Statistics of Mesoscopic Fluctuations of Quantum Capacitance

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Abstract

The Thouless formula $G = (e^2/h)(E_c/\Delta)$ for the two-probe dc conductance G of a d-dimensional mesoscopic cube is re-analysed to relate its quantum capacitance C_Q to the reciprocal of the level spacing Δ. To this end, the escape time-scale τ occurring in the Thouless correlation energy $E_c = \hbar/\tau$ is interpreted as the *time constant* $\tau = RC_Q$ with $RG \equiv 1$, giving at once $C_Q = (e^2/2\pi\Delta)$. Thus, the statistics of the quantum capacitance is directly related to that of the level spacing, which is well known from the Random Matrix Theory for all the three universality classes of statistical ensembles. The basic questions of how intrinsic this quantum capacitance can arise purely quantum-resistively, and of its observability *vis-a-vis* the external geometric capacitance that combines with it in series, are discussed.

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The non-self-averaging nature of quantum conductance, the transmittance and the energy-level spacing of a mesoscopic system, due ultimately to quantum coherence and the interference, has motivated extensive studies of the statistics of their sample-to-sample fluctuations.[1] For a given sample, however, these reproducible fluctuations manifest as random functions of some externally controlled parameters such as the Fermi energy, or the Aharonov-Bohm magnetic flux, etc. Microscopic analysis of the statistics of these fluctuations in disordered electronic structures having diffusive transport has been based on the impurity-diagrammatic perturbative technique, while the macroscopic approaches generally appealed to the random matrix theory.[1] Fluctuations of the wave transmission coefficient and of the eigenvalue spacing for ballistic microstructures (classically chaotic cavities) have been treated through the semiclassical and the random matrix approaches.[2,3] Very recently, quantum capacitance[4,5] has been added to the growing list of these non-selfaveraging quantities, where the authors[6] have calculated the statistics of the mesoscopic capacitance fluctuations for the case of a chaotic cavity coupled capacitatively to a backgate. This has involved calculating the Wigner delay time which is given by the energy derivative of the phase associated with the scattering matrix. The latter in turn was obtained from a global random matrix theory for all the three universality (symmetry) classes of the statistical ensembles. It may be noted in passing that, for the case of a single channel, the distribution of the time delay for a disordered mesoscopic system was obtained earlier using the invariant imbedding approach.[7] In the

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present work we have addressed the problem of calculating the statistics of the quantum capacitance of a mesoscopic disordered conductor *directly* in terms of a generally valid argument predicated on the Thouless expression for the conductance $G = (e^2/h)(E_c/\Delta)$ for a d-dimensional cube. Here Δ is the energy level spacing at the Fermi level and $E_c = \hbar/\tau$ the Thouless (correlation) energy, where τ is the time scale of escape out of the sample. Results for the *RC-time constant* for the ballistic, the diffusive as well as for the possibly anomalous diffusive regime at the mobility edge are reported. We also address the physical question of how a reactive element such as the capacitance is determined resistively through the Thouless conductance expression.

Starting from the Thouless expression for the conductance $G = (e^2/h)(E_c/\Delta)$, and recalling that $E_c = \hbar/\tau$, where $\tau \equiv RC_Q$ is to be interpreted as the *RC-time constant* with C_Q as the effective (quantum) capacitance, and $R(\equiv 1/G)$ the series quantum resistance, we get at once $C_Q = (e^2/2\pi\Delta)$. Here the level spacing Δ scales as L^{-d} for a d-dimensional sample. Thus, we notice immediately that the quantum capacitance C_Q is a volume effect a (proportional to L^{-3} in three dimensions), quite unlike the classical (geometrical) capacitance that scales as L^{-1} in three dimensions.

Now the problem of finding the statistics of C_Q reduces to that of finding the level-spacing distribution. Defining the dimensionless capacitance $x = C_Q/\bar{C}_Q$, with $\bar{C}_Q$ as the ensemble averaged capacitance, we can at once write down the probability distribution $P_\beta(x)$ for the three

universality classes, labelled by the symmetry parameter $\beta = 1$, 2 and 4 corresponding, respectively, to the Gaussian Orthogonal Ensemble (GOE), the Gaussian Unitary Ensemble (GUE) and the Gaussian Symplectic Ensemble (GSE), as

$$P_1(x) = \frac{1}{2\pi x^3} exp[-(1/\pi x^2)] \; , \tag{1}$$

with the average capacitance $\bar{C}_Q = (e^2/4 < \Delta >)$;

$$P_2(x) = \frac{\pi}{2x^4} exp[-(\pi/4x^2)] \; , \tag{2}$$

with the average capacitance $\bar{C}_Q = (2e^2/\pi^2 < \Delta >)$; and

$$P_4(x) = (81\pi^2/128x^6) exp[-(9\pi/16x^2)], \tag{3}$$

with the average capacitance $\bar{C}_Q = (16e^2/9\pi^2 < \Delta >)$. In all the three cases above, the quantum capacitance distribution has a single peak and a long tail. In fact the variance diverges (weakly, i.e., logarithmically) for the GOE case. For a mesoscopic sample of interest, we have typically $\Delta \sim$ 10 mK, and this gives $C_Q \sim 20$ fF.

The fact that the quantum capacitance is determined resistively via the Thouless formula has several consequences for the RC relaxation time of the mesoscopic sample, coupled capacitatively to a backgate. This can be seen as follows for the various transport regimes of interest.

Ballistic regime

Here the Thouless energy $E_c \equiv (\hbar^2/2m) \int | \bigtriangledown \psi |^2 d^d x$, (being the extra

kinetic energy associated with the phase-twisting of the wavefunction ψ from the periodic to the anti-periodic boundary condition) scales as $E_c \sim L^{-1}$. Recalling that $\Delta \sim L^{-d}$, we get at once $G \sim L^{d-1}$ (the two-probe Sharvin conductance of an ideal metal with L^{d-1} transverse channels). The corresponding RC-time constant τ scales as $\tau \sim L$.

Diffusive limit

Arguing as above, we have $E_c \sim 1/L_{arc}$, where L_{arc} is the arc- length of a random walk with the end-to-end distance $\sim L$ (sample size). Thus, $L_{arc} \sim L^2$ giving RC-time constant $\tau \sim L^2$.

Anomalous diffusion

At and near the mobility edge when the sample size $L \ll \xi$ (the coherence length), we can expect the diffusion to become anomalous in that $L_{arc} \sim L^{2\eta}$ with $\eta \neq 1$. Reasoning as above, this would give RC time constant $\tau \sim L^{2\eta}$.

The above clearly shows the characteristically different scaling behaviour of τ for the three different transport regimes.

We now return to the question of how a reactive element such as the quantum capacitance can arise concomitantly with the quantum resistance. The whole point is that on the one hand the quantum capacitance (unlike its classical geometrical counterpart) involves the energy cost of promoting the piled-up electrons across the energy-level spacing, while on the other the latter also represents the typical energy mismatch (obstructing the electron diffusion) which is involved in determining the con-

ductance via the Thouless formula. Indeed, the larger the level spacing the greater the promotional energy cost, and hence the smaller the capacitance. Concomitantly, the larger energy spacing implies larger resistance. The two are functionally related, and conspire to give the RC-time constant that scales with the sample size as derived above for the three metallic regimes.

As for the observability of the quantum capacitance, we note that it combines with the classical (geometrical) capacitance in series.[5] Thus, it is effective only for sufficiently small samples inasmuch as $C_Q = e^2/2\pi\Delta \sim L^3$. while $C_{classical} \sim L$ for a three-dimensional mesoscopic sample.

Finally, our present approach to mesoscopic quantum capacitance, based as it is on the Thouless conductance formula, shares the latter's generality inasmuch as the single electron coupling and the level spacing may be generalized to the coupling of correlated electron excitations and the associated excitation energy-spacings.[8]

In conclusion, we have derived the statistics of mesoscopic capacitance fluctuations basing directly on the generally valid Thouless conductance formula by identifying the escape time as the *RC-time constant*. Inasmuch as this quantum capacitance combines in series with the classical geometrical capacitance, the speed of a mesoscopic circuit element must be limited ultimately by this quantum *RC-time constant*.

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