DIRECTIONAL VARIATION OF GEOMAGNETIC CUT-OFF RIGIDITY AROUND HYDERABAD, INDIA

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Abstract

The geomagnetic cut-off rigidities for cosmic ray particles arriving at the top of the atmosphere over Hyderabad, India (geographic latitude 17·6° N. and longitude 78·5° E.), as a function of zenith and azimuthal angles and the vertical cut-off rigidities for a few neighbouring locations, have been made using the sixth degree simulation of the geomagnetic field by the C.D.C. 3600 Computer.

1. Introduction

Pioneer calculations of the effect of the earth's magnetic field on the motion of charged particles have been made by Stormer[1] in his study of aurorae using a dipole field. Later by taking into account the eccentricity of the earth's magnetic centre, Vallarta[2] has studied the longitudinal variation of cosmic ray intensity. The minimum energy needed for a charged particle to arrive at a given geomagnetic latitude (the cut-off rigidity) has been studied extensively by Alpher[3] for various zenith and azimuthal angles using only the dipole terms of the magnetic field. Dipole simulation of the geomagnetic field is rather a poor approximation to the actual situation and hence higher degree simulations of the field have been used by Quenby and Webber[4] to calculate the vertical cut-off rigidities; later these calculations were modified by Quenby and Wenk[5] in 1962. The cut-off rigidities given by the above authors have been widely used till now. McCracken *et al.*[6] have calculated the asymptotic directions of cosmic rays far removed from the earth using sixth degree expansion of the geomagnetic field. These calculations simplify the computation of cut-off rigidities for various arrival directions of cosmic ray particles at the top of the atmosphere using higher order simulation of the earth's magnetic field.

During recent years an extensive balloon launching facility has been set up by the Tata Institute of Fundamental Research at Hyderabad (geographic latitude 17·4° N. and longitude 78·5° E.). In addition to the Tata Institute, scientists from many other countries have also taken advantage of this facility to make balloon launches for cosmic ray and upper atmosphere experiments. Therefore we have made here calculations of the cut-off rigidities as a function of azimuthal and zenith angles over Hyderabad using the latest information on the geomagnetic field. Since the balloons at the ceiling altitudes could drift a few degrees in latitude and longitude, vertical cut-off calculations have also been made in the neighbourhood of Hyderabad. These calculations have been made using the sixth degree simulation of the geomagnetic field by the C.D.C. 3600 computer of the Tata Institute of Fundamental Research.

2. Geomagnetic Theory and Earth's Magnetic Field

The motion of a cosmic ray particle in the earth's magnetic field is described by the Lorentz equation

$$m \frac{d^2\vec{R}}{dt^2} = \frac{Ze}{c}\left(\frac{d\vec{R}}{dt} \times \vec{H}\right) \tag{1}$$

where H is the magnetic field; m and Ze are the relativistic mass and charge of the particle; and C is the velocity of light. Equation (1) can be written in terms of spherical co-ordinates as

$$\begin{aligned}
\frac{\partial v_r}{\partial t} &= \frac{Ze}{mc}(v_\theta H_\phi - v_\phi H_\theta) + \frac{v_\theta^2}{r} + \frac{v_\phi^2}{r} \\
\frac{\partial v_\theta}{\partial t} &= \frac{Ze}{mc}(v_\phi H_r - v_r H_\phi) - \frac{v_r v_\theta}{r} + \frac{v_\phi^2}{r \tan\theta} \\
\frac{\partial v_\phi}{\partial t} &= \frac{Ze}{mc}(v_r H_\theta - v_\theta H_r) - \frac{v_r v_\phi}{r} - \frac{v_\theta v_\phi}{r \tan\theta}
\end{aligned} \tag{2}$$

where v_r, v_θ, v_ϕ are the components of the particle velocity along the radial, co-latitude and east longitude directions and are given in terms of zenith and azimuthal angles by (3).

$$\begin{aligned}
v_r &= \frac{dr}{dt} = v \cos Z \\
v_\theta &= r\frac{d\theta}{dt} = -v \sin Z . \cos A \\
v_\phi &= r \sin\theta \frac{d\phi}{dt} = v \sin Z . \sin A
\end{aligned} \tag{3}$$

where azimuthal angle A is measured east of north and r is measured from the centre of earth and is given by (4).

$$r = b\,(1 - \epsilon^2 \cos^2\lambda) + r'. \tag{4}$$

Here b and ϵ are the polar radius and eccentricity of the earth; and λ and r' are the geographic latitude and the vertical distance from the surface of the earth.

One needs to know the components of magnetic field H_r, H_θ and H_ϕ to solve the sets of simultaneous linear differential equations (2) and (3). For points outside the earth, the geomagnetic potential V due to sources of magnetism inside the earth may be written as

$$V = a \sum_{n=0}^{\infty} \sum_{m=1}^{n} \left(\frac{a}{r}\right)^{n+1} P_n^m (\cos\theta)\,(g_n^m \cos m\phi + h_n^m \sin m\phi) \tag{5}$$

where a is the average radius of the earth; $P_n^m(\cos\theta)$ are the partially normalised associated Legendre functions and g_n^m and h_n^m are the gauss coefficients. The components of earth's magnetic field along the r, θ, ϕ directions are obtained from the equation (5) through

$$H_r = \frac{\partial V}{\partial r}, \quad H_\theta = \frac{1}{r}\frac{\partial V}{\partial \theta}$$

and

$$H_\phi = \frac{1}{r \sin\theta}\frac{\partial V}{\partial \phi}. \tag{6}$$

Experiments in studying the magnetic field, connected with possible external source and with upper-air magnetic properties (in general, using satellites) have been conducted primarily by using magnetometers measuring only the magnitude of the intensity and not its direction. So the gauss coefficients can be calculated using the method of scalar magnetic intensity.[7] Table I gives the value of the gauss coefficients calculated by Jensen and Cain[8] from the measurements of magnetic intensity since 1940, using sixth degree expansion of the equation (5).

3. Method and Procedure

The method employed in the present calculations is similar to that of McCracken *et al.*[6] who have calculated the asymptotic direction of cosmic rays at a distance far removed from the earth's surface for various arrival directions at the top of the atmosphere. The sub-routine given by the above authors to determine the components of geomagnetic field, using sixth degree expansion of the geomagnetic potential, is used in the present computer programme.

TABLE I

Values of the gauss coefficients given by Jesnen and Cain

n	m	$g_n^m \times 10^5$	$h_n^m \times 10^5$	n	m	$g_n^m \times 10^5$	$h_n^m \times 10^5$
1	0	30411·2	0	5	0	1625·6	0
1	1	2147·4	−5798·0	5	1	−3440·7	−79·6
2	0	2403·5	0	5	2	−1944·7	−200·0
2	1	−5125·3	3312·4	5	3	−60·8	459·7
2	2	−1338·1	−157·9	5	4	277·5	242·1
3	0	−3151·8	0	5	5	69·7	−121·8
3	1	6213·0	1487·0	6	0	−1952·3	0
3	2	−2489·8	−407·5	6	1	−485·3	−575·8
3	3	−649·6	21·0	6	2	321·2	−873·5
4	0	−4179·4	0	6	3	2141·3	−340·6
4	1	−4529·8	−1182·5	6	4	105·1	−11·8
4	2	−2179·5	1000·6	6	5	22·7	−111·6
4	3	700·8	43·0	6	6	111·5	−32·5
4	4	−204·4	138·5				

The main approach to the problem is to calculate the minimum momentum needed for a proton to reach the earth in a given arrival direction from infinity. Since the magnetic jaw as defined by Quenby and Wenk[8] occurs at a distance $2/\cos^2\lambda c$ earth radii from the centre of a pure geomagnetic dipole where λc is the geomagnetic latitude, it is sufficient to trace the particle up to 10 earth radii near the geomagnetic equator. The trajectory of the particles can be traced by solving the sets of linear differential equations (2) and (3). These equations are solved by using the Gill modification of the Runge-Kutta integration process.[9]

The initial point on the trajectory is fixed by equations (4) and (7)

$$\theta = \frac{\pi}{2} - \lambda \quad \text{and} \quad \phi = \eta. \tag{7}$$

The initial value of r' in equation (4) is taken to be 35 km. in the present calculations since most of the experiments involving the measurement of primary fluxes are conducted at an altitude $\sim$35 km. from sea-level. The initial components of the velocity of the particle is fixed by the momentum and the given zenith and azimuthal angles. Suitable step-lengths for the integration process are given for various momenta and at various regions of the trajectory, so as to enable the calculations to be accurate as well as fast.

At each step of the integration process the position and the radial velocity of the particle are examined. If the particle intercepts the earth, the initial momentum of the particle is increased by steps of 2·0 GeV/c. till the particle successfully traces its trajectory up to 10 earth radii. Once the particle crosses this limiting point, the momentum is reduced by half the value of the initial step and the integration is repeated from the initial point. If the particle again crosses the limiting point successfully with the new momentum, the value of the momentum is reduced, or is increased if the particle intercepts the earth, by half the value of the earlier step, and the integration is repeated again. This process is continued till an accuracy of 0·01 GeV/c. in the momentum is achieved. The final value of the momentum of the particle gives the cut-off rigidity for protons, for a given zenith and azimuthal angle.

To check for forbidden bands, a wide momentum region around the cut-off rigidity has been scanned with a momentum interval of 0·05 GeV/c. It is found that the penumbral bands at this latitude are negligible.

The above calculations are done for various zenith and azimuthal angles by changing the components of the initial velocity given by equation (3). The vertical cut-off rigidities around Hyderabad have been computed by changing the initial point on the trajectory given by the equations (4) and (7).

For negatively charged particles, equation (1) can be re-written in the form

$$m \frac{d^2\vec{R}}{dt^2} = -\frac{Ze}{c}\left(\frac{d\vec{R}}{dt} \times H\right). \tag{8}$$

Using this, we have computed the cut-off rigidities for negatively charged particles in the east-west plane over Hyderabad.

RESULTS

The computed values of the cut-off rigidities for various arrival directions of cosmic ray particles at the top of the atmosphere over Hyderabad are presented in Table II. The vertical cut-off rigidities in the neighbourhood of Hyderabad are listed in Table III. It is also found that the variation of cut-off rigidities with zenith and azimuthal angles around Hyderabad is

TABLE II

Cut-off rigidities in GV for various zenith and arimuthal angles for positively charged particles over Hyderabad, India (Geog. Lat. 17·6° N. and long. 78·5° E.)

Vertical cut-off rigidity = 16·92 GV

Azimuth	Zenith							
	10°	20°	30°	40°	50°	60°	70°	80°
0°	17·45	18·20	19·20	20·52	22·17	24·22	26·67	30·39
10°	17·70	18·77	20·08	21·78	23·92	26·61	29·95	34·92
20°	17·95	19·27	20·92	23·02	25·67	29·05	33·36	39·61
30°	18·17	19·73	21·70	24·20	27·34	31·41	36·70	44·20
40°	18·36	20·14	22·39	25·23	28·86	33·58	39·77	48·36
50°	18·52	20·47	22·95	26·09	30·11	35·39	42·36	51·89
60°	18·61	20·70	23·34	26·72	31·05	36·73	44·33	54·77
70°	18·66	20·83	23·58	27·08	31·59	37·55	45·58	56·59
80°	18·66	20·84	23·59	27·14	31·72	37·78	45·95	57·28
90°	18·61	20·73	23·42	26·89	31·41	37·39	45·48	56·77
100°	18·51	20·52	23·08	26·36	30·66	36·39	44·14	54·97
110°	18·36	20·20	22·53	25·58	29·52	34·78	41·95	52·02
120°	18·16	19·77	21·84	24·53	28·03	32·70	39·03	47·95
130°	17·95	19·28	21·03	23·33	26·30	30·22	35·58	43·09
140°	17·70	18·73	20·14	21·98	24·39	27·55	31·77	37·66

TABLE II (*Contd.*)

Azimuth	Zenith							
	10°	20°	30°	40°	50°	60°	70°	80°
150°	17·42	18·17	19·20	20·61	22·45	24·83	27·91	32·14
160°	17·14	17·58	18·27	19·23	20·52	22·20	24·33	27·02
170°	16·86	17·02	17·36	17·92	18·73	19·80	21·14	22·77
180°	16·59	16·45	16·52	16·73	17·14	17·72	18·45	19·33
195°	16·23	15·73	15·39	15·22	15·16	15·22	15·39	15·58
210°	15·92	15·14	14·53	14·08	13·73	13·52	13·34	13·27
225°	15·70	14·72	13·92	13·30	12·83	12·45	12·16	11·97
240°	15·58	14·47	13·59	12·89	12·34	11·92	11·64	11·45
255°	15·53	14·41	13·53	12·83	12·30	11·91	11·64	11·52
270°	15·59	14·53	13·72	13·09	12·64	12·33	12·14	12·09
285°	15·77	14·84	14·16	13·66	13·33	13·14	13·11	13·20
300°	15·98	15·30	14·83	14·52	14·39	14·41	14·58	14·86
315°	16·30	15·91	15·70	15·67	15·83	16·14	16·58	17·11
330°	16·67	16·64	16·77	17·11	17·66	18·36	19·22	19·95
345°	17·08	17·42	17·98	18·80	19·84	21·14	22·64	24·73

TABLE III

Vertical cut-off rigidities in GV for various geographic latitudes and longitudes around Hyderabad

Latitude	Longitude					
	73°	75°	77°	79°	81°	83°
15°	16·94	17·00	17·05	17·10	17·14	17·18
16°	16·88	16·94	17·00	17·05	17·09	17·13
17°	16·81	16·87	16·93	16·98	17·03	17·06
18°	16·72	16·79	16·84	16·89	16·94	16·97
19°	16·61	16·68	16·74	16·79	16·84	16·88
20°	16·49	16·56	16·62	16·68	16·72	16·76
21°	16·35	16·42	16·48	16·54	16·59	16·63
22°	16·19	16·26	16·33	16·38	16·43	16·48
23°	16·01	16·09	16·16	16·21	16·27	16·30

similar to that at Hyderabad. The cut-off rigidities for negatively charged particles in the east-west plane over Hyderabad for various zenith angles are presented in Table IV.

TABLE IV

Cut-off rigidities in GV for various zenith angles in the east-west plane for negatively charged particle over Hyderabad, India

Vertical cut-off rigidity = 16·10 GV

Zenith angle	Cut-off rigidities	
	East	West
5°	15·45	16·82
10°	14·87	17·63
15°	14·35	18·54
20°	13·89	19·56
25°	13·48	20·71
30°	13·11	22·00
35°	12·79	23·43
40°	12·52	25·11
45°	12·28	26·99
50°	12·07	29·13
55°	11·89	31·59
60°	11·75	34·43
65°	11·64	37·72
70°	11·57	41·56
75°	11·51	46·08
80°	11·49	51·44

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