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Quantum beats from channeling relativistic electrons

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Abstract. Possibility has been explored of periodic time variation of intensity (quantum beats) of radiation emitted from relativistic electrons channeling along the ⟨110⟩ axis of a crystal of f.c.c. (diamond) lattice structure.

Keywords. Quantum beats; electron channeling; channeling radiation.

1. Introduction

After the exploration of possibility of emission of a new radiation—the channeling radiation—from relativistic electrons and positrons moving along major crystal axes or in between major crystal planes (Kumakhov 1976), much theoretical progress has been made to understand the characteristics of this radiation. In particular Kumakhov (1977), Kumakhov and Wedell (1977), Wedell (1978) and Pantell and Alguard (1979) have contributed much towards the understanding of the channeling radiation. But theoretical treatments of these (and other) authors are valid only for zero-time resolution (infinite time uncertainty) which is not suitable to study the coherent character of the channeling radiation. The purpose of the present paper is to explore the possibility of coherent character of the channeling radiation when relativistic electrons pass along the ⟨110⟩ axis of a f.c.c. (diamond) crystal.

2. Potential field around a ⟨110⟩ row

Projection of some ⟨110⟩ rows of a f.c.c. (diamond) crystal on the transverse (110) crystal plane has been shown in figure 1. We take the z-axis of the cartesian coordinate system (x, y, z) along the ⟨110⟩ rows so that the set (x, y) specifies the transverse space. We take the origin (0, 0) of the transverse space (x, y) at the crystal row R, and choose the y-axis along the ⟨001⟩ crystal direction (see figure 1). Let (x_i, y_i) or equivalently (D_i, ϕ_i), be the (transverse) coordinates of the (⟨110⟩) row R_i. Here,

$$D_i^2 = (x_i^2 + y_i^2), \quad \phi_i = \tan^{-1}(y_i/x_i). \tag{1}$$

We treat the potential of a ⟨110⟩ row as a static continuum potential. In the Lindhard's continuum field approximation (Gemmell 1974), the potential of the row R_i, say V_{R_i}, at the point (r, ϕ) is given by

$$V_{R_i}(r, \phi) = -(2Ze^2/d) \ln (1 + 3a^2/\xi_i^2)^{1/2}, \tag{2}$$

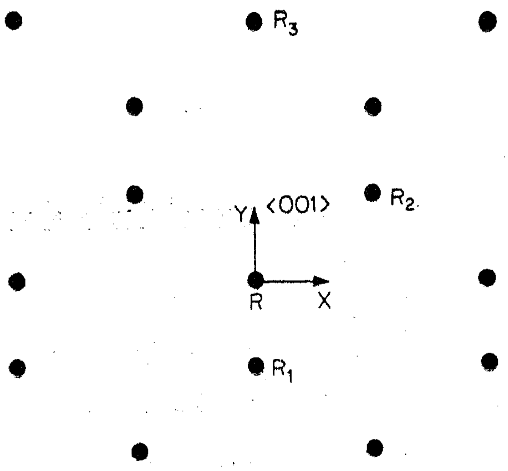

Figure 1. Projection of ⟨110⟩ rows of a f.c.c. (diamond) crystal on the (100) lattice plane ($RR_1 = D$, $RR_2 = \sqrt{3}\, D$, $RR_3 = 3D$).

where,

$$\xi_i^2 = r^2 + D_i^2 + 2rD_i f_i(\phi), \tag{3}$$

with

$$f_i(\phi) = -(\cos\phi_i \cos\phi + \sin\phi_i \sin\phi). \tag{4}$$

In equation (2), Z is the atomic number of the crystal atom, e is the electronic charge, d is the lattice constant, and a is the screening radius. The polar coordinates r and ϕ are related with the cartesian coordinates x, y as follows:

$$x = r\cos\phi, \qquad y = r\sin\phi. \tag{5}$$

For $\xi_i > \sqrt{3}\, a$, the logarithmic term in (2) may be expanded as a series, and for

$$(r^2 + D_i^2) \gg 2rD_i f_i(\phi), \tag{6}$$

$1/\xi_i^2$ may be approximated by

$$1/\xi_i^2 \simeq [1/(r^2 + D_i^2) - 2rD_i f_i(\phi)/(r^2 + D_i^2)^2]. \tag{7}$$

It may be seen that except the row $R \equiv (0, 0)$, all rows satisfy the condition (6) and $\xi_i > \sqrt{3}\, a$ in the vicinity of the row R (r small). Therefore, for the row $R_i \neq R$, we may put (2) (for small r) in the form

$$V_{R_i} \simeq Q \sum_{p=1}^{\infty} \sum_{m=0}^{p} (-1)^{p+m} (E_{pm}/m!)\, A^p B^m f_i^m(\phi), \tag{8}$$

where

$$Q = Ze^2/d, \; E_{pm} = (p-1)!/(p-m)!,$$

$$A = 3a^2/(r^2 + D_i^2), \quad B = 2rD_i/(r^2 + D_i^2). \tag{9}$$

The crystal potential $U_R(r, \phi)$ is given by (see Gemmell 1974)

$$U_R(r, \phi) = \sum_{i=\text{all rows}} V_{R_i}(r, \phi) - U_R^{\min}, \tag{10}$$

where $U_R^{\min}$ is the minimum value of $U_R(r, \phi)$, and specifies the choice of zero of $U_R(r, \phi)$. [The subscript R serves to mean that the origin of (r, ϕ) is at R].

Let us study the effects of the potentials V_{R_l} of the ⟨110⟩ rows which contribute to $U_R(r, \phi)$. We first consider the rows R_1 and R. We note that the separation D of the rows R and R_1 (see figure 1) is much smaller than the interatomic distance d along a ⟨110⟩ row. In fact $d = 2\sqrt{2}\, D$. [The second nearest neighbourer of R, R_2, lies at a distance $\sqrt{3}\, D$]. Due to this fact the crystal potential in a certain region between two consecutive atoms of the row R is dominated by an atom of the row R_1, namely that which is nearest to this region. This has been shown in figure 2a for the central region of two atoms A and B of the row R. The triangular section LMN is the central region in a $(1\bar{1}0)$ plane where the potential of the atom A_1 of the row R_1 dominates over the potential of the rest of the crystal (considered in the same plane). It is clear from the figure that in the atomic row R there are regions where the potential of the atoms of this row do not dominate. A similar consideration of the fields of the atoms of other rows (R_2, R_3, etc., see figure 2b) shows that around the row R (r small), the potential field of the atoms of other (than R_1) rows do not dominate. Thus we conclude that around the row R (for r small) the contribution of the atoms of the row R_1 to the crystal potential is such that the dynamics of the channeling electron is directly influenced. Therefore, in determining the continuum potential around the row R effect of the row R_1 must essentially be taken into account. Keeping this in mind, for small r values ($r < D/2$), we neglect in (10) contributions of all but the rows R and R_1. This leads to the following approximate form of the crystal potential.

$$U_R(r, \phi) \simeq U_R^{(1)}(r, \phi) = V_R + V_{R_1}, \qquad (11)$$

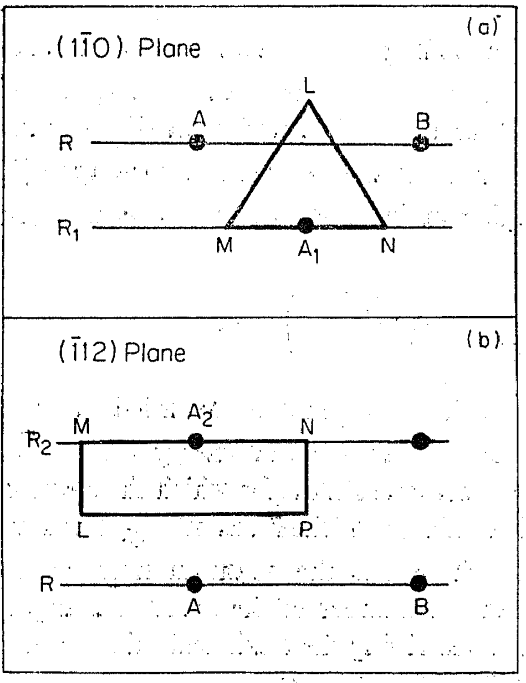

Figure 2. Regions where the potentials of the atoms of nearer ⟨110⟩ rows of R dominate in the channels of two rows (one R, other R_1, R_2). (a) Region of the dominating field (the triangle LMN) of an atom A_1 of the nearest ⟨110⟩ row R_1 shown in the (110) plane. (b) Region of the dominating field (the rectangle $LMNP$) of an atom A_2 of the ⟨110⟩ row R_2 shown in the (112) plane.

where V_R is obtained from (2) on taking $D_i=0$, and V_{R_1} is obtained from (8) by taking $(D_i, \phi_i)=(D, 3\pi/2)$.

3. Quantum beats

In order to study the states of the electron moving in the transverse potential $U_R^{(1)}$, we separate the circular (function of r only) and non-circular (function of r and ϕ) parts of $U_R^{(1)}$. For this purpose, we write $U_R^{(1)}$ as

$$U_R^{(1)}=W_C(r)+W_{NC}(r, \phi), \tag{12}$$

where we define $W_C(r)$ by

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} U_R^{(1)}\, d\phi = V_R+\frac{1}{2\pi}\int_{-\pi}^{\pi} V_{R_1}\, d\phi. \tag{13}$$

In (12), subscripts C and NC refer to circular and non-circular respectively.

From (12), (13), and (8) we obtain

$$W_{NC}=Q\sum_{p=0}^{\infty}\sum_{m=1}^{p}(-1)^{p+m}\, E_{pm}\, A^p B^m \left\{\frac{1}{m!}\sin^m\phi - (-1)^{m/2}\, 2^{-m}\,[(m/2)!]^{-2}\,\delta_{m,\,\text{even}}\right\}. \tag{14}$$

Here $\delta_{m,\,\text{even}}=0$ or 1 depending upon whether m is an odd or an even (including zero) integer. Note that, for $m=0$, the quantity in square bracket of (14) vanishes. Then at least, the $m=0$ term of (8) is absorbed in W_C (equation (13)).

Since, in (14), the minimum value of m is 1, according to (9), for $r=0$, W_{NC} would be zero, while for small r values ($r \ll D$), $m=1$ term of (14) would dominate over other m terms, so that

$$W_{NC} \xrightarrow[r\ll D]{} -\left(\frac{2QD}{D^2+3a^2}\right) r\sin\phi. \tag{15}$$

Equation (15) implies that W_{NC} may be regarded, at least for low r values, as a perturbation against the circular potential energy W. It is therefore reasonable to describe the electronic transverse states (for which the electron is closer to the row R) in terms of the statistical mixture of the states of W_C. The states of W_C are completely specified by n and L, where n is the quantum number of the transverse hamiltonian; and L is the quantum number of the transverse angular momentum. Therefore, the density matrix describing the statistical mixture of the electronic transverse states, for low n, is given by (Fano 1957)

$$\rho=\sum_{LL'} |\, nL)\,(nL\,|\,\rho\,|\,nL')\,(nL'\,|. \tag{16}$$

Since the density matrix (16) is obtained for those electronic states in which the electron moves (in bound states) closer to the row R (r small, or equivalently, n small), it would be useful for low n states only. In this sense in the following we shall confine our interest to low n values only.

The time evolution of the density matrix $\rho(t)$ is governed by

$$\hbar \frac{d\rho}{dt} = -i[H_t + W_{NC}, \rho] - \Gamma\rho, \tag{17}$$

where $H_t = -(\nabla^2/2\gamma m) + W_C$, with γm as relativistic mass of the electron, is the transverse hamiltonian

$$\left(\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right);$$

and Γ is the rate of spontaneous decay. $2\pi\hbar$ is the Plank's constant.

Equation (17) has the following solution for matrix elements of $\rho(t)$ between two angular momentum states (say L and L').

$$(nL \mid \rho(t) \mid nL') = (nL \mid \rho(0) \mid nL') \exp\,[(-iw_{nL', nL} - \Gamma)\, t/\hbar], \tag{18}$$

where $$w_{nL', nL} = (nL \mid H_t + W_{NC}) \mid nL) - (nL' \mid H_t + W_{NC} \mid nL') \tag{19}$$

is the difference of first order (perturbation) energies of the states $|nL)$ and $|nL')$. In eq. (18) L-states other than L and L' have been assumed uncoupled.

Channeling radiation is emitted when the electron makes a transition from higher energy states to lower energy states of the transverse space. The selection rule for an allowed $nL \rightarrow n'L'$ transition in the dipole approximation is $L' = L \pm 1$. [Here, by 'allowed' we mean those transitions which are allowed in the dipole approximation.] Thus transitions from all the mixture L-states (described by ρ; see (16)) of a higher energy (higher n) to some final L-state of a low n cannot occur. For example, for an $n = 2 \rightarrow n = 1$ transition, only $|21) \rightarrow |10)$ transition is allowed; the $|20) \rightarrow |10)$ transition being forbidden. But according to (18) the density matrix of the higher n state contains an oscillatory factor which causes the electron to move in the (mixture) L-states of the n. But since all these L-states cannot decay to one final L-state, the intensity of the radiation would vary periodically in time. And therefore there will be occurrence of quantum beats [see, for example Berry 1977 and Macek 1969 and 1970].

For experimental observation of the quantum beats, whose possibility is explored above, the incident electron beam must be made coherent either by applying electric field or by applying magnetic field. A discussion of such techniques is given in Berry (1977). In particular, the suggestion of Eck (1973) for observing quantum beats from mixed parity states (see also Gaupp *et al* 1974 and Sellin *et al* 1973) will facilitate the experimental task. [The parity of the $|nL)$ is state$(-1)^L$].

The experimental observation of quantum beats would depend upon the time resolution Δt of the measuring apparatus. The time resolution must be such that $\Delta t \sim \hbar/w_{nL', nL}$ which depends, through $w_{nL' nL}$, upon the energy of the incident electron. The dependence is such that Δt varies inversely with γ, where γ is the ratio of the relativistic mass to the rest mass of the electron. Thus the higher the energy of the incident electron, the higher time resolution will be required for the observation of the quantum beats. This fact, from the experimental viewpoint, sets up an upper limit on the incident electron energy for observation of quantum beats.

4. Conclusions

We have seen that under certain circumstances (§ 2) there may be an occurrence of quantum beats in the channeling of relativistic electrons in crystals. While the study has been made here for a particular case (<110> rows of a f.c.c. (diamond) crystal) it may be extended for other crystal structures and for other axes and planes also. The only requirement is that there must be a presence of a perturbation whose effect is to couple the various angular momentum (or spin) states.

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