interaction and fragmentation respectively; further an attenuation mean free path of $18 \cdot 3 \pm 1 \cdot 2$ cm. in emulsion[4] has been used. The values thus obtained are shown in Table I. The errors quoted include those arising from the number of events observed and uncertainties in the parameters used.

In order to take into account the variation of flux with cut-off rigidity within the eight angular bins, an effective threshold rigidity R_e was defined for each bin as given by Equation. (1), assuming an integral spectrum of form $N\ (> R) = aR^{-\gamma}$.

$$R_e = \frac{\iint aR^{-\gamma+1}(\theta_p, \delta)\, G(\theta_p, \delta)\, d\theta_p\, d\delta}{\iint aR^{-\gamma}(\theta_p, \delta)\, G(\theta_p, \delta)\, d\theta_p d\delta} \qquad (1)$$

where $\iint G(\theta_p, \delta)\, d\theta_p\, d\delta$ is the geometrical factor for a given bin. Here a and γ are constants, θ_p is the projected angle and δ the dip angle of the track. Equation (1) was evaluated using the geomagnetic cut-off rigidities calculated by Daniel and Stephens[5] for different arrival directions over Hyderabad. The value of γ used in this equation has been obtained from an analysis of the east-west asymmetry of the helium nuclei as given in Table II, which is independent of the corrections described earlier; it can be seen that R_e is rather insensitive to small changes in the value of γ. Further one sees from Table II that the most probable value of γ should lie close to $1 \cdot 7$; this may be compared with the value of $1 \cdot 74$ determined from a regular plot of the flux values (Fig. 1). The effective cut-off rigidities corresponding to the eight angular bins have also been shown in Table I.

We have also determined the vertical intensity of the helium nuclei at the top of the atmosphere over Hyderabad by making use of a total of 1487 tracks within 30° to the vertical. After taking into account all relevant corrections, the flux of primary helium nuclei above an effective threshold rigidity of $16 \cdot 73$ GV is found to be $15 \cdot 0 \pm 0 \cdot 45$ (m.^{2}sr.sec.)$^{-1}$; this is by far the best flux value determined so far at this latitude.[6-9] For comparison, we have shown in Table III, the other determinations made from this latitude, and it can be seen that the intensity obtained from a recent experiment using Cerenkov scintillator[9] is in good agreement with

TABLE II

Estimate of the exponent of the integral rigidity spectrum of helium nuclei using the east-west asymmetry

Zenith angle interval	Calculated ratio of N_W/N_E				Observed ratio
	$\gamma=1\cdot6$	$\gamma=1\cdot7$	$\gamma=1\cdot8$	$\gamma=1\cdot9$	
10–65	2·55	2·70	2·86	3·00	$2\cdot65\pm0\cdot13$
20–65	3·06	3·27	3·51	3·75	$3\cdot24\pm0\cdot21$
30–65	3·72	4·01	4·30	4·73	$4\cdot20\pm0\cdot35$
40–65	4·55	4·95	5·47	6·00	$5\cdot42\pm0\cdot62$

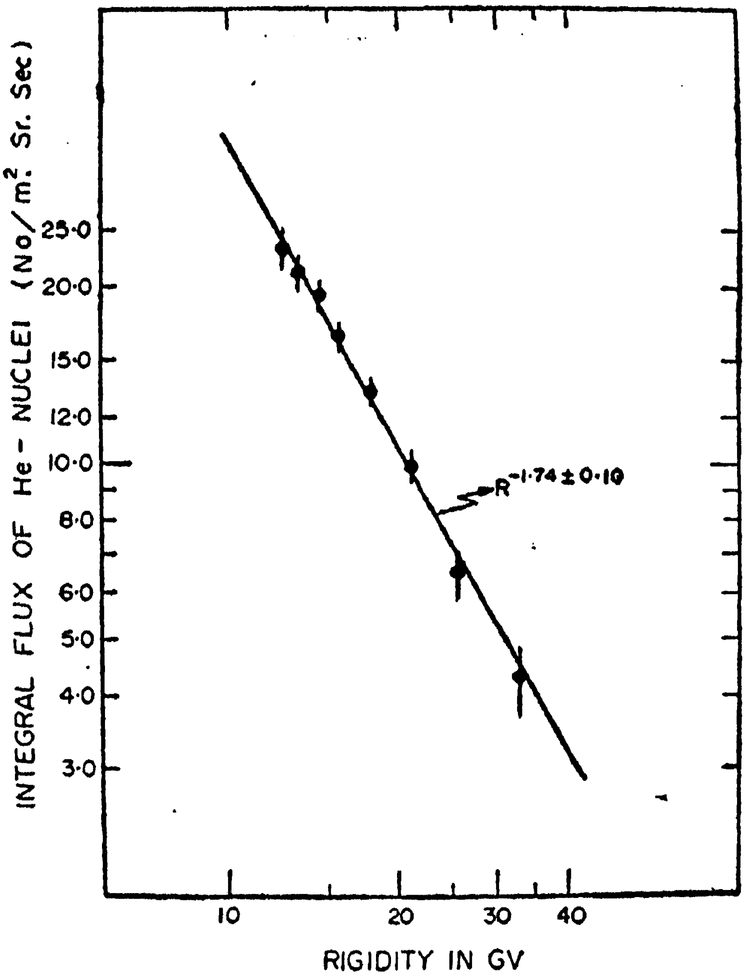

FIG. 1. Integral rigidity spectrum of the primary helium nuclei using the present experimental values.

the present result. The earlier estimates seem to be somewhat higher than the value obtained from this experiment; this may be due to statistical and/or various systematic errors in the experimental and extrapolation procedures involved in those determinations.

TABLE III

Comparison of the measurements of the vertical intensity of primary helium nuclei at the top of the atmosphere over Hyderabad

Date of flight	Technique	Ceiling altitude x in g.cm^2. of air	J (>R) particles per m^2. sr.sec. at $x=0$	Reference
March 1960 ..	Emulsion	8·6	17·9 ± 1·9	Kajarekar[6]
March 1961 ..	Cerenkov-Scintillator	10·0	19·1 ± 1·2	Balasubramanian *et al.*[7]
March 1962 ..	Emulsion	16·0	16·3 ± 1·7	Shukla[8]
April 1963 ..	Emulsion	10·2	15·0 ± 0·45	Present experiment
April 1965 ..	Cerenkov-Scintillator	7·0	14·7 ± 2·0	Agrawal *et al.*[9]

The flux values obtained for the eight angular bins have been plotted in Fig. 1 against their respective effective cut-off rigidities. The integral rigidity spectrum thus obtained can be represented by a power law of the type

$$N(> R) = 1990\, R^{-1\cdot74 \pm 0\cdot11}\, (\mathrm{m.}^2\mathrm{sr.sec.})^{-1} \qquad (2)$$

There are two other experiments[8, 10] which have been made in the past to determine the power index γ in the rigidity region of 5–15 GV; in both the values of γ were obtained between 1·3–1·5. This apparent discrepancy between our measurements and those from the two investigations can be understood in terms of the slow flattening of the spectrum below 10 GV as will be shown in 4.1.

4. DIFFERENTIAL RIGIDITY SPECTRA OF COSMIC RAY NUCLEI AT SOLAR MINIMUM

It can be seen from Fig. 2 that for the first time we have acquired from various investigations reliable information on the rigidity spectrum of He-nuclei applicable to the year of minimum solar activity from very low rigidities upto about 50 GV. We have therefore considered it to be profitable to compare this spectrum with similar spectra for protons and the S-nuclei ($Z \geqslant 6$) in order to deduce information on the relative spectral shapes. In what follows in Section 4, this will be described in some detail. However, notice that in all the ensuing analysis, data for $R < 10$ GV relate to measurements in 1965 while for $R > 10$ GV, where modulation effects are negligible, all data available have been made use of.

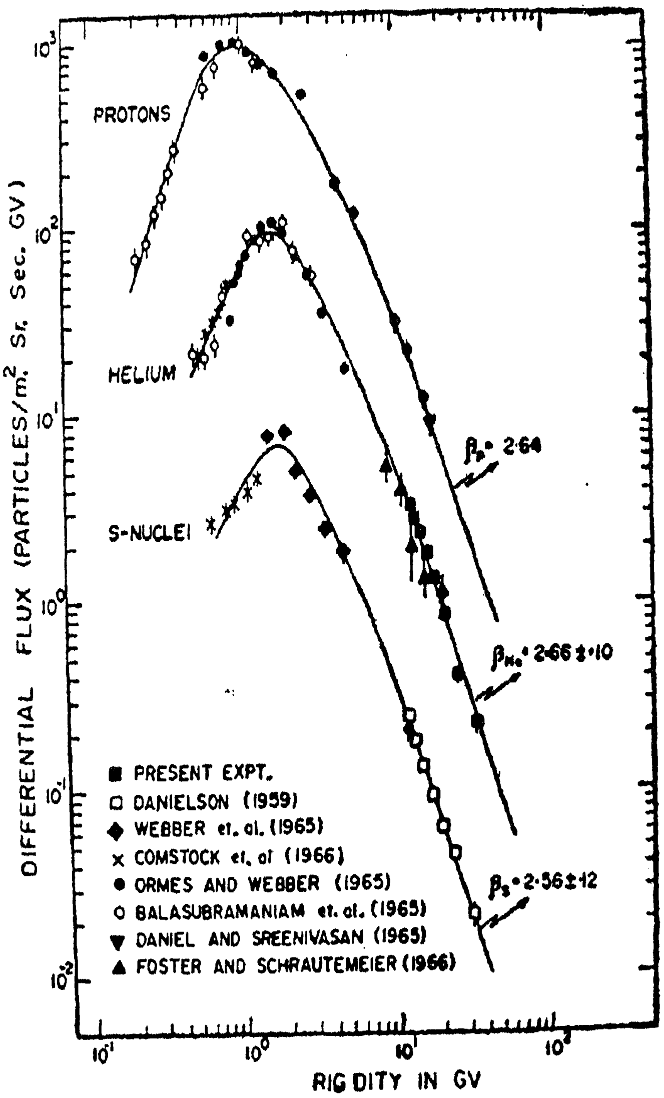

FIG. 2. Differential rigidity spectrum of protons, helium nuclei and S-nuclei of the primary cosmic radiation at solar minimum. The errors are not indicated when they are small compared to the size of the point. Only statistical errors have been associated with Danielson's flux values.

4·1. *Helium nuclei.*—During 1965, the year of solar minimum, differential flux measurements in the rigidity interval 500 MV–5 GV have been made with high statistical accuracy using satellite[11, 12] and balloon borne[10] detectors; such data along with those from the present experiment are shown in Fig. 2. The differential flux values from the present experiment were obtained by multiplying the integral flux values by a factor γ/R; also shown in Fig. 2 are values obtained from Coulomb scattering measurements[13] made on tracks due to primary helium nuclei recorded in nuclear emulsions. The curve drawn for He-nuclei in Fig. 2 is the best fit for all the experimental values. It is found that the spectrum above 10 GV can be represented by a single power law with an exponent $\beta_{He} = 2 \cdot 66 \pm 0 \cdot 10$. The spectrum slowly flattens below 10 GV; this behaviour can be attributed to the effect of residual solar modulation which becomes increasingly important at lower rigidities.

4·2. *Protons.*—We have also plotted in Fig. 2 the differential flux values measured for protons during 1965 by Balasubramanian *et al.*[11] in the region 200 MV to 2·5 GV using satellite-borne instruments, and by Ormes and Webber[10] in the region 600 MV–6 GV using balloon-borne detectors. In the rigidity interval of 10–17 GV, the differential flux values were derived from the integral flux values measured by Ormes and Webber[10] and Daniel and Sreenivasan.[14] The best fit line drawn through all these points is shown in Fig. 2. A spectrum with a power index of $\beta_p = 2 \cdot 64$ is found to be the best fit for all observations $\geqslant$ 10 GV.

4·3. *S-nuclei.*—In view of the paucity of data for the subgroups (namely the M and H groups) comprising the S-nuclei for the year 1965, we have considered here only the S-group of nuclei. The data at low rigidities according to Comstock *et al.*[12] and Webber *et al.*[15] are shown in Fig. 2. At higher rigidities only two measurements exist. The first one is an integral flux measurement at 12·1 GV[15]; the second one is due to Danielson[16] and contains elaborate measurements made in an oriented horizontal emulsion stack exposed over Guam where the vertical cut-off rigidity is 16·3 GV. Since the raw data obtained from this experiment has been published, it has become possible for us to deduce the integral fluxes of S-nuclei from about 12 GV to 30 GV with good accuracy; the procedure followed and the flux values obtained are given below.

Danielson[16] has tabulated the number of S-nuclei observed at flight altitude for all zenith and azimuthal directions. Since, however, (i) the near vertical particles leave steep tracks in the emulsion and hence pose difficulties of charge identification, and (ii) the particles arriving at very large zenith angles traverse large thicknesses of atmosphere, we made use of the data for zenith angles between 15° and 75°, but all azimuthal angles. This surface area was then divided into a large number of angular bins, each of size 10° (zenith) × 20° (azimuth). Then the respective flux values at the top of the atmosphere were estimated first in arbitrary units for each bin using an extrapolation procedure as given in Appendix II. The cut-off rigidity corresponding to each angular bin was calculated using the sixth degree simulation of earth's magnetic field in a manner similar to the one used by Daniel and Stephens.[1] The angular bins were then grouped according to 7 rigidity intervals. Since, however, the collecting areas in Danielson's experiment were not given, we obtained the true flux values at the top of the atmosphere by normalising the value for the right rigidity interval in Danielson's experiment to the vertical integral flux of S-nuclei over Hyderabad, India,[17] which has a geomagnetic latitude very close to that of Guam. The integral flux values thus obtained are $1 \cdot 95 \pm 0 \cdot 16$, $1 \cdot 56 \pm 0 \cdot 11$, $1 \cdot 25 \pm 0 \cdot 11$, $1 \cdot 04 \pm 0 \cdot 07$, $0 \cdot 83 \pm 0 \cdot 07$, $0 \cdot 70 \pm 0 \cdot 07$ and $0 \cdot 43 \pm 0 \cdot 06$ (m.^{2}sr.sec.)$^{-1}$ at rigidities 12·2, 13·4, 14·6; 13·3, 14·6, 16·8, 19·9, 23·7 and 31·1 GV respectively. The differential fluxes shown in Fig. 2 were then deduced from the above-mentioned values using the procedure referred to earlier. A best fit curve has been drawn through the esperimental points; for $R \gtrsim 10$ GV a straight line fit is in good agreement with the data and yields a value $\beta_S = 2 \cdot 56 \pm 0 \cdot 12$.

5. DISCUSSION

5·1. *The rigidity spectra.*—From a careful examination of Fig. 2, the following observations can be made:

(i) In the case of He-nuclei, for which the most abundant measurements are available upto about 40 GV, the spectrum (for $R \gtrsim 2$ GV) continuously steepens upto about 10 GV; for $R \gtrsim 10$ GV and upto 40 GV the exponent remains constant and has a value of $2 \cdot 66 \pm 0 \cdot 10$.

(ii) Again for S-nuclei, a straight line fit with $\beta_S = 2 \cdot 56 \pm 0 \cdot 12$ is found to yield the best fit for the data at $R \gtrsim 10$ GV. As in case of He-nuclei the data at lower rigidities, though relatively meagre for S-nuclei, is consistent with a steady steepening of the spectrum from 2 to 10 GV.

(iii) In case of protons, on the other hand, the data for low rigidities are more extnsive than that for $R \gtrsim 10$ GV. In spite of this, a slow steepening of the spectrum at 1–10–GV and a straight line fit with $\beta_P = 2 \cdot 6$ at $R \gtrsim 10$ GV would be the best assignment one could make with the existing data.

It is thus seen that the data available for $R \gtrsim 10$ GV for all the three components considered above, fit within experimental errors a single power spectrum with a spectral index of 2·6. The data, summarised in Fig. 2, further suggest strongly that for all these components, the spectrum continuously steepens from about 2 GV to about 10 GV. This observation would imply that even at the time of solar minimum, measurable residual modulation exists upto almost 10 GV. It would therefore be necessary that in problems relating to cosmic ray propagation in space and solar modulation, normalisation of spectra be made at R > 10 GV.

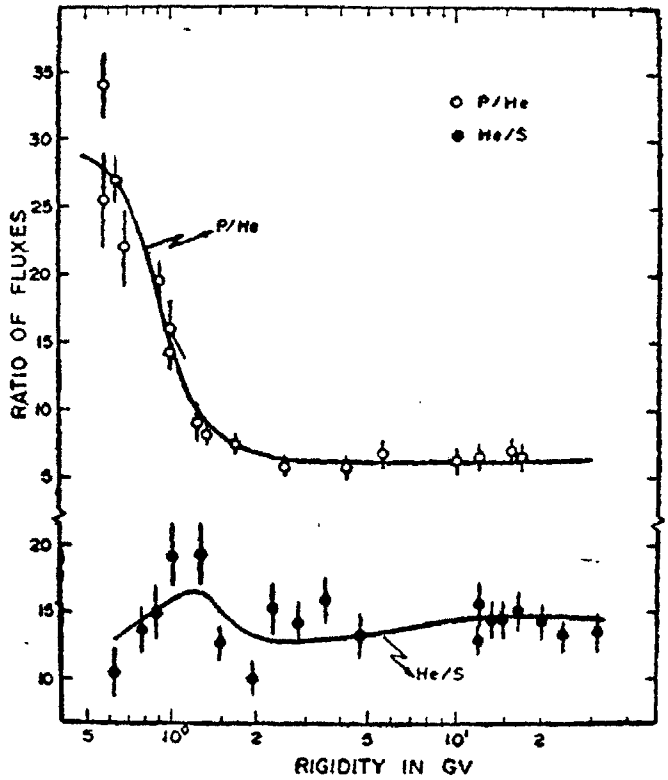

FIG. 3. The ratios of differential fluxes P/He and He/S have been shown against rigidity. The two curves in this figure represent the ratios obtained from the corresponding curves in Fig. 2.

5.2. *Ratios of flux of different components.*—In order to make a comparison between the spectral shapes of the P, He and S-nuclei, we have studied the ratios of the differential fluxes P/He and He/S as a function of rigidity; this is demonstrated in Fig. 3. The points shown in Fig. 3 were obtained by taking the ratios of the relevant differential flux values; the curves shown in Fig. 3 were obtained from the best fitting curves given in Fig. 2. It is seen from the data on the ratio P/He, that it has a constant value of 6·3 for R > 2 GV; the increasing value of the ratio with decreasing rigidity is likely to be due to ionisation losses in space. As for the ratio He/S, though the scatter of points is rather large, it will not be inconsistent with a constant value of 14 over the rigidity interval of 1–30 GV.

It would be pertinent to mention here that observations extended upto at least a few hundred GV would be of further interest; it is hopefully expected that such measurements would be made in the near future using, independently, gas cerenkov counters and magnetic deflection in balloon borne equipment.

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7. REFERENCES

1. Anand, K. C., Daniel, R. R., Stephens, S.A., Bhowmik, B., Krishna, C. S., Mathur, P. C., Aditya, P. K. and Puri, R. K. — *Proc. Int. Conf. on Cosmic Rays, London,* 1965, **1**, 362.
2. ———— .. *Proc. Symposium on Cosmic Rays, Bombay,* 1965, **69**.
3. Webber, W. R., and Ormes, J. F. — *Proc. Int. Conf. on Cosmic Rays, Jaipur,* 1963, **3**, 3.
4. Apparao, M. V. K., Daniel, R. R. and Neelakantan, K. A. — *Proc. Ind. Acad. Sci.,* 1956, **42**, 181.
5. Daniel, R. R. and Stephens, S. A. — *Ibid.,* 1966, **63**, 275.
6. Kajarekar, P. J. .. *Ibid.,* 1966, **64**, 123.
7. Balasubramanian, V. K., Ganguli, S. N., Gokhale, G. S., Kameswara Rao, N., Kunte, P. K., Menon, M. G. K. and Swami, M. S. — *Jour. Phys. Soc., Japan,* 1962, **17** (Suppl. 3 A), 8.

8. Shukla, P. G. ..	*Proc. Int. Conf. on Cosmic Rays, London*, 1965, **1**, 360.
9. Agrawal, P. C., Damle, S. V., Gokhale, G. S., Joseph, G., Kunte, P.K., Menon, M. G. K. and Sunderrajan, R.	*Ibid.*, 1965, **1**, 457.
10. Ormes, J. F. and Webber, W. R.	*Ibid.*, 1965, **1**, 349.
11. Balasubramanian, V. K., Hagge, D. E., Ludwig, G. H. and McDonald, F. B.	*Ibid.*, 1965, **1**, 427.
12. Comstock, G. M., Fan, C. Y. and Simpson, J. A.	*Ap. J.*, 1966, **146**, 51.
13. Foster, F. and Schrautemier, B. E.	*Nuovo Cimento*, Series x **47**, **189**, **1967**.
14. Daniel, R. R. and Sreenivasan, N.	*Nuovo Cimento*, 1965, **35**, 391.
15. Webber, W. R., Ormes, J. F. and Rosenvinge, T. V.	*Proc. Int. Conf. on Cosmic Rays, London*, 1965, **1**, 407.
16. Danielson, R. E. ..	*Phys. Rev.*, 1959, **113**, 1311.
17. Badhwar, G. D., Durgaprasad, N. and Vijayalakshmi, B.	*Proc. Ind. Acad. Sci.*, 1965 *b*, **62**, 17.
18. Noon, J. H. and Kaplon, M. F.	*Phys. Rev.*, 1955, **97**, 789.
19. Badhwar, G. D., Durga prasad, N. and Vijayalakshmi, B.	*Proc. Ind. Acad. Sci.*, 1965 *a*, **61**, 374.

APPENDIX I

Evaluation of Collecting Power of the Stack for Helium Nuclei during the Ascent of the Balloon

The emulsion stack was rotated through 180° about an axis perpendicular to the emulsion surface. Hence particles which entered the stack from east (or west) while the balloon was ascending, and emerged out of the stack without interacting would look as if they entered from east (or west) at the scan line. Therefore only the number of He-nuclei which entered the stack at the time of ascent and left it without interacting need be calculated.

In Fig. 4, the Y–*Z* plane is the plane of the emulsions in the stack; it is also the east–west plane. Scanning was carried out along a line DE 7 mm. from the top edge AB. Let F (θ) be the flux of helium nuclei above the atmosphere at an angle θ to the zenith in the east–west plane and dy be an element of scan-length at a distance y from the side OA. The number of particles that reach dy after traversing the stack at a direction θ and at time t during the ascent is given by

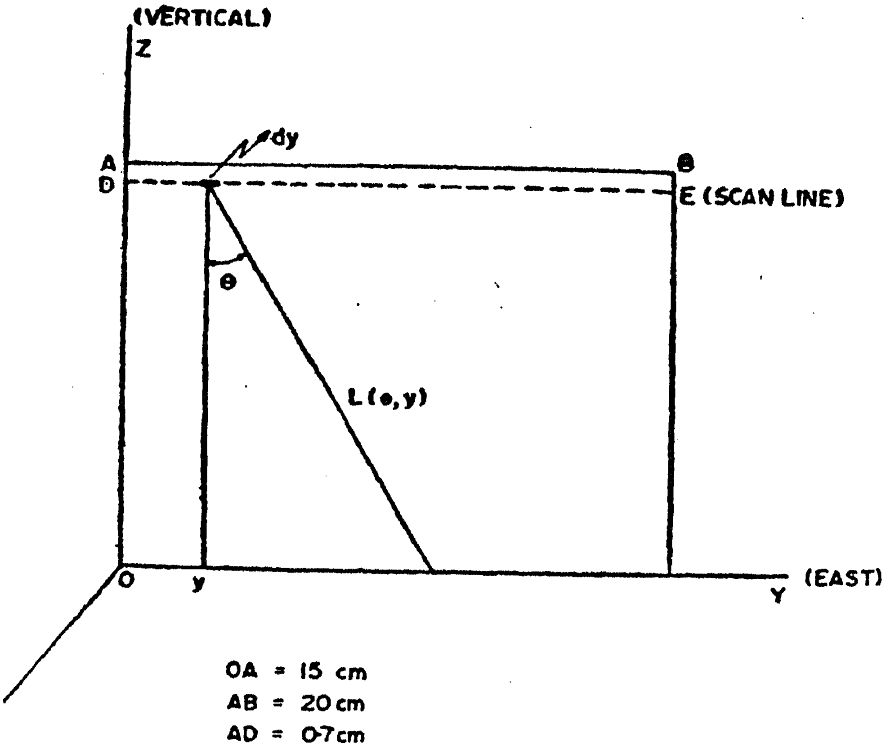

FIG. 4. Geometric representation of the particle trajectory in the emulsion plane.

$$\mathrm{N}\,(\theta)\,d\theta\,dt\,dy = \mathrm{F}\,(\theta)\,\mathrm{G}\,(\theta)\,e^{-s(t)\sec.\theta/\Lambda}\,e^{-\mathrm{L}\,(\theta,y)/\lambda}\,d\theta\,dt\,dy \qquad (3)$$

where $G(\theta)\,d\theta dy$ is the geometrical factor for accepting a track at y, $x(t)$ is the amount of vertical matter over the stack at time t [here $x(t)$ is obtained from the flight curve] and $L(\theta, y)$ is the path length traversed by the particle in the stack before reaching the scan line. Further we have used $\Lambda = 52 \pm 4$ g./cm.^{2} as the attenuation mean free path in air and $\lambda = 17{\cdot}5 \pm 1{\cdot}1$ cm. as the interaction mean free path in emulsion.

Since almost the entire scanning was carried out in emulsions situated in the middle of the stack, we can write

$$L(\theta, y) = \text{OD.Sec. } \theta \qquad \text{for } (y + \text{OD.tan } \theta) \leq \text{AB}$$

$$L(\theta, y) = (\text{AB} - y) \text{ cosec } \theta \qquad \text{for } (y + \text{OD}\cdot\text{tan } \theta) > \text{AB}$$

Hence the total number of helium nuclei that would be recorded at the scan line during the entire time of ascent in an angular interval $d\theta$ at an angle θ to the zenith is

$$N(\theta)\,d\theta = \int_{y_1}^{y_2}\int_{t_1}^{t_2} F(\theta)\,G(\theta)\,d\theta\, e^{-L(\theta, y)/\lambda}\, e^{-x(t)\,\text{sec.}\,\theta/\Lambda}\, dy\; dt \tag{4}$$

By definition the collection power

$$c(\theta)\,d\theta = \frac{N(\theta)\,d\theta}{F(\theta)}$$

$$\therefore\quad C(\theta)\,d\theta = \int_{y_1}^{y_2}\int_{t_1}^{t_2} G(\theta)\,d\theta\; e^{-L(\theta, y)/\lambda}\, e^{-x(t)\,\text{sec.}\,\theta/\Lambda}\, dy\, dt. \tag{5}$$

Equation 5 has been evaluated numerically and it is found that the correction is close to 5% of the total number of helium nuclei incident on the track at the ceiling altitude.

APPENDIX II

Extrapolation Procedure used for S-nuclei

The procedure often adopted to extrapolate the flux values of heavy nuclei observed at the flight altitude to the top of the atmosphere is by making use of the diffusion equations originally given by Noon and Kaplon.[18] However, the approximations used in solving these equations are such that the solutions thus obtained are not strictly valid at large atmospheric depths; this becomes quite important for atmospheric depths $\gtrsim \lambda$. Since in the present analysis, we have made use of all particles of charge $\geqslant 6$ with zenith angles between 15° and 75° from Denielson's experiment which was conducted under $9\cdot3$ g.cm.$^{-2}$ of air, it becomes necessary to estimate the growth of these nuclei rather accurately upto large atmospheric depths. For this purpose, the S-nuclei were subdivided into H_1 $(Z \geqslant 20)$, H_2 $(16 \leq Z \leq 19)$, H_3 $(10 \leq Z \leq 15)$ and M $(6 \leq Z \leq 9)$ groups and the growth of these groups of nuclei in the atmosphere computed using the following relation, which is valid at all depths.

$$J_i(x) = J_i(o)\, e^{-x/\Lambda_i} + \sum_{j<i} \int_0^x P_{ij}\, J_j(y)\, e^{-(x-y)/\Lambda_i} \frac{dy}{\lambda_j} \tag{3}$$

where

$J_i(0)$ and $J_i(x)$ are the fluxes of *i*-th nuclei at depths 0 and x g.cm.$^{-2}$;

Λ_i is the attenuation mean free path of the i-th type;

$J_j(y)$ is the flux of *j*-th type of nuclei at a depth y g.cm.$^{-2}$;

Λ_j and λ_j are the attenuation and interaction mean free paths of *j*-th type.

P_{ji} is the fragmentation parameter for a *j*-th type of nucleus to give rise to an *i*-th type.

The first term on the right-hand side of Equation (3) corresponds to the attenuation of the *i*-th type of nuclei while the second term is the contribution from the fragmentation of nuclei belonging to the higher charge groups. Here $P_{ji}\, J_j(y)\, (dy/\lambda_j)$ is the number of *i*-th type of nuclei from the fragmentation of *j*-th type at a depth y, and $e^{-(x-y)/\Lambda}$ is the survival probability for the *i*-th type during a traversal of $(x - y)$ g.cm.$^{-2}$

Equation (3) has been evaluated numerically for all the subgroups; the relative abundances of H_1, H_2, H_3 and M nuclei at the top of the atmosphere and the values of Λ, λ and P_{ji} used here were taken from Badhwar *et al.*[17, 18] From the above calculations we obtained the ratio $J_S(0)/\Sigma J_i(x)$ as a function of depth and then extrapolated the observed number of S-nuclei to the top of the atmosphere.