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Efficiency and current reversals in spatially inhomogeneous ratchets

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Efficiency of generation of net unidirectional current in an adiabatically driven symmetric periodic potential system is studied. The efficiency shows a maximum, in the case of an inhomogeneous system with spatially varying periodic friction coefficient, as a function of temperature. The ratchet is not most efficient when it gives maximum current. The direction of current may also be reversed as a function of noise strength when, instead, an asymmetric periodic potential is considered.

Motion of a particle in a medium generally involves frictional resistance. The frictional resistance or the friction coefficient influences the dynamical properties, such as the rate of relaxation of the system, but does not affect the equilibrium properties . In some inhomogeneous systems the friction coefficient could be space dependent and such systems are not rare in nature. The motion of particles in such inhomogeneous system is difficult to describe theoretically because the effect of the space dependent friction cannot be incorporated as due to an effective potential. The motion of a Brownian particle in an inhomogeneous system with periodically varying space-dependent friction coefficient is of much recent interest [1–12] .

Recently it has been shown that mobility of over-damped Brownian particles in washboard potential exhibit stochastic resonance in inhomogeneous systems without the application of periodic input signal [8,9], i.e, the probability current shows a maximum as function of temperature. The mobility also exhibits a resonance like phenomenon as a function of external field strength (constant in time) and noise induced slowing down of particle in an appropriate parameter regime [9]. These effects could not have been possible in over-damped homogeneous systems with uniform friction coefficient. Also, it has been shown that in these inhomogeneous systems it is possible to obtain net unidirectional current in periodic potential systems in a thermal bath but subjected to an external zero mean time-periodic field or random forces [3–5,7]. The efficiency of such rocked thermal ratchet system to achieve unidirectional current has been examined. For this we have followed the method of stochastic energetics [13–15]. Earlier it was claimed that thermal fluctuations monotonically reduce the efficiency of an adiabatically rocked thermal ratchet [14]. In the present work we mainly explore the possibility of obtaining a current in an inhomogeneous medium with varying space-dependent friction coefficient when quasi-statically rocked by a zero-time-averaged external field. Also, the dependence of the efficiency of operation of such systems on thermal fluctuations present in the system is examined. In these systems we show that thermal fluctuations facilitate energy conversion by ratchet systems , contrary to earlier claims [14]. Moreover the direction of current may also be reversed as a function of noise strength in the presence of asymmetric periodic potential in the same quasi-static limit.

As usual, the efficiency of a machine(system) is defined as the ratio of useful work accomplished by the system to the total energy expended on the system to get the work done. We study the motion of an over-damped Brownian particle in a potential $V(q)$ subjected to a space dependent friction coefficient $\gamma(q)$ and an external force field $F(t)$ at temperature T. The motion is described by the Langevin equation [5,8–10,16]

$$\frac{dq}{dt} = -\frac{(V'(q) - F(t))}{\gamma(q)} - k_B T \frac{\gamma'(q)}{[\gamma(q)]^2} + \sqrt{\frac{k_B T}{\gamma(q)}}\xi(t), \tag{1}$$

where $\xi(t)$ is a randomly fluctuating Gaussian white noise with zero mean and correlation : $<\xi(t)\xi(t')>= 2\delta(t-t')$. We take $V(q) = V_0(q) + qL$, where $V_0(q+2n\pi) = V_0(q) = -sin(q)$, n being any natural integer. L is a constant force (load) representing the slope of the washboard potential against which the work is done. Also, we take the friction coefficient $\gamma(q)$ to be periodic : $\gamma(q) = \gamma_0(1 - \lambda \sin(q + \phi))$, where ϕ is the phase difference with respect to $V_0(q)$. The equation of motion is equivalently given by the Fokker-Planck equation

$$\frac{\partial P(q,t)}{\partial t} = \frac{\partial}{\partial q}\frac{1}{\gamma(q)}[k_B T \frac{\partial P(q,t)}{\partial q} + (V'(q) - F(t))P(q,t)]. \tag{2}$$

This equation can be solved for the probability current j when $F(t) = F_0 =$ constant, and is given by [8,9,17]

$$j = \frac{k_B T(1 - \exp(-2\pi(F_0 - L)/k_B T))}{\int_0^{2\pi} \exp(\frac{-V_0(y)+(F_0-L)y}{k_B T})dy \int_y^{y+2\pi} \gamma(x) \exp\frac{V_0(x)-(F_0-L)x)}{k_B T}}dx. \tag{3}$$

It may be noted that even for $L = 0$, $j(F_0)$ may not be equal to $-j(-F_0)$ for $\phi \neq 0, \pi$. This fact leads to the rectification of current (or unidirectional current) in the presence of an applied a.c field $F(t)$. We assume $F(t)$ changes slowly enough, i.e, its frequency is smaller than any other frequency related to relaxation rate in the problem. For a field $F(t)$ of a square wave amplitude F_0, an average current over the period of oscillation is given by, $<j> = \frac{1}{2}[j(F_0) + j(-F_0)]$. This particle current can even flow against the applied load L and thereby store energy in useful form. In the quasi-static limit following the method of stochastic energetics it can be shown [14,15] that the input energy R (per unit time) and the work W (per unit time) that the ratchet system extracts from the external noise are given by $R = \frac{1}{2}F_0[j(F_0) - j(-F_0)]$ and $W = \frac{1}{2}L[j(F_0) + j(-F_0)]$ respectively. Thus the efficiency (η) of the system to transform the external fluctuation to useful work is given by

$$\eta = \frac{L[j(F_0) + j(-F_0)]}{F_0[j(F_0) - j(-F_0)]}. \tag{4}$$

Earlier, it has been found that for homogeneous systems, $\gamma(q) = \gamma_0, \eta$ monotonically decreases as T is increased from zero in the quasi-static limit [14]. We find, however, that in the case of inhomogeneous systems, when $\gamma(q)$ is periodic, it is possible to maximize η (see below) as a function of temperature by suitably choosing the parameters ϕ, λ and F_0 for given L.

It is found, numerically, that the effect of λ, the amplitude of periodic modulation of $\gamma(q)$, for instance on $<j>$ is more pronounced for larger value of $\lambda (0 \leq \lambda < 1)$. We choose $\lambda = 0.9$ throughout our work. Contrary to the homogeneous friction case where the net current goes to zero , the detailed analysis of the Eqn. (3) shows that the net current $<j>$ quickly saturates monotonically to a value equal to $\frac{-\lambda}{2}\sin\phi$ as a function of F_0 irrespective of the value of temperature T. All the interesting features are captured at smaller values of F_0 itself. We shall confine ourselves to $F_0 < 1$, the threshold value at which the barrier of the potential $V_0(q) = -\sin q$ just disappears. ϕ determines the direction of asymmetry of the ratchet. For $\phi > \pi$, we have a forward moving ratchet (current flowing in the positive direction) and for $\phi < \pi$ we have the opposite, in the presence of external quasi-static force $F(t)$. When the system is homogeneous $(\lambda = 0)$, $<j>= 0$ for $L = 0$ and for all values of F_0 and T, because the potential $V(q)$ is symmetric. However, when $\lambda \neq 0, <j> \neq 0$ for all $\phi \neq 0, \pi$. Fig.(1) clearly illustrates this. In fig.(1) we have plotted $<j>$ (in dimensionless units) for various values of F_0 at $\phi = 1.3\pi$ as a function of temperature T (in dimensionless units). Henceforth all our variables like $< j >, F_0, T$ are in dimensionless units [17]. The inset shows the behaviour of $< j >$ with temperature for various ϕ when $F_0 = 0.5$. We have chosen $L = 0.02$ (corresponding to a positive mean slope of $V(q)$) and $F_0 = 0.3$. One would expect, for $\lambda = 0$, $< j >< 0$ for all values of F_0 and $T \neq 0$. Fig.(1) shows that $< j >> 0$ for all values of ϕ (the net current being up against the mean slope of $V(q)$). Though the result is counterintuitive, but it is still understandable. For $j(-F_0)$ the particle is expected to have acquired higher mean velocity before it hits the large $\gamma(q)$ region and hence faces maximum resistive force and therefore though $j(-F_0)$ is negative it is small in magnitude compared to $j(F_0)(> 0)$, where the particle faces just the reverse situation. The average current $< j >$ exhibits maximum as a function of temperature. At large value of temperature, $< j >$ becomes negative. In this regime of high temperature the ratchet effect disappears and the current which is negative is in response to the load. The peaking behaviour of $< j >$ as a function of T is due to the synergetic effect of the thermal fluctuations and the space dependent friction coefficient $\gamma(q)$. Of course, this result is obtained in the quasi-static limit when the time scale of variation of $F(t)$ is much larger compared to any other relevant time scales involved in the system. Fig.(2) shows the efficiency η of flow of current against the load (slope) L. The parameter values chosen for figs.(1 and 2) are same. *η shows a maximum as a function of temperature.* It is to be noted that the temperature corresponding to maximum efficiency is not close to the temperature at which the current $< j >$ is maximum. From this we conclude that *the condition of maximum current does not correspond to maximum efficiency of current generation.* This fact has been pointed out earlier but in a different system [15]. Fig.(3) shows how the temperature $T = T_P^j$ at which $< j >$ peaks vary vis-a-vis the temperature $T = T_P^\eta$ (inset) at which η peaks as a function of F_0 for various values of ϕ as shown in the figure. We find that $T_P^\eta \neq T_P^{<j>}$ for any value of F_0 and ϕ.

In fig.(4) we have plotted the average current $< j >$ in the absence of load as function of temperature for various values of F_0 for given $\phi = 1.3\pi$. It shows that in the absence of load L, $< j >$ never changes sign from positive to negative. It is possible, however, to get a current reversal as a function of temperature if, instead of a symmetric potential one considers an asymmetric potential $V_0(q) = -\sin q - \frac{\mu}{4}\sin 2q$ (where μ lies between 0 and 1, describes an asymmetry parameter) [6]. In fig.(5) we have plotted $< j >$ in the absence of load as function of temperature with asymmetry parameter $\mu = 1$. In fig.(5) we notice that the current reverses its direction as function of temperature or noise strength. This current reversal with temperature in the adiabatic limit is possible only when the system is inhomogeneous. In homogeneous systems (but asymmetric potential) current reversal is possible when the frequency of the field swept $F(t)$ is large [18]. The initial rise in the value of $< j >$ in fig. (5), in the asymmetric potential case, can mainly be attributed to the existence of two separate threshold values of F_0 for flow of current in the two directions. Here the current reversals arise due to the combined effect of ϕ and μ. It can be noted that in the parameter range that we have considered the direction of current for $\phi = 0$ and $\mu \neq 0$ is opposite to that when $\phi \neq 0$ and $\mu = 0$.

We, thus, conclude that it is possible to achieve reasonable efficiency in obtaining net unidirectional current in an adiabatically rocked ratchet in an inhomogeneous system with space dependent friction coefficient by suitably tuning the parameter values of the system. The efficiency shows a maximum as a function of temperature as does the net current $< j >$, though they do not occur at the same temperature. In our case thermal fluctuations facilitate the energy conversion. Also current reversal is possible in inhomogeneous systems even in an adiabatically rocked but asymmetric potential ratchet system.

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FIGURE CAPTIONS .

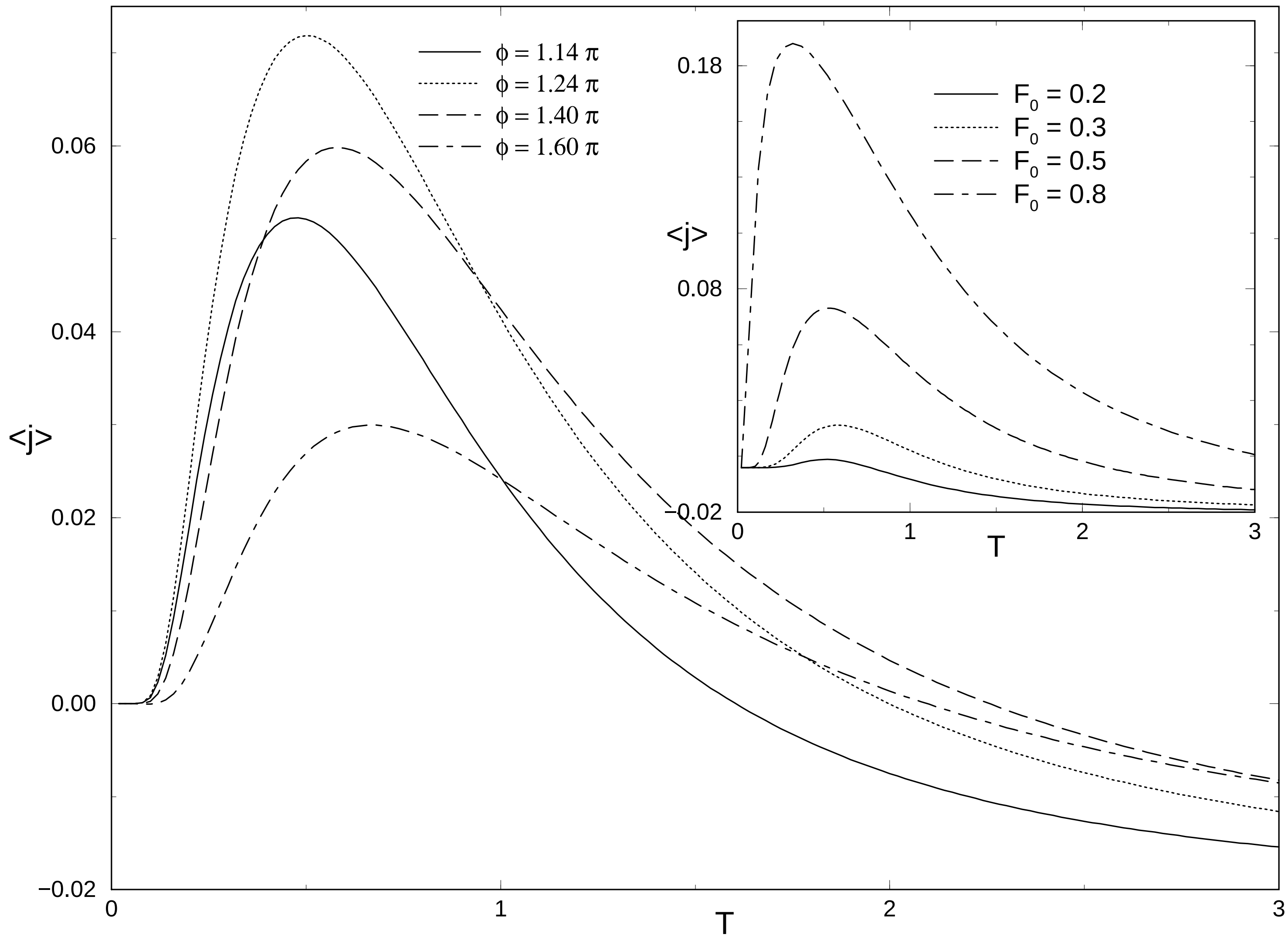
Fig. 1. The net current $< j >$ as a function of T for various values of ϕ at $F_0 = 0.5$. The inset shows the variation of $< j >$ with T for different F_0 at $\phi = 1.3\pi$. $L = 0.02$ for both the figures.

Fig. 2. Efficiency η as a function of T for various values of ϕ at $F_0 = 0.5$. The inset shows the variation of η as a function of T for different values of F_0 at $\phi = 1.3\pi$. $L = 0.02$ for both the figures.

Fig. 3. Temperature T_p^j, corresponding to peak current as a function of F_0 for various values of ϕ. The inset shows the variation of temperature T_p^η, corresponding to peak efficiency as a function of F_0 for various values of ϕ. $L = 0.02$ in this case

Fig. 4. Current $< j >$ as a function of T for various values of F at $\phi = 1.3\pi$. $L = 0$ in this case.

Fig. 5. Current $< j >$ in asymmetric potential for various values of F (for diferent values of ϕ in the inset) at $\phi = 1.3\pi$(at $F = 0.5$ in the inset) when load $L = 0$.



<j>
T
0.06
0.04
0.02
0.00
−0.02
0
1
2
3
ϕ = 1.14 π
ϕ = 1.24 π
ϕ = 1.40 π
ϕ = 1.60 π
0.18
0.08
−0.02
F0 = 0.2
F0 = 0.3
F0 = 0.5
F0 = 0.8

FIG. 1.

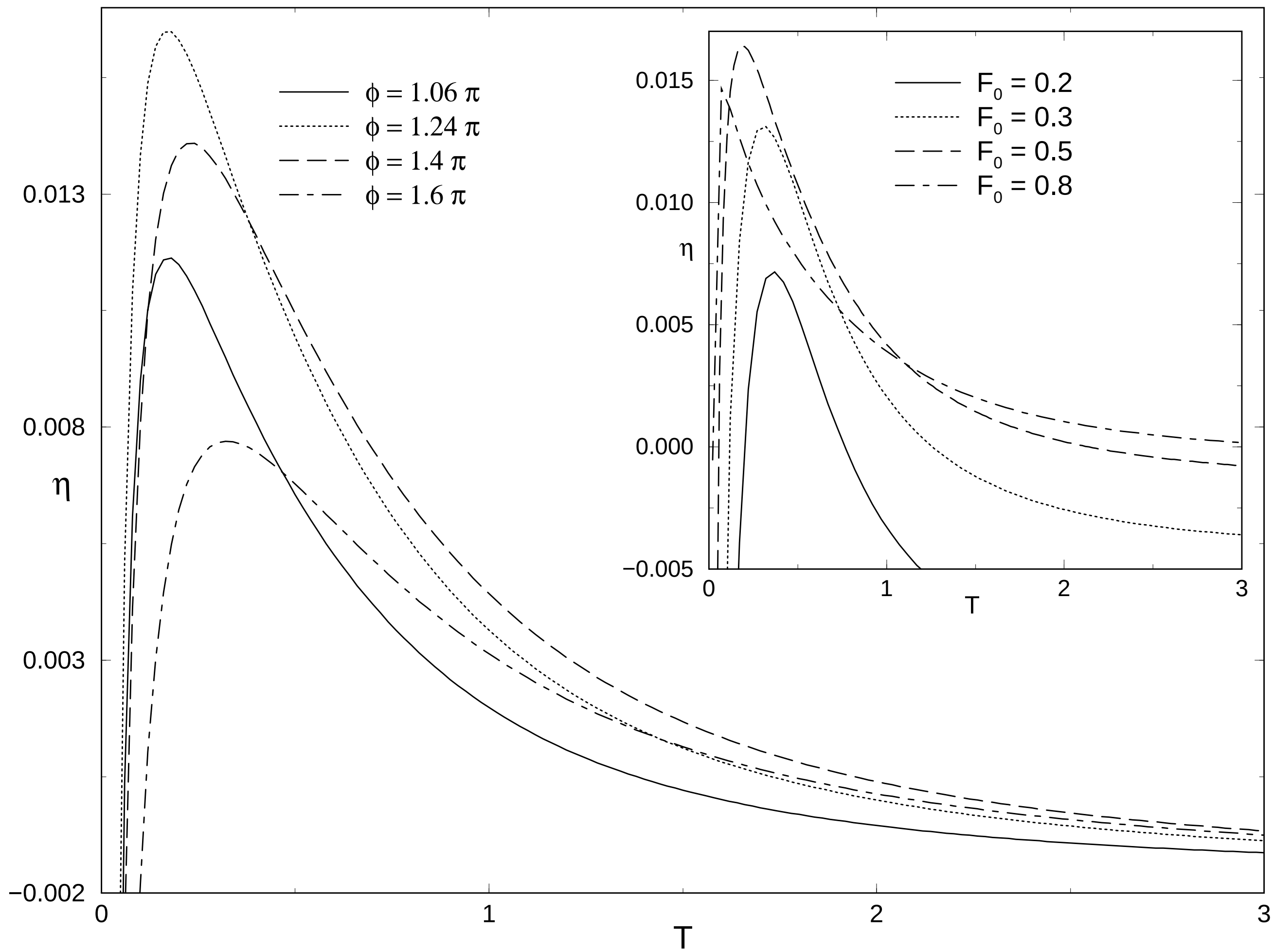

φ = 1.06 π
φ = 1.24 π
φ = 1.4 π
φ = 1.6 π
η
T
0.013
0.008
0.003
−0.002
0
1
2
3
F0 = 0.2
F0 = 0.3
F0 = 0.5
F0 = 0.8
0.015
0.010
0.005
0.000
−0.005

FIG. 2.

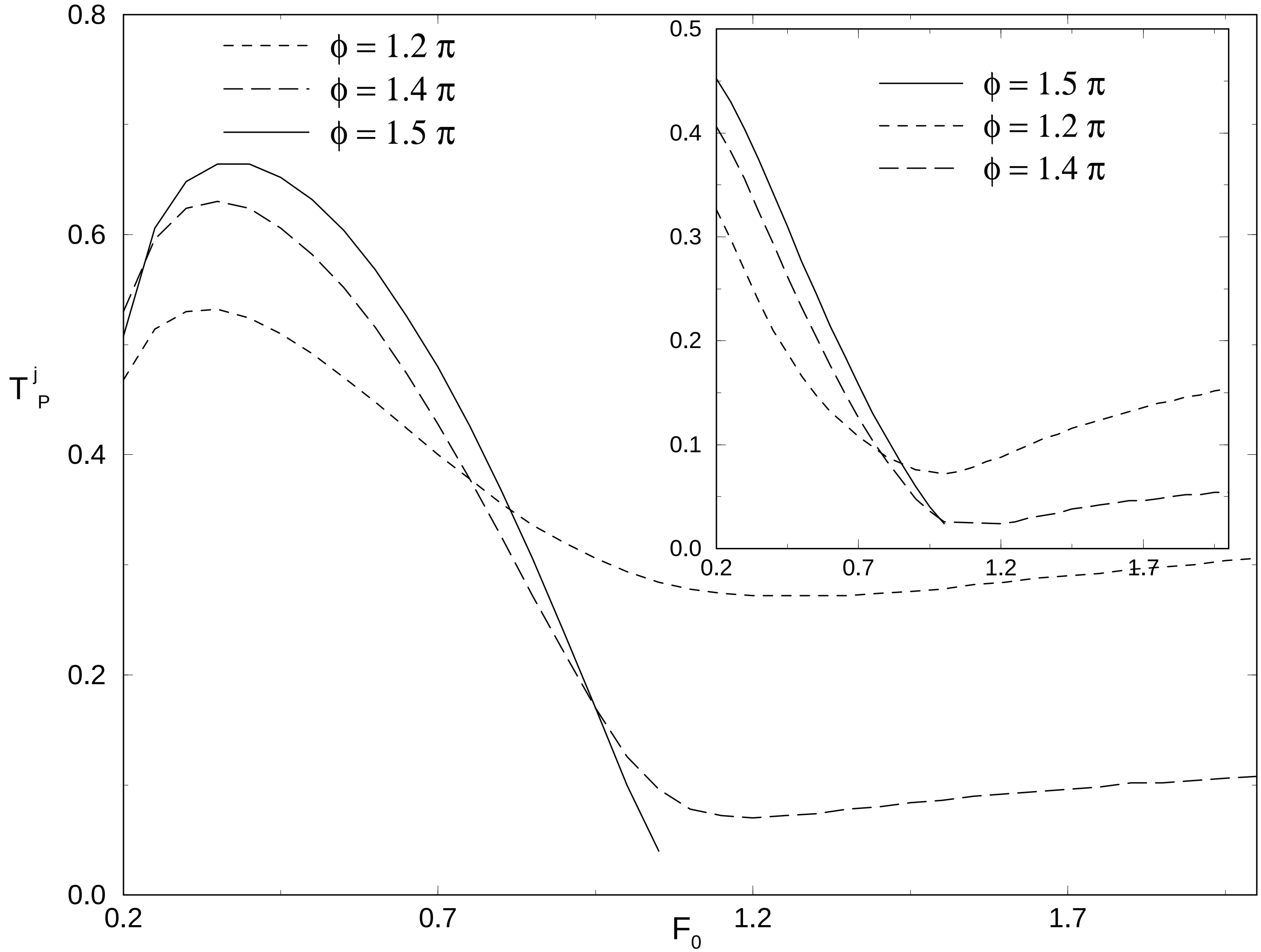

ϕ = 1.2 π
ϕ = 1.4 π
ϕ = 1.5 π
0.8
0.6
0.4
0.2
0.0
0.2
0.7
1.2
1.7
T^{j}_{P}
F_0
0.5
0.4
0.3
0.2
0.1
0.0
0.2
0.7
1.2
1.7
ϕ = 1.5 π
ϕ = 1.2 π
ϕ = 1.4 π

FIG. 3.

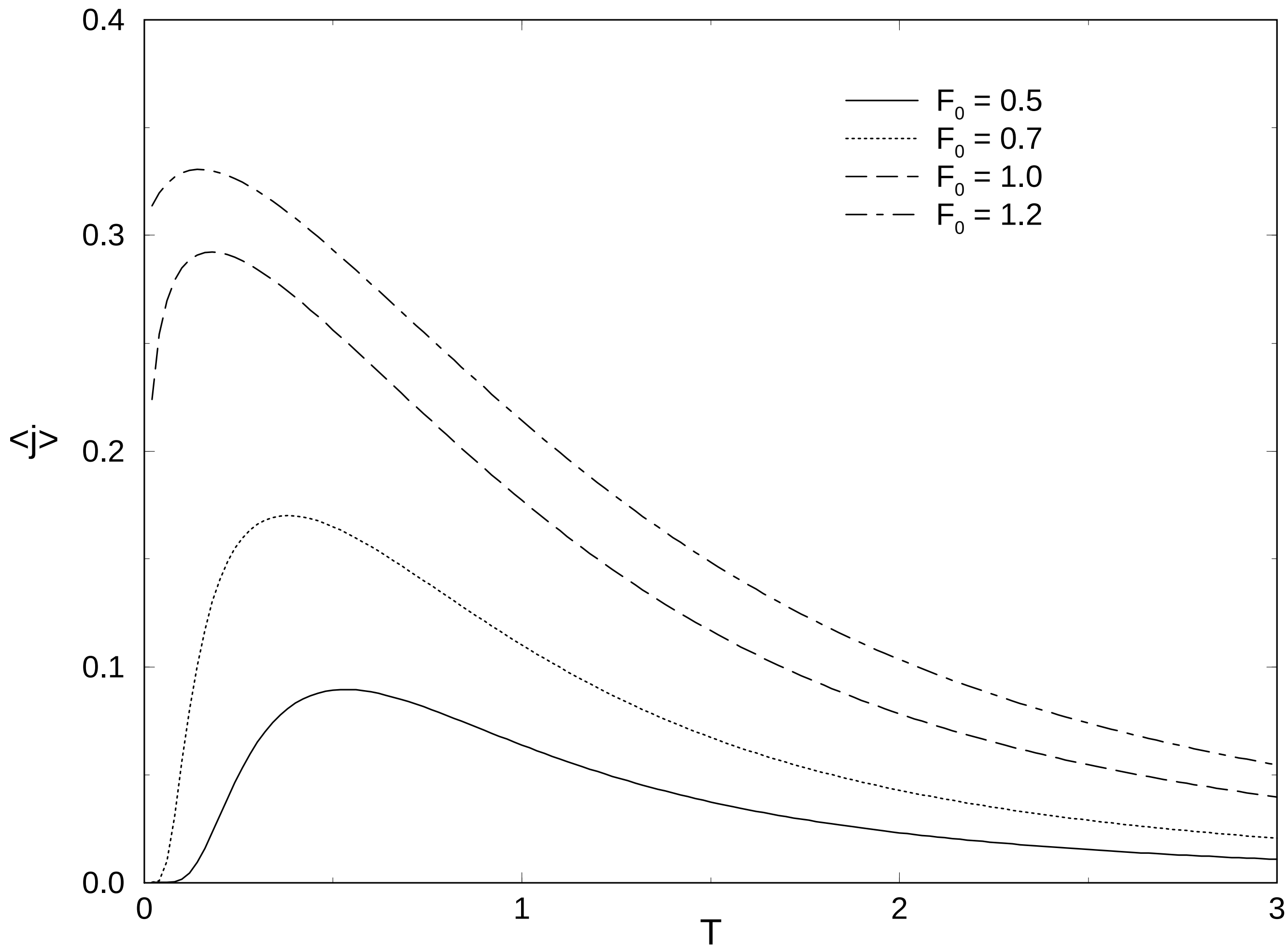
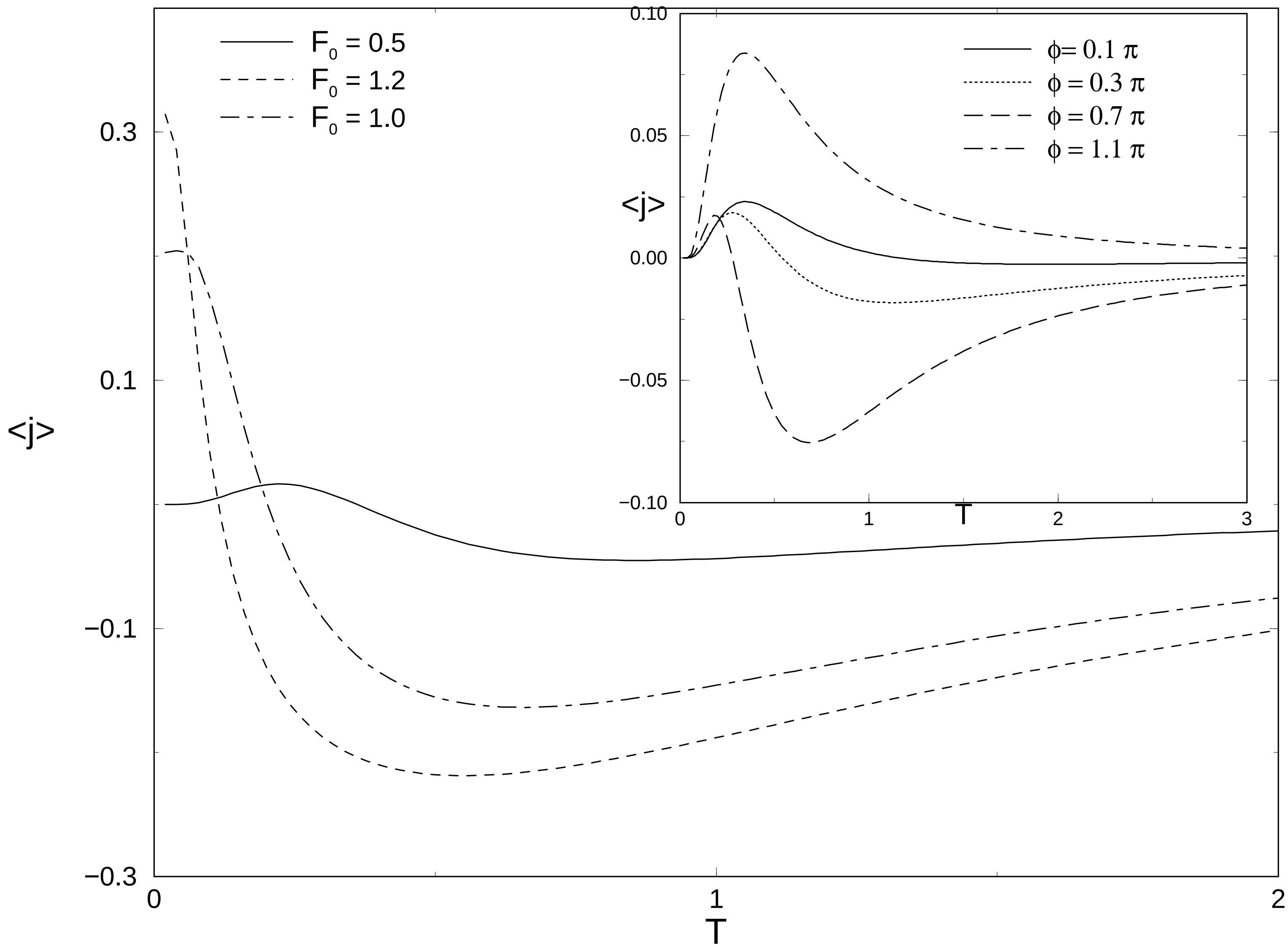


FIG. 4.



F0 = 0.5
F0 = 1.2
F0 = 1.0
<j>
T
0.3
0.1
−0.1
−0.3
0
1
2
φ= 0.1 π
φ = 0.3 π
φ = 0.7 π
φ = 1.1 π
0.10
0.05
0.00
−0.05
−0.10
3

FIG. 5.