Some stochastic phenomena in a driven double-well system

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Abstract

We study the overdamped motion of a Brownian particle in a driven double-well system to understand various physical phenomena observed experimentally. These phenomena include hysteresis, stochastic resonance, and net unidirectional motion in a symmetric periodic system. We argue that the area of the hysteresis loop so defined is a good measure of synchronization (with respect to the applied field) of passages between the two wells. We find that stochastic resonance may be relevant even in case of large amplitude driving field due to a recently discovered phenomena of noise induced stability of unstable states. We try to find relation between some of these apparently different phenomena.

Keywords: double-well system, stochastic resonance, synchronization, noise induced stability, hysteresis.

PACS numbers: 82.20.Mj, 05.40.+j,75.60.Ej

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I. INTRODUCTION

There are several phenomena in physics and chemistry which are simply seen as an event of potential barrier crossing. The examples include the motion of point defects in crystalline solids, transitions in bistable optical devices, and chemical reactions. The event of crossing over the potential barrier is aided by fluctuating forces and the problem was formally dealt with initially by Kramers [1]. The transition from one local equilibrium situation to the other finds a simple theoretical description in the study of two-well potential systems. In these nonlinear deterministically bistable systems the inclusion of noise gives rise to interesting dynamical effects. The stochastic dynamics in a static two-well potential has been studied extensively and is fairly well understood [2]. However, it is very difficult to find analytical solutions when the system is driven by an external field. We study the driven two-well system numerically as a simple model to understand various experimentally observed phenomena such as hysteresis, stochastic resonance [3], and net unidirectional motion in a symmetric periodic potential [4].

When the response of a system lags behind an externally applied field the system is said to show hysteretic behaviour *vis-a-vis* the applied field. The most familiar example of hysteresis being the variation of magnetization of a ferromagnetic system as an external magnetic field is varied in time. The phenomenon, however, is quite general and any system that shows metastability exhibits hysteresis. In the present work we treat it as a result of competition between various rates [6] (time scales). In particular, in the two-well system we have the rate of relaxation at the bottom of the wells, the mean rate at which passage takes place between the two wells across the potential barrier and the rate at which the system is being driven by the external field; the former two rates depending on the external field. Depending on the situation the external field could be taken as varying monotonically in time or varying periodically. In order to assign a single rate (or at most two rates) of variation of the external field we either consider a linear sweep [7] (monotonic) or a saw-tooth type of (periodic) field sweep [8]. In the case of periodic field sweep we show (section

II) that hysteresis loop area shows maximum as a function of the strength of the fluctuating forces. Moreover, we argue that the hysteresis loop area is a good measure of degree of synchronization of passages from one well to the other. Thus, the more the passages get synchronized with the applied field the larger is the hysteresis loop area. The same numerical data, namely the distribution of field values (or time intervals) at which passages take place from one well to the other, are used to understand the rest of the phenomena listed in the previous paragraph.

In the recent past stochastic resonance has been studied extensively. Initially it was discovered theoretically to understand the reccurrence of ice ages [9] at certain regular intervals of time ($\approx 10^5$ years). Subsequently the phenomenon was observed experimentally [3] in various systems such as electronic trigger circuits, two-mode ring lasers, and mammalian neuronal networks. The optimization of the response of a system to an input signal as a function of the input noise strength is termed as stochastic resonance. It is a nonlinear effect. The response of the system is measured as the ratio of output signal (at the input signal frequency) to (the output background) noise strength. This signal-to-noise ratio (SNR) (obtained from the power spectrum of the output signal) shows a peak as a function of input noise strength. For a simple understanding the nonlinear system can be modeled as a two-well potential. The residence time distribution of a Brownian particle in one of the two wells, as the system is driven by an input periodic signal, is then taken as the response. The simple model system shows stochastic resonant behaviour (section III) given a small amplitude periodic signal. This resonance is seen to occur as a result of synchronized passages across the potential barrier in the low "frequency" limit establishing a close relationship between these two phenomena. Stochastic resonance, however, can also be seen in case of large amplitude (overcritical) input periodic signals.

The two-well system can be represented by the usual Landau potential (Fig. 1). When the strength of the external field $h(t)$ becomes larger (Eqs. 2 and 3) than a critical value h_c one of the two wells disappears and one single well remains. Therefore a Brownian particle with its instantaneous position at the now nonexistent well will roll down towards the single

remaining well. When the large amplitude field is periodic each well in turn will disappear and then reappear exactly once in a period. Therefore, if the field sweep is slow enough a Brownian particle should pass from one well to the other exactly once in a period when the noise strength is almost zero. The scenario, however, changes as the strength of the noise is gradually increased. Instead of the average number of passages per cycle (ANPPC) increasing from one it, in fact, decreases initially and then on increasing the noise strength further it attains a minimum value and then begins to increase and ultimately becoming larger than one corresponding to the case of noise dominated dynamics. Similar effects have been predicted recently in other systems also [10]. This decrease of the ANPPC from 1 as the noise strength is increased from zero (or the noise induced stability of an unstable state) helps to obtain stochastic resonance in the case of large amplitude input signals (section IV).

The phenomena discussed in the last two paragraphs were studied taking the external periodic field sweep to be symmetric in time. The saw-tooth type periodic field sweep can easily be taken as temporally asymmetric keeping the average force per cycle, due to the external field, zero. We propose a physical model for a net preferentially unidirectional motion of a Brownian particle in a symmetric periodic potential when the system is subjected to such a temporally asymmetric periodic field. The fluctuating forces (noise) necessary to achieve such a motion need not be colored. A symmetric Gaussian white noise will be sufficient. We provide numerical support to this model by studying the two-well system and calculating the net accumulation of particles in each of the two wells when subjected to temporally asymmetric saw-tooth type periodic fields (section V). We further show that such a physical model can be made more efficient by suitably choosing the strength of the fluctuating forces. The motivation for the proposition of such a model comes from an experimentally observed phenomenon of net (statistically) preferentially unidirectional motion of protein motor (kinesin) molecules along microtubules [4]. And the idea [5] of using two time scales for the field sweep comes from the two flagellar strokes (power and reverse) executed by sperm cells for their propulsion. This subject is being persued intensively in

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recent years under the title of correlation ratchets, thermal ratchets or the Maxwell's demon-type information engines.

II. HYSTERESIS

We calculate hysteresis from the first-passage-time (FPT) distribution of a Brownian particle in the double-well potential under the influence of an external field. We also calculate it from the residence time distributions in each of the two wells. The motion of the particle is studied by solving, numerically, the overdamped Langevin equation

$$\dot{m} = -\frac{\partial \Phi}{\partial m} + \hat{f}(t), \tag{1}$$

where m is the order parameter (in this case position of the particle) and the dot over it denotes derivative with respect to time.

$$\Phi(m, t) = U(m) - mh(t), \tag{2}$$

(Fig. 1) where, $U(m)$ is the Landau two-well symmetric potential

$$U(m) = -\frac{a}{2}m^2 + \frac{b}{4}m^4 \tag{3}$$

and $h(t)$ is the external time dependent field sweep. $\hat{f}(t)$ are randomly fluctuating forces and are taken to be Gaussian with statistics

$$< \hat{f}(t) >= 0, \tag{4}$$

and

$$< \hat{f}(t)\hat{f}(t') >= 2D\delta(t - t'). \tag{5}$$

Here $< ... >$ represents an ensemble average over all realizations of the random forces.

A. The first-passage times

In the beginning (at time $t = 0$) we take the Brownian particle at the bottom of one of the two wells and then monitor its motion as time progresses. As soon as the particle reaches the bottom of the other well we record the time $t = \tau$ and stop the process. We begin the process all over again from the bottom of the first well and repeat the earlier procedure. By repeating the process for a large number of times (for meaningful averages) we obtain the density distribution $\rho(\tau)$ of the FPT's, τ. $\rho(\tau)$ depends on how the field $h(t)$ is swept and the strength of the fluctuating forces D.

When the field $h(t)$ is swept linearly,

$$h(t) = h_0 + \dot{h}t, \tag{6}$$

where $\dot{h}$ is a fixed constant. By taking, for example, $h_0 \gg h_c$ (the critical field $= 2(a/3)^{3/2}$ for $b = 1$; we take $a = 2$ in our numerical calculation throughout) and $\dot{h} < 0$ and start from the bottom of the right side well one can obtain $\rho(\tau)$ and get a sensible hysteresis loop $M(h)$.

$$\frac{M(h)}{h_c} = 1 - 2 \int_h^{h_0} \rho(h')dh', \tag{7}$$

where $\rho(h)$ is the distribution of field values at τ's. Equation (7) gives the upper half of the hysteresis loop. The other half is obtained by symmetry [7]. The linear sweep of the field has physical analogues in, for example, sweeping the temperature T (as a function of time) from above the freezing temperature of a melt to down below that temperature, where $M(T)$ would then correspond to the degree of nucleation of crystallites in the melt. However, D would now depend on T. By suitably modelling $D(T)$ one can obtain the qualitative features of the usual (T-T-T) curves [11]. Also it should be noted that by taking $\dot{h}$ small one can check whether the hysteresis loop area so obtained follows any power-law behaviour [12] in terms of $\dot{h}$. In an earlier work we, however, do not find any universal power law behaviour [7].

When the field $h(t)$ is varied periodically (in our case as a saw-tooth) in time the distribution $\rho(\tau)$ shows (Fig. 2) gradually diminishing peaks occurring periodically [8]. The periodicity roughly matches the periodicity of $h(t)$. Since the field oscillates between h_0 and $-h_0$, we can again find the distribution $\rho(h)$ of passage fields $h(\tau)$ and the hysteresis loop $M(h)$. We have shown earlier that the hysteresis loop area acquires a maximum as a function of $\dot{h}$ as well as D [8]. The area of the hysteresis loop as defined in the present work (see below) is a good measure of the degree of synchronization of passages (from one well to the other) with respect to the external field sweep $h(t)$.

For periodic $h(t)$ with amplitude $h_0 < |h_c|$, a Brownian particle on the right well (Eq. 2) will encounter the least barrier of passage to the left well when $h = -h_0$. Therefore, the probability of passage from the right well to the left well will be the largest when $h(t) = -h_0$. If we consider passage to take place only at $h(t) = -h_0$ from the right well to the left and only at $h(t) = h_0$ from the left well to the right, we have the case of perfect synchronization of passages with respect to the variation of the field. This will, however, correspond to $\rho(\tau)$ having sharp peaks occurring periodically at $t : h(t) = -h_0$ if we begin from the right well. And the corresponding $\rho(h)$ will have a single sharp peak at $h = -h_0$. Therefore, from Eq. 7 we will have a rectangular hysteresis loop and hence with the largest possible loop area. On the other hand if the passages are completely random (the case of least synchronization) we will have a uniform $\rho(h)$ from h_0 to $-h_0$ and the hysteresis loop will consequently have the smallest area (zero). From the discussions of these two extreme cases we arrive at the conclusion that hysteresis loop area, indeed, provides a good measure of degree of synchronization of passages from one well to the other as the field is periodically swept. Therefore the earlier result [8] shows that the passages become most synchronized at optimum values of noise strength D and field sweep rate $\dot{h}$. Similar results are obtained when hysteresis is calculated from the residence time distribution (see below).

B. Residence time distributions

Instead of studying a large number of identical replicas of a system one can study the same system for a long time and observe the passages a Brownian particle executes in the course of time. This way one can find the residence times and their distributions $\rho_1(\tau)$ and $\rho_2(\tau)$, in each of the two (left and right) wells, respectively, (Fig. 3). Also one can find the passage field distributions $\rho_{12}(h)$ and $\rho_{21}(h)$ for passages from the left well to the right well and from right well to the left well, respectively. However, the first-passage field distribution $\rho(h)$ from right well to the left well, for example, is not expected to be exactly the same as $\rho_{21}(h)$. This is because for FPT calculation one starts from the same initial condition $h(t=0)$, whereas for $\rho_{21}(h)$ $\rho_{12}(h)$ acts as the distribution of initial field values, *etc*. Also, in the previous (FPT based) work $\rho(h)$ was obtained for variation from h_0 to $-h_0$. However, the proper book-keeping of the field values of passages from one well to the other throughout the cycle of $h(t)$, namely from h_0 to $-h_0$ and then back to h_0, is required. Taking all these aspects into account we calculate the probability of the Brownian particle being in the right well $(m_2(h))$ when the field value is h:

$$m_2(h) = m_2(h - \Delta h) - m_2(h - \Delta h)\rho_{21}(h)\Delta h + m_1(h - \Delta h)\rho_{12}(h)\Delta h, \qquad (8)$$

and similarly for $m_1(h)$. The difference $m(h) = m_2(h) - m_1(h)$ is analogous to magnetisation in magnetic systems. The stationary (closed) hysteresis loop $m(h)$ is obtained by iteration. Note that there is a qualitative difference in the hysteresis loops calculated using the two methods. In the earlier case (FPT based) the hysteresis loop was always saturated by construction. In the present case the hysteresis loop need not be saturated and indeed one obtains saturation only for $h_0 > |h_c|$ together with small $\dot{h}$ and large D cases (Fig. 4). The area of hysteresis loop so obtained shows a maximum [13] as a function of D and also as a function of $\dot{h}$ (Fig. 5). As in case of hysteresis loop area from first-passage field distribution (Eq. 7) one can argue that in this case too the hysteresis loop area is a good measure of degree of synchronization of passages. Thus the passages, again, become most synchronized at an optimum value of the noise strength D and also at an optimum value of $\dot{h}$.

III. STOCHASTIC RESONANCE

We use the residence time distributions $\rho_1(\tau)$ or $\rho_2(\tau)$ to study stochastic resonance. As mentioned in the introduction the signal-to-noise ratio (SNR) in the power spectrum of these distributions is used to study it. We adopt the following procedure to obtain the power spectrum [14]. We calculate the histogram of ,say, $\rho_2(\tau)$ (Since the periodic drive is symmetric the two distributions are identical.) dividing the period T of $h(t)$ into 10 equal parts. We assign the mid-point of each time interval $\Delta = T/10$ the value equal to the height of the histogram at the interval. Usually the number of such data points is not large. We augment the data points obtained from the histogram by zeroes at similar equal intervals so that the maximum number of paded data points, unless mentioned otherwise, is 16384 $(= 2^{14})$. We then calculate the Fourier transform of this data using the Fast-Fourier-Transform (FFT) routines. The number of (doctored) data points is taken to be large in order to obtain Fourier trasforms at close intervals of frequency. Square the suitably normalized Fourier transformed data to give the power spectrum of $\rho(\tau)$. The normalization is done correctly such that the spectral density is one for zero frequency. We obtain sharp peaks at frequencies equal to $\frac{2\pi n}{T/2}$ for $n = 1, 2, \ldots$. We measure the first peak height and the background value of the spectrum at the same frequency. The background value of the spectrum is usually small and the possibility of commiting error in assigning a value to it is correspondingly large. The background value of spectrum is termed as noise and the peak height as the signal. We thus calculate the SNR for each value of $(\dot{h}, D)$. The SNR shows maximum as a function of D as well as $\dot{h}$ (Fig. 6). It should be noted that the measurement of SNR is not very accurate (large errors) but their variation as a function of D or $\dot{h}$ is so large that the trend of their variation as a function of D or $\dot{h}$ is not affected. (Note that our conclusions pertain mostly to the trends of variation rather than to the actual values.)

The maximum of SNR as a function of D indicates stochastic resonance. We find the value of $D = D_{SNR}^m$ at which the maximum occurs for various values of $(h_0, \dot{h})$. Similarly, we note the value $D = D_a^m$ at which the hysteresis loop area becomes maximum for the same

set of values $(h_0, \dot{h})$. The plots of D_a^m as well as D_{SNR}^m as a function of $\dot{h}$ for fixed values of h_0 is shown in Fig. 7. It is noteworthy that eventhough D_a^m and D_{SNR}^m differ widely for large $\dot{h}$, as $\dot{h}$ is decreased towards zero D_a^m and D_{SNR}^m become closer and closer [13]. As mentioned earlier hysteresis loop area is a good measure of degree of synchronization of passages from one well to the other. Therefore, at small $\dot{h}$ stochastic resonance occurs when the synchronization is close to its largest value. This result indicates that there is a close relationship between synchronization and stochastic resonance. This aspect of stochastic resonance had not been explored so far.

SNR also peaks [13] as a function of $\dot{h}$ as does the hysteresis loop area. It indicates that there is a time scale of maximum response of the system depending on the values of h_0 and D. The occurrence of this maximum as a function of frequency is closer to the conventional resonance phenomena. There has been some controversy in literature about whether the stochastic resonance is a bonafied resonance [15] at all. In Ref. [15] an effort has been made to resolve the controversy by measuring the strength of the first peak of the distribution $\rho_{12}(\tau)$ and noting their variation. Here our treatment takes account of all the peaks of $\rho_{12}(\tau)$ in order to calculate the hysteresis loop area which is used to measure the degree of synchronisation of passages. The maximum of SNR as a function of frequency (equivalent to $\dot{h}$ in our case) has been found [16] only recently (analytically) to occur in a double (square)-well potential system.

IV. NOISE-INDUCED STABILITY OF UNSTABLE STATES

In the previous section we studied the response of the two-well system to a small amplitude (subcritical) input signal in order to obtain stochastic resonance. We now study the response of the same system to a large amplitude (overcritical) input periodic signal. In particular, we take $h_0 = 1.4 h_c$. As can be seen from Eq. 2 the left well disappears and gives way to a steep potential line for $h(t) > h_c$ and only the deepened right well remains (Fig. 1). The reverse situation results when $h(t) < -h_c$. We find that when $D = 0$ and $\dot{h}$ less than a

threshold value $\dot{h}_{th}$ (in our case $\approx 0.384h_c$ per unit time) a particle rolls down alternately to both the wells exactly once in a period of $h(t)$. Therefore, the average number of passages per cycle (ANPPC) equals 1. However, as $\dot{h}$ is taken larger than $\dot{h}_{th}$ the ANPPC goes to zero very sharply (Fig. 8). However, the situation changes as D is gradually increased from zero.

Figure 9 shows the variation of ANPPC as a function of D for various values of $\dot{h}$. For any $\dot{h} < \dot{h}_{th}$ increasing D from zero initially decreases ANPPC. It attains a minimum value and then begins increasing ultimately becoming larger than 1 for large D. For all $\dot{h} > \dot{h}_{th}$ ANPPC monotonically increases from a small value (zero) as D is increased from zero (Fig. 9). Therefore, the range $0 < \dot{h} < \dot{h}_{th}$ provides an interesting interval within which the passage of the Brownian particle from an unstable situation is slowed down because of the presence of fluctuating forces. We, therefore, see the gradual emergence of more than one peaks of $\rho_{1(2)}(\tau)$ as D is increased from zero (Fig. 10) which on further increament of D get broadened. Interestingly, on calculating the power spectrum of $\rho_2(\tau)$ we find corresponding evolution of broad peaks (centred at the periodic field frequency and at its harmonics) as D is increased from zero. (No peaks for $D = 0$.) However, on increasing D further the peaks gradually disappear (Fig. 11). This is clearly a signature of stochastic resonance in this case of large amplitude forcings. Thus, the slowing down of the passage of the Brownian particle in a two-well system makes it meaningful to study SR as a response to large amplitude input signal.

The role of noise in the two cases of small and large amplitude forcings are quite the opposite. In the large amplitude case the observance of stochastic resonance is because the presence of noise slows down the decay of the unstable states whereas in the small amplitude cases the noise helps to overcome the potential barrier for passage. In the case of large amplitude forcings the passages take place mostly when the intervening potential barrier is missing. The passages are not noise induced, rather they are noise controlled. Correspondingly the hysteresis will have a different nature. For analogy, the process in this

case will be more akin to large amplitude temperature cycling across the freezing point of a liquid than field cycling well within the ferromagnetic regime. The calculated hysteresis loop area does not show maximum at the same frequencies or noise strengths where SR appears.

V. TWO-WELL SYSTEM SUBJECTED TO TEMPORALLY ASYMMETRIC PERIODIC FORCING

It is quite contrary to common sense perception to observe asymmetric motion of a particle in a symmetric periodic potential without any obvious external bias. Several physical models [17] were proposed to obtain asymmetric motion in a periodic potential. In most cases the periodic potential is taken to have ratchetlike asymmetry within a period and some correlated fluctuating forces are applied. We propose that asymmetric motion can be obtained even in a symmetric (nonratchetlike) periodic potential by subjecting the particle to a Gaussian white noise of zero mean. This is made possible by the application of a temporally asymmetric (within a period) periodic forcing. We consider a saw-tooth type periodic field $h(t)$ but having the magnitude of the two slopes different. We consider the external field $h(t)$ to be such as to exert a zero mean force on an otherwise free particle implying thereby that the particle motion is not biased on the average by the external field.

We take the amplitude h_0 of the external field $h(t)$ to be small ($h_0 < h_c$) and monitor the motion of a particle [18] on the two-well potential subjected to white noise (fluctuating forces). We then find the mean values ($\overline{\tau}_1$ and $\overline{\tau}_2$) of residence times in each of the two wells. We find that when $h(t)$ is asymmetric (defined by the asymmetry parameter $\Delta = \frac{T_1 - T_0/2}{T_0/2}$, where T_1 is the time the field takes to decrease from h_0 to $-h_0$ and T_0 is the period of $h(t)$)) the two mean values $\overline{\tau}_1$ and $\overline{\tau}_2$ (corresponding to left and right wells, respectively) are not equal. This simply implies that the particle spends unequal time in each of the two wells. Moreover, $\overline{\tau}_1 + \overline{\tau}_2 > T_0$. If $\overline{\tau}_1 + \overline{\tau}_2$ were equal to T_0 we would conclude that eventhough $\overline{\tau}_1 \neq \overline{\tau}_2$ the particle makes on the average equal number (=1) of passages from left to right and from right to left in a cycle of $h(t)$. However, the inequality implies that

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the average number of passages from the left well to the right is not equal to the average number of passages from the right well to the left per cycle of $h(t)$. This observation helps us to make the important inference that in a periodic potential system a similar asymmetry in passages will ensue between the two adjacent valleys resulting in a net current across the periodic system. It is noteworthy that the passages considered here are in a symmetric (nonratchetlike) potential unlike in the work of Magnasco [17] who considers ratchetlike asymmetric potential. Also here the passages are due to periodic forcings of amplitude smaller than the critical amplitude h_c necessitating the presence of noise unlike in the work of Ajdari, et al where large amplitude forcings are used without the noise [19].

Figure 12 shows how $m = \overline{\tau}_2 - \overline{\tau}_1$ vary as a function of Δ. The monotonic rise is quite evident. Moreover, from Fig. 13 we see that given an asymmetry parameter Δ, m acquires a maximum as a function of D. This shows that the net asymmetric current can be optimized by suitably choosing the noise strength D. Again, the area of the hysteresis loops becomes maximum (Fig. 14) as a function of D for a given Δ. But the position of this maximum does not coincide with the position of the maximum of m as a function of D. We can also see how the hysteresis loop area vary as Δ is changed.

VI. CONCLUSION

In this work we have tried to understand the phenomena of hysteresis, stochastic resonance, synchronization and asymmetric passages in a model symmetric bistable system. We find that stochastic resonance, at least at low signal frequencies, could be a result of synchronised response of the system to the input periodic signal. The degree of synchronization itself being optimized by the noise strength as well as the signal "frequency". Also, we obtain stochastic resonance in the response to a large amplitude periodic forcing as a result of noise induced slowing down of a Brownian particle rolling down an inclined potential surface. By studying the processes in the bistable system we could also come to the conclusion that a net unidirectional current can be obtained even in a symmetric (nonratch-

etlike) periodic system subjected to Gaussian white noise when acted upon by a temporally asymmetric but zero averaged periodic forcing. The unidirectional current is also optimized by the noise strength. It is noteworthy that our model Landau (two-well) potential system is a very simple nonlinear system yet its study provides valuable understanding of all the above mentioned experimentally observed phenomena.

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FIGURES

FIG. 1. Plots of (a) $\Phi(m)$ as a function of m for $h(t) = 0.6h_c$, and (b) m_{xtrm}, the extrema points of $\Phi(m)$ as a function of h, for $a = 2.0$, and $b = 1.0$.

FIG. 2. Shows (a) the time variation of the external field $h(t)$, (b) the first-passage-time distribution $\rho(\tau)$, (c) the passage field distribution $\rho(h)$, and (d) the corresponding hysteresis loop calculated from $\rho(h)$, for $h_0 = 0.7h_c$, $D = 0.3$, and the period of $h(t)$, $T_0 = 28.0$.

FIG. 3. Shows (a) the residence time distribution $\rho_1(\tau)$ in the well 1, and (b) the passage field distribution $\rho_{21}(h)$ for passage from the right well 2 to the left well 1, for $h_0 = 0.9h_c$, $D = 0.2$, and period of $h(t)$, $T_0 = 36.0$.

FIG. 4. Hysteresis loops calculated from $\rho_i(\tau)$ and $\rho_{ij}(h)$, $i, j = 1, 2$, for various values of $h_0 = 1.4h_c$ (outer), $0.9h_c$(middle) and $0.7h_c$ for $\dot{h} = 0.05h_c$ and $D = 0.2$.

FIG. 5. Plots of hysteresis loop area A (a) as a function of D for $\dot{h} = 0.05h_c(\circ)$, $0.1h_c(\square)$, $0.2h_c(\diamond)$, $0.4h_c(\triangle)$, and $0.6h_c(\triangledown)$, and (b) as a function of field sweep rate $\dot{h}$ for $D = 0.1(\circ)$, $0.15(\square)$, $0.2(\diamond)$, and $0.3(\triangle)$ for $h_0 = 0.9h_c$. In this and in the rest of the figures the lines joining the points are only to guide the eye.

FIG. 6. Plots of signal-to-noise ratio (SNR) for $h_0 = 0.9h_c$ (a) as a function of D, for $\dot{h}/h_c = 0.05(\circ)$, $0.1(\square)$, $0.2(\diamond)$, $0.4(\triangle)$, $0.5(\triangleleft)$, $0.6(\triangledown)$, and $0.72(\triangleright)$, and (b) as a function of $\dot{h}$ for various values of $D = 0.1(\circ)$, $0.15(\square)$, and $0.2(\diamond)$.

FIG. 7. Plots the peak positions of the plots of hysteresis loop area A versus $D(\circ)$ and those of SNR versus $D(\square)$, for $h_0 = 0.7h_c$ (empty symbols, solid joining lines) and for $h_0 = 0.9h_c$ (filled symbols, dashed joining lines).

FIG. 8. Average number of passages per cycle (ANPPC) is plotted as a function of $\dot{h}$ for $D = 0.00001$. Averaging is done over 5000 cycles. The line is drawn to guide the eye.

FIG. 9. Shows ANPPC as a function of D for various $\dot{h}$ values:(a) $0.05h_c$($\circ$), (b) $0.2h_c$($\square$), (c) $0.28h_c$($\diamond$), (d) $0.35h_c$($\triangle$), and (e) $0.4h_c$($\triangleleft$). For $\dot{h} \widetilde{>} 0.392h_c$ the curves, for example the curve for $\dot{h} = .4h_c$, show monotonic behaviour starting from ANPPC=0. The lines are drawn to guide the eye.

FIG. 10. Shows $\rho(\tau)$ for $h_0 = 1.4h_c$, $\dot{h} = 0.028h_c$ and $D = 0.001$ (bottom), $D = 0.1$ (middle; shifted up by 0.25) and $D = 0.4$ (shifted up by 0.5).

FIG. 11. Shows power spectral density (psd) for three values of D: (a) $D = .001$ (dotted), (b) $D = .1$(solid line), and (c) $D = .4$ (dash-dotted), at $\dot{h} = .028h_c$. We have taken, for each curve, 256 (augmented with zeroes) data points at a time interval of 2.0.

FIG. 12. Shows the variation of accumulation M in a well as a function of Δ for (a) $D = 0.15$($\square$), (b) $D = 0.2$($\diamond$), (c) $D = 0.5$($\triangle$), and (d) $D = 0.7$($\triangledown$).

FIG. 13. Plot of M versus D for (a) $\Delta = 0.5$($\circ$), and (b) $\Delta = \frac{13}{14}$($\square$).

FIG. 14. Plot of hysteresis area as a function of D for (a) $\Delta = 0.5$($\circ$), and (b) $\Delta = \frac{13}{14}$($\square$).

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(b)