Motion in a rocked ratchet with spatially periodic friction

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We present a detailed study of the transport and energetics of a Brownian particle moving in a periodic potential in the presence of an adiabatic external periodic drive. The particle is considered to move in a medium with periodic space dependent friction with the same periodicity as that of the potential but with a phase lag. We obtain several results, most of them arising due to the medium being inhomogeneous and are sensitive to the phase lag. When the potential is symmetric we show that efficiency of energy transduction can be maximised as a function of noise strength or temperature. However, in the case of asymmtertic potential the temperature may or may not facilitate the energy conversion but current reversals can be obtained as a function of temperature and the amplitude of the periodic drive. The reentrant behaviour of current can also be seen as a function of phase lag.

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I. INTRODUCTION

The search for the possibility of unidirectional motion in a periodic system without the application of any obvious bias is of current research interest [1–15]. Such possibility requires the system to be out of equilibrium in order for the process to be consistent with the second law of thermodynamics. Several physical models have been proposed to obtain such motion. In all the models noise plays the central role. One of the most discussed models is the one in which an asymmetric periodic potential system is adiabatically rocked [3,5] by applying constant forces F and $-F$ at regular intervals of time. One obtains unidirectional motion because in such a system the current $j(+F) \neq -j(-F)$. Eventhough the time averaged applied force over a period vanishes the averaged current $< j >= 0.5[j(+F) + j(-F)]$ becomes finite in the presence of noise (thermal fluctuations). Moreover, the average current $< j >$ was found to peak at an intermediate noise strength (or temperature). In this model it has been further shown that by suitably choosing the asymmetric periodic potential one may obtain current reversal [16] as a function of temperature provided the rocking frequency is high. Similar results, however, can be obtained in the presence of a unbiased colored noise instead of the oscillating force. There are several other interesting models to obtain unidirectional motion including models where potential barriers themselves are allowed to fluctuate [6] or models wherein symmetric potential system is driven by temporally asymmetric forces [9–11], etc.

The result that thermal noise helps to obtain unidirectional current in a periodic system was quite important. But later on it was pointed out that obtaining mere current does not necessarily mean that the system does work efficiently [17]. Doing work involves flow of current against load and hence one must, in the spirit of the model, obtain current up against a tilted (but otherwise periodic) potential system. Analysis shows, however, that the efficiency of an adiabatically rocked system (ratchet) monotonically decreases with temperature. Therefore though such a ratchet system helps extract a large amount of work at an intermediate temperature (where the current peaks) the work is accomplished at a larger expense of input energy; thermal fluctuation does not facilitate efficient energy conversion in this model ratchet system. In a subsequent work [18] this deficiency was rectified but in a different model wherein the asymmetric potential oscillates in time, instead of its slope being changed (rocked) between $+F$ and $-F$ adiabatically. In both these models the friction coefficient was constant and uniform in space. The present work makes a detailed study of the rocked ratchet system with nonuniform friction coefficient which varies periodically in space [13–15,19–28].

In this work we take the friction coefficient to vary with the same periodicity as the potential but with a phase difference, ϕ. The phase difference ϕ, the amplitude λ of variation of friction coefficient, the amplitude F_0 of rocking, the load, etc. affect the functioning of the ratchet in an intricate and nontrivial manner. The two of the important results we obtain are: (1) The efficiency of the adiabatically rocked ratchet shows a peak as a function of temperature, though the peak (which may or may not exist in case of spatially asymmetric potentials) position does not coincide with the temperature at which the current peaks, and (2) the current could be made to reverse its direction as a function of noise strength and the amplitude F_0 even at low frequencies of rocking. These attributes are solely related to the medium being inhomogeneous with space dependent friction. It is worth noting that the introduction of space dependent friction, though does not affect the equilibrium properties (such as the relative stability of the locally stable states), changes the dynamics of the system in a nontrivial fashion. Recently it has been shown that these systems exhibit noise induced stability, it shows stochastic resonance in washboard potentials without the application of external periodic input signal, [23,24] and also unidirectional motion in periodic symmetric potential (non ratchet-like) systems. [15,21,22,29]

In the next section we describe our model and obtain an expression for current and efficiency in the quasi-static limit. In Sec. III we present our results.

II. EQUATION OF MOTION IN INHOMOGENEOUS SYSTEMS

The nature of correct Fokker-Planck equation in the presence of space-dependent diffusion coefficient (inhomogeneous medium) was much debated earlier. Later on the correct expression was found from a microscopic treatment of system-bath coupling. The motion of an overdamped particle, in a potential $V(q)$ and subject to a space dependent friction coefficient $\gamma(q)$ and an external force field $F(t)$ at temperature T is described by the Langevin equation [15,23–25,28]

$$\frac{dq}{dt} = -\frac{(V'(q) - F(t))}{\gamma(q)} - k_B T \frac{\gamma'(q)}{[\gamma(q)]^2} + \sqrt{\frac{k_B T}{\gamma(q)}}\xi(t), \tag{1}$$

where $\xi(t)$ is a randomly fluctuating Gaussian white noise with zero mean and correlation : $< \xi(t)\xi(t') >= 2\delta(t-t')$. Here $< .. >$ denotes an ensemble average over the distribution of the fluctuating noise $\xi(t)$. The primes in Eq. (1) denote the derivative with respect to the space variable q. It should be noted that the above equation involves a multiplicative noise with an additional temperature dependent drift term. The additional term turns out to be essential in order for the system to approach the correct thermal equilibrium state. We take $V(q) = V_0(q) + qL$, where $V_0(q + 2n\pi) = V_0(q) = -V sin(q)$, n being any natural integer. L is a constant force (load) representing the slope of the washboard potential against which the work is done. Also, we take the friction coefficient $\gamma(q)$ to be periodic :
$\gamma(q) = \gamma_0(1 - \lambda \sin(q + \phi))$, where ϕ is the phase difference with respect to $V_0(q)$. The equation of motion is equivalently given by the Fokker-Planck equation

$$\frac{\partial P(q,t)}{\partial t} = \frac{\partial}{\partial q}\frac{1}{\gamma(q)}[k_B T \frac{\partial P(q,t)}{\partial q} + (V'(q) - F(t))P(q,t)]. \tag{2}$$

This equation can be solved for the probability current j when $F(t) = F_0 =$ constant, and is given by [23,24,31]

$$j = \frac{k_B T(1 - \exp(-2\pi(F_0 - L)/k_B T))}{\int_0^{2\pi} \exp(\frac{-V_0(y)+(F_0-L)y}{k_B T})dy \int_y^{y+2\pi} \gamma(x) \exp \frac{V_0(x)-(F_0-L)x)}{k_B T}dx}. \tag{3}$$

In the presence of space dependent friction and the phase lag $\phi \neq 0, \pi$ and in the absence of load $j(F_0) \neq -j(-F_0)$ even for a spatially periodic symmetric potential. Thus when the system is subjected to an external ac field $F(t)$ the unidirectional particle flow (or rectification of the current) takes place. The phase lag ϕ brings in the intrinsic asymmetry in the dynamics of the system. When an externally applied ac force changes slowly enough (quasi-static or adiabatic limit) i.e., when the time scale of variation of $F(t)$ is much larger compared to any other time scales involved in the system we can readily obtain an expression for the unidirectional current. For a field $F(t)$ of a square wave amplitude F_0, an average current over the period of oscillation is given by, $< j > = \frac{1}{2}[j(F_0) + j(-F_0)]$. This particle current can even flow against the applied load L and thereby store energy in useful form. In the quasi-static limit following the method of stochastic energetics [32] it can be shown [17,18] that the input energy E_{in} (per unit time) and the work W (per unit time) that the ratchet system extracts from the external noise are given by $E_{in} = \frac{1}{2}F_0[j(F_0) - j(-F_0)]$ and $W = \frac{1}{2}L[j(F_0) + j(-F_0)]$ respectively. Thus the efficiency (η) of the system to transform the external fluctuation to useful work is given by

$$\eta = \frac{L[j(F_0) + j(-F_0)]}{F_0[j(F_0) - j(-F_0)]}. \tag{4}$$

Henceforth all our variables like $< j >, E_{in}, W, F_0, T$ are made dimensionless. The amplitude of potential V is set to unity as all other energy scales are scaled with respect to V. We evaluate $< j >, W, E_{in}$, and η numerically [30] using Eq. (3).

III. RESULTS AND DISCUSSIONS

First, we present our results for average (net) unidirectional current in a symmetric periodic potential induced by adiabatic rocking and in the absence of load. We emphasize here that to obtain these currents the system must be inhomogeneous. The phase lag ϕ (except for $\phi = 0, \pi$, for which unidirectional current is not possible) plays an important role in determining the direction and magnitude of $< j >$. For $2\pi > \phi > \pi$, in the presence of external quasistatic force $F(t)$ and in the absence of load L, we have a forward moving ratchet (current flowing in the positive direction) and for $0 < \phi < \pi$ we have the opposite. For instance, if we examine the effect of friction coefficient close to the minimum of the potential two different situations are encountered depending on the value of ϕ. When $\phi > \pi$ and $F_0 > 0$ the particle experiences lower friction near the barriers in the direction of acquired velocity. The situation is reverse when $F_0 < 0$. From Eq. (3) it follows that in a static force F_0, $j(F_0) \neq -j(-F_0)$, hence rectification of current occurs in the presence of external adiabatic drive. Moreover, it should be emphasized that the magnitude of current or mobility, in the static field F_0, depends sensitively on the potential and the frictional profile over the entire period. Depending on the system parameters the current or mobility can be much larger or smaller than the current or mobility of a particle moving in a homogeneous medium characterised by the space averaged frictional coefficient. In fact, in the intermediate values of temperature and F_0 the mobility can be made much larger than their asymptotic limits. This leads to stochastic resonance in a washboard potential in the absence of ac signal [23,24].

In Fig. 1 the average unidirectional current $< j >$ is presented as a function of ϕ and T. For this figure the value of amplitude of square-wave ac field $F_0 = 0.05$, and the amplitude of frictional modulation $\lambda = 0.9$. It can be seen that $< j >$ changes sign as a function of ϕ and current exhibits either a minimum or a maximum as a function of temperature depending on the value of ϕ. At the two limits of temperature (0 and inf) the currents vanish. This is the case for the value of F_0 less than the critical value F_c of F_0 where the barrier to motion in either direction vanishes. In our case the critical value of F_0 is equal to 1. This stochastic resonance-like phenomenon has been observed in rocked ratchet systems characterised by asymmetric periodic potentials [3,5,16]. From the contours of the plot it is clear that as we move away from phase shift $\phi = \pi$ positive (or negative) direction the temperature at which maxima (minima) occurs shifts to a larger value and the absolute value of the current at the peak decreases. However, the present symmetric potential situation does not lead to multiple current reversals as a function of temperature in the quasi-static limit of an external drive.

The current as a function of T and F_0 is shown in Fig. 2 for $\phi = 1.3\pi$ and $\lambda = 0.9$. For smaller fields F_0 compared to the critical field F_c the current exhibits a maximum as a function of temperature. As we increase F_0 the temperature at which the peak occurs decreases. For fields larger than F_0 the current, however, decreases monotonically with temperature because the barrier to motion disappears. In this high field region mobility of a particle decreases with increase of temperature [24]. However, as we increase the field the net current $< j >$ monotonically changes and saturates to a finite value. This is in contrast $< j >$ vanishes for large F_0 in the ratchet subjected to a rocking force in the absence of space dependent friction. In Fig. 3 we have plotted $< j >$ versus $< F_0 >$ and ϕ for fixed values of $T = 0.5$ and $\lambda = 0.9$. It can again be seen clearly that the current monotonically varies and saturates to a value given by $-\frac{\lambda}{2} sin(\phi)$ independent of temperature. This result [24] follows from the analysis of Eq.(3). As expected current reversal can be seen as a function of ϕ for large F_0. It can also be verified that the current $< j >$ increases monotonically as a function of the amplitude λ of the friction coefficient and hence we do not present variation of $< j >$, etc., with λ.

We now discuss the efficiency of a symmetric periodic potential system in the presence of space dependent friction driven by an adiabatic periodic field. The efficiency of such a system with uniform friction has been studied earlier and it has been shown that temperature does not facilitate the efficiency (η) of energy conversion in the system [17]. To calculate η we make use of Eqs. (3) and (4). In our analysis the load L is applied against the direction of net current (in the absence of load). In this situation particle current can flow against the applied load L less than some critical value L_c thereby storing energy in useful form. For $L > L_c$ one cannot talk meaningfully the concept of efficiency as the current flows in the direction of the load and hence no storage of useful energy takes place. In Fig. 4 we have plotted efficiency η, input energy E_{in} and work done W (scaled up by a factor 60 for convenience of comparison) as a function of T for the parameter values, $F_0 = 0.5$, $\phi = 1.3\pi$, $\lambda = 0.9$, and the load $L = 0.04$. The figure shows that the efficiency exhibits a maximum as a function of temperature indicating that thermal fluctuation facilitates energy conversion. This in contrast to the case of uniform friction coefficient where η decreases monotonically with the increase of temperature in the same adiabatic limit [17]. It is to be mentioned that the temperature corresponding to the maximum efficiency is not the same as the temperature at which the average current $< j >$ becomes maximum in the absence of load. The temperature at which the extracted work maximizes is not the same as the temperature at which the efficiency becomes maximum for the same parameter values. The input energy increases with temperature monotonically and saturates at the high temperature limit. η, W, and E_{in} show similar qualitative behaviour for other parameter values. The above important observation of temperature facilitating the energy conversion is applicable for the spatially symmetric potential. In general in adiabatically rocked systems with frictional nonuniformities the increasing thermal noise need not increase the efficiency. The efficiency is sensitive to the qualitative nature of the periodic potential (and also to the nonuniformity of friction). For instance, asymmetric potential exhibits quite complex behaviour of η and $< j >$. To illustrate this we take $V_0(q) = -\sin q - \frac{\mu}{4}\sin 2q$ (where μ lies between

-1 and 1, and is the asymmetry parameter). With this potential we discuss three separate cases: Case A - system in a symmetric potential ($\mu = 0$) in an inhomogeneous medium ($\lambda \neq 0$), Case B - system in an asymmetric potential ($\mu \neq 0$) in an inhomogeneous medium ($\lambda \neq 0$), and Case C - system in an asymmetric potential ($\mu \neq 0$) in a homogeneous medium ($\lambda = 0$).

In Fig.5, we have presented results of η versus T for all the three cases described above. For this we have taken $F = 0.5$, $L = 0.002$, and $\phi = 1.3\pi$. For case A $\lambda = 0.9$, for case B $\mu = 0.08$ and $\lambda = 0.9$, and for case C $\mu = 0.08$. As discussed earlier for the case A temperature maximizes the efficiency. Case C, where the medium is homogeneous, efficiency monotonically decreases with temperature. These observations have been emphasized in earlier literature. The case B, where potential asymmetry and frictional inhomogeneity are present, the efficiency decreases monotonically in this parameter regime. In general whether the temperature facilitates the energy conversion in case B depends sensitively on the system parameters. In some limited parameter range the peaking behaviour is seen as in Fig.6. The parameter values are indicated in the caption. The presence of asymmetry in the potential may or may not help in enhancing the efficiency of the system. This can be seen from Figs. 5 and 6. In all these cases the work done W and the input energy E_{in} show similar qualitative features as shown in the inset of Fig. 1. Now, we discuss the variation of efficiency as a function of load.

On general grounds it is expected that the efficiency too exhibits maximum as a function of load. It is obvious that the efficiency is zero when load is zero. At the critical value L_c (beyond which current flows in the direction of the load) the value of current is zero and hence the efficiency vanishes again. In between these two extreme values of load the efficiency exhibits maximum. Beyond $L = L_c$ the current flows down the load and therefore the idea of efficiency becomes invalid. In Fig. 7, we have plotted η versus load for all the three cases for chosen values of parameters as mentioned in the figure. In all these cases current monotonically decreases as a function of load. The work done against load W exhibits a maximum as a function of load. The load at which W shows maximum does not coincide with the load at which η becomes maximum. The input energy E_{in} as a function of load varies non monotonically exhibiting a minimum. However, depending on the case under consideration the value of the load at which the minimum in the input energies observed may be larger than L_c above which efficiency is not defined.

In Fig. 8, we have plotted the efficiency versus the amplitude of the adiabatic forcing F_0 for all the three cases. It can be seen from the figure that for the system in an inhomogeneous medium, namely for cases A and B, η exhibits a maxima and saturate to the same value in the large amplitude limit. In contrast, for the case C after exhibiting maximum η goes to zero. This follows from the simple fact that in the large amplitude limit in the absence of frictional inhomogeneities the net unidirectional current tends to zero. The peculiar feature of saturation of efficiency in inhomogeneous media is related to the fact that the average current saturates to a constant value in the high amplitude limit as discussed earlier. This somewhat counter-intuitive result is typical to inhomogeneous media. Having discussed efficiency of energy conversion we now study the nature of net current $< j >$ in the presence of spatially asymmetric potential to examine if current reversals take place in the adiabatic limit in the absence of load. It is known from the earlier literature [16] that in an adiabatically rocked asymmetric potential ratchet system net current does not exhibit reversals as a function of T. In these systems current reversals are possible when the frequency of the applied ac field is large. We show here that in the presence of frictional inhomogeneities in addition to asymmetry in the potential one can observe current reversal as a function of thermal noise. In Fig. 9, we have plotted the magnitude of net current $< j >$ versus T for all the three cases A, B, and C. The corresponding parameter values are mentioned in the caption of the figure. The cases A and C do not exhibit current reversal. This is a general result independent of parameter values. However, in case B current reversal is observed. To obtain current reversal both asymmetry in potential and nonuniform friction coefficient are essential, that is, current reversals arise due to the combined effect of ϕ and μ. Moreover, it should be noted that to observe current reversals the parameter range should be such that the net current in case A is in the opposite direction to that in case C. For the case B for which current reversal is observed, the plot of efficiency separates into two disjoint branches as the load should be reversed keeping the magnitude same when the current reversal takes place. In the presence of $\mu \neq 0$ and $\lambda \neq 0$, where the current reversals are observed, the efficiency as a function of temperature is less than the maximum value of efficiency in either of the two cases A and C. That is, $\eta(\mu \neq 0, \lambda \neq 0) < max[\eta(\mu = 0, \lambda \neq 0), \eta(\mu \neq 0, \lambda = 0)]$. To further analyze the nature of current reversals, in Fig. 10, we have plotted $< j >$ as a function of ϕ and T for fixed values of $\mu = 1$, $\lambda = 0.9$, and $F_0 = 0.5$. From the contour plots it is clear that as a function of ϕ the current reverses sign twice in the intermediate temperature range. Thus the current exhibits reentrant behaviour as a function of phase ϕ which is special to the case B. As we decrease the asymmetry in the potential the $< j >= 0$ contour line shifts towards $T = 0$ thereby enhancing the domain of current reversal to a lower value of temperature as a function of ϕ. As a function of temperature the current reversals occur in a definite range of phase ϕ which , in turn, depends on other material parameters. The qualitative behaviour of $< j >$ remains unaltered for different F_0 as long as F_0 is less than the critical value.

In Fig. 11, $< j >$ is plotted as a function of T and F_0 for $\mu = 1$, $\phi = 1.3\pi$, and $\lambda = 0.9$. As opposed to the case of symmetric potential (Fig. 2), currents in the small temperature regime do exhibit maxima and then saturate to a constant value as noted earlier. As a function of T the current exhibits similar features as in the case A (Fig. 2). Figure 12, shows $< j >$ as a function of F_0 and ϕ for $T = 0.1$, $\mu = 1$, and $\lambda = 0.9$. As opposed to case A (see Fig. 3) $< j >$ shows current reversal as a function of F_0 in the range $0 < \phi < \pi$. However, in the asymptotic limit of F_0 current saturates to a value $-\frac{\lambda}{2} sin(\phi)$ independent of the value of the asymmetry parameter μ. From the contour plot it follows that as a function of phase ϕ we observe the reentrant behaviour of current at high values of F_0. As we decrease μ the $< j >= 0$ contour shifts towards smaller values of F_0 thus making it possible to observe the double reversals at even smaller values of F_0. This reentrant behaviour as a function of ϕ and the current reversal as a function of F_0 is very specific to the case B alone (compare Fig. 3).

Thus we conclude from our studies that the dynamics of a particle in an inhomogeneous medium is rich and complex. In the presence of adiabatic forcing and asymmetry in the potential current reversals can be observed as a function of T and F_0. And depending on the system parameters thermal fluctuations facilitate the energy conversion. The above behaviour cannot be seen in the homogeneous medium in the same adiabatic limit. However, it is possible to observe these in homogeneous media in the presence of finite frequency ac drive (nonadiabatic regime). This seems to suggest that ϕ may play the characteristic role of frequency in our model in the absence of nonadiabatic ac drive. This has been noted earlier in the context of observation of stochastic resonance phenomena in inhomogeneous media in the presence of static tilt alone [24]. As a function of phase ϕ, we observe reentrant behaviour for the current which arises because of interplay between asymmetry, inhomogeneity, thermal noise, and strength of the adiabatic forcing. Some of the phenomena can be understood at best at a qualitative level only. The effect of nonadiabatic forcing may be of further interest. Work on this line is under investigation.

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FIGURE CAPTIONS .

Fig. 1. The net current $< j >$ as a function of ϕ and T, for parameter values $F_0 = 0.5$, $\lambda = 0.9$. In the base plane contour of surface plot $< j > (\phi, T)$ are given, dotted line indicates $< j >= 0$ contour.

Fig. 2. $< j >$ as a function of temperature T and the amplitude of the rocking force F_0, for $\phi = 1.3\pi$, $\lambda = 0.9$.

Fig. 3. $< j >$ as a function of the amplitude of the rocking force F_0, and phase ϕ, for $T = 0.5$.In the base plane contour of surface plot is given.

Fig. 4. Efficiency η, E_{in} and W as a function of T for $\phi = 1.3\pi$, $F_0 = 0.5$, $\lambda = 0.9$, and $L = 0.4$. W has been scaled up by a factor 60 to make it comparable with η and E_{in}. Y-axis is in dimensionless units.

Fig. 5. Efficiency versus temperature for (i) case A ($\mu = 0$, and $\lambda = 0.9$), (ii) case B ($\mu = 0.08$, $\lambda = 0.9$), and (iii) case C ($\mu = 0.08$, $\lambda = 0.0$), for fixed $F_0 = 0.5$, $\phi = 1.3\pi$, and $L = 0.002$.

Fig. 6. Efficiency η versus temperature T, for different values of μ and for $F_0 = 0.5$, $\lambda = 0.9$, $L = 0.1$, and $\phi = 1.3\pi$.

Fig. 7. Efficiency versus load L for (i) case A ($\mu = 0$, and $\lambda = 0.9$), (ii) case B ($\mu = 1.0$, $\lambda = 0.9$), and (iii) case C ($\mu = 1.0$, $\lambda = 0.0$), for fixed $F_0 = 0.5$, $\phi = 1.3\pi$, and $T = 0.1$.

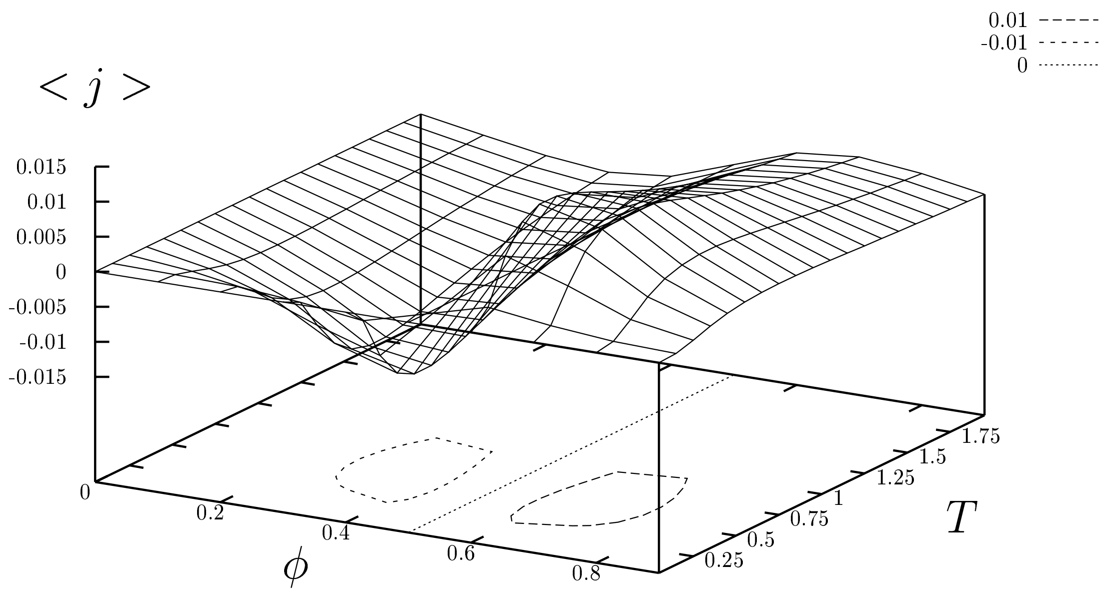
Fig. 8. Efficiency versus F_0 for (i) case A ($\mu = 0$, and $\lambda = 0.9$), (ii) case B ($\mu = 1.0$, $\lambda = 0.9$), and (iii) case C ($\mu = 1.0$, $\lambda = 0.0$), for fixed $L = 0.02$, $\phi = 1.3\pi$, and $T = 0.1$.

Fig. 9. Current $< j >$ versus temperature T for (i) case A ($\mu = 0$, and $\lambda = 0.9$), (ii) case B ($\mu = 1.0$, $\lambda = 0.9$), and (iii) case C ($\mu = 1.0$, $\lambda = 0.0$), for fixed $F_0 = 0.5$, $\phi = 0.3\pi$, and $L = 0.0$.

Fig. 10. The net current $< j >$ as a function of ϕ and T, for parameter values $F_0 = 0.5$, $\lambda = 0.9$, and $\mu = 1.0$. In the base plane contour of surface plot $< j > (\phi, T)$ are given, dotted line indicates $< j >= 0$ contour.

Fig. 11. $< j >$ as a function of temperature T and the amplitude of the rocking force F_0, for $\phi = 1.3\pi$, $\lambda = 0.9$, and $\mu = 1.0$.

Fig. 12. $< j >$ as a function of the amplitude of the rocking force F_0, and phase ϕ, for $T = 0.5$, and $\mu = 1.0$. In the base plane contour of surface plot $< j > (\phi, T)$ are given, dotted line indicates $< j >= 0$ contour.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T

FIG. 1.

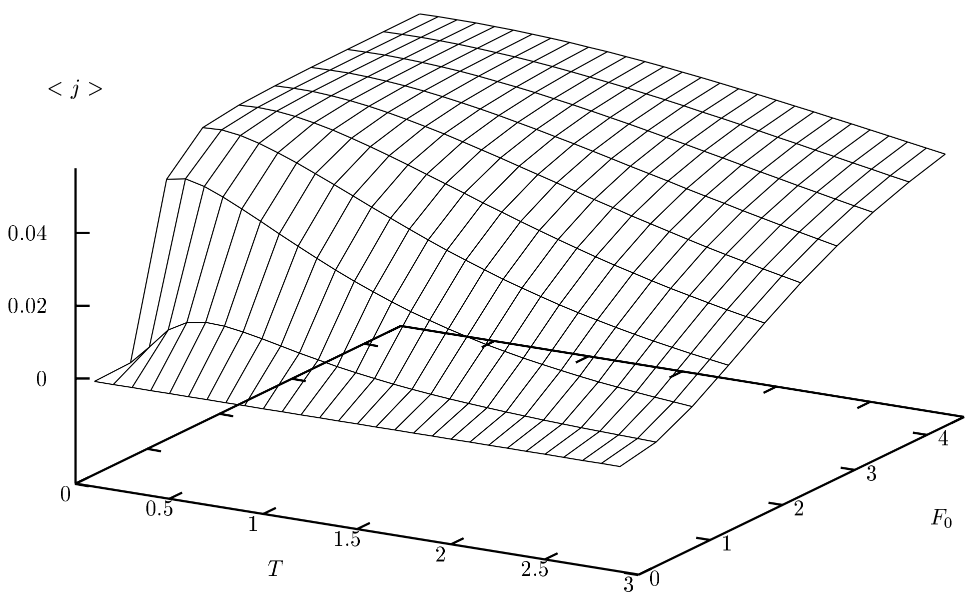
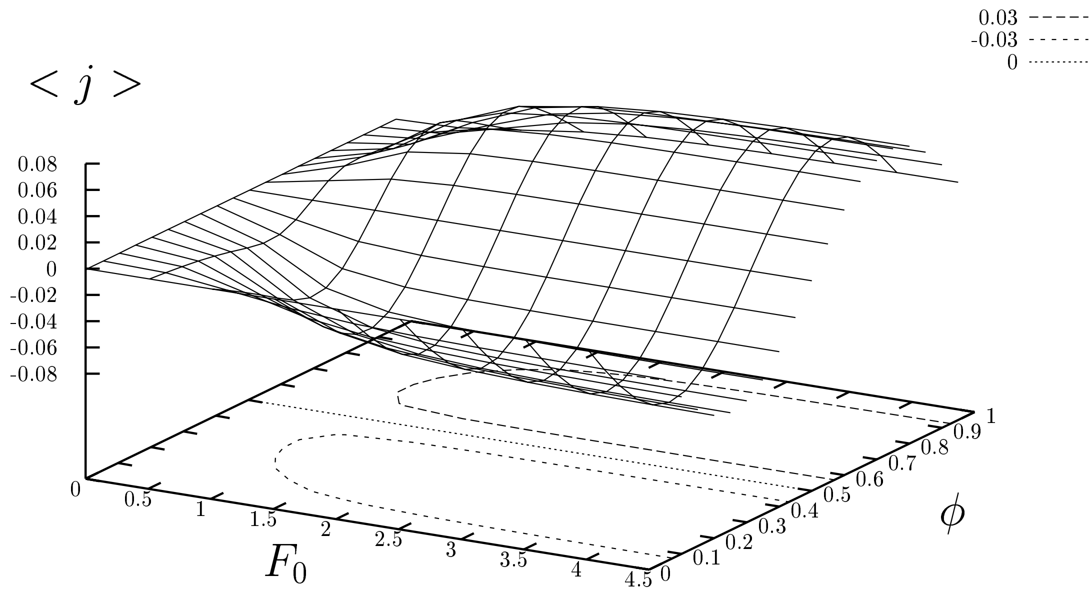


FIG. 2.



0.03
-0.03
0
$< j >$
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
F_0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
ϕ

FIG. 3.

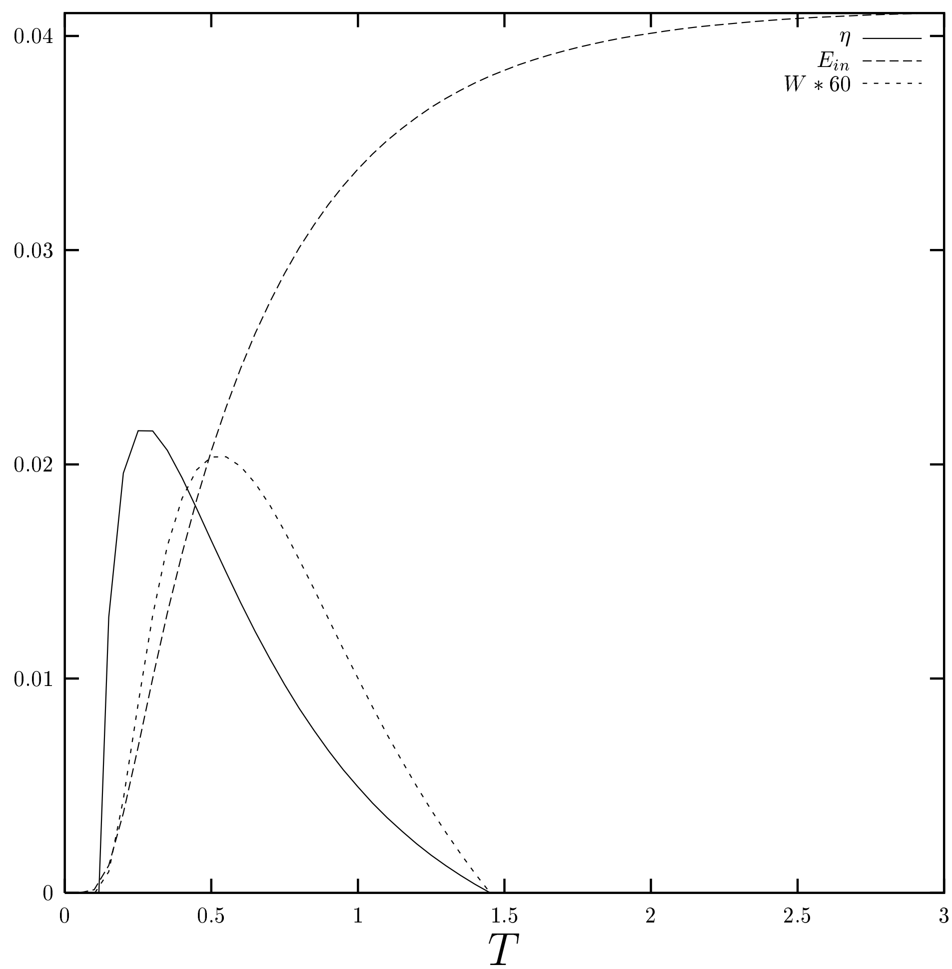


FIG. 4.

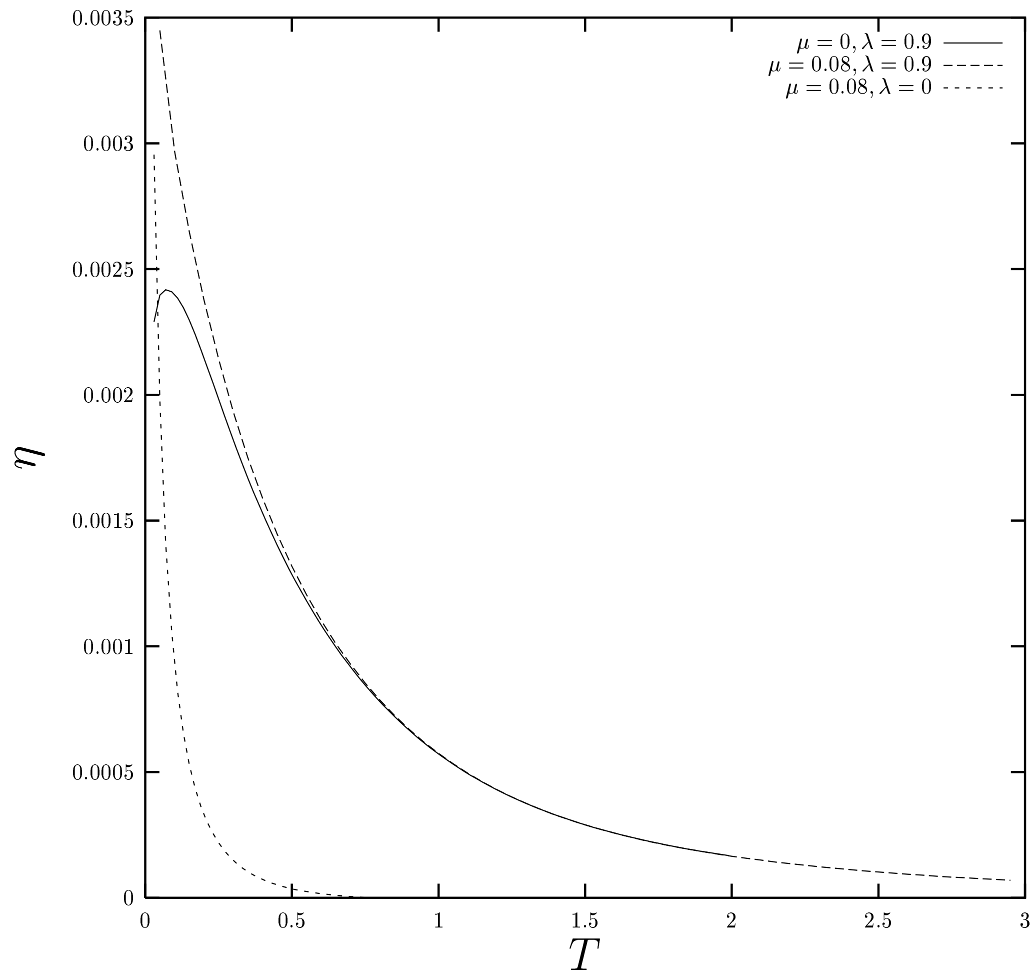

0.0035
0.003
0.0025
0.002
0.0015
0.001
0.0005
0
η
0
0.5
1
1.5
2
2.5
3
T
$\mu = 0, \lambda = 0.9$
$\mu = 0.08, \lambda = 0.9$
$\mu = 0.08, \lambda = 0$

FIG. 5.

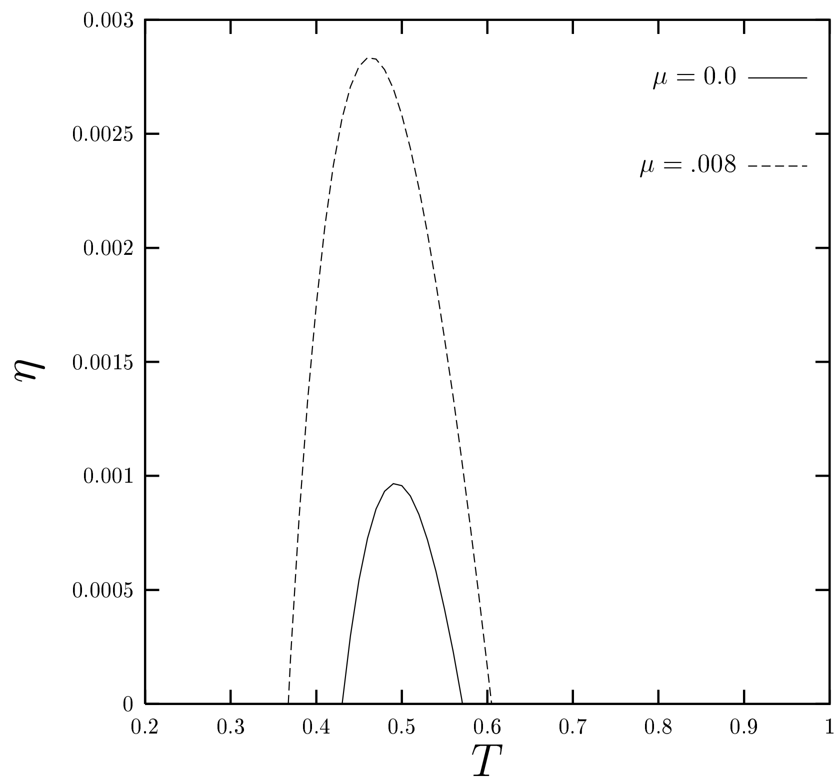

0.003
0.0025
0.002
0.0015
0.001
0.0005
0
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
η
T
$\mu = 0.0$
$\mu = .008$

FIG. 6.

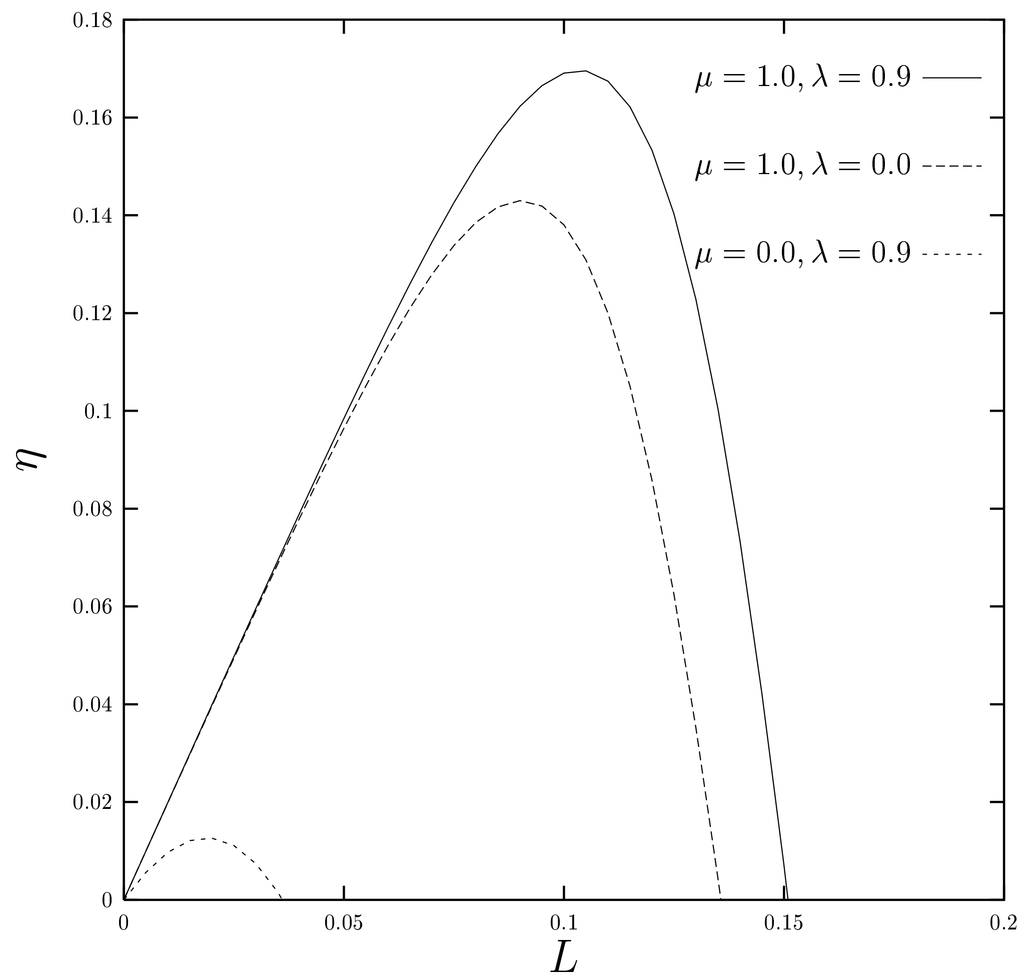
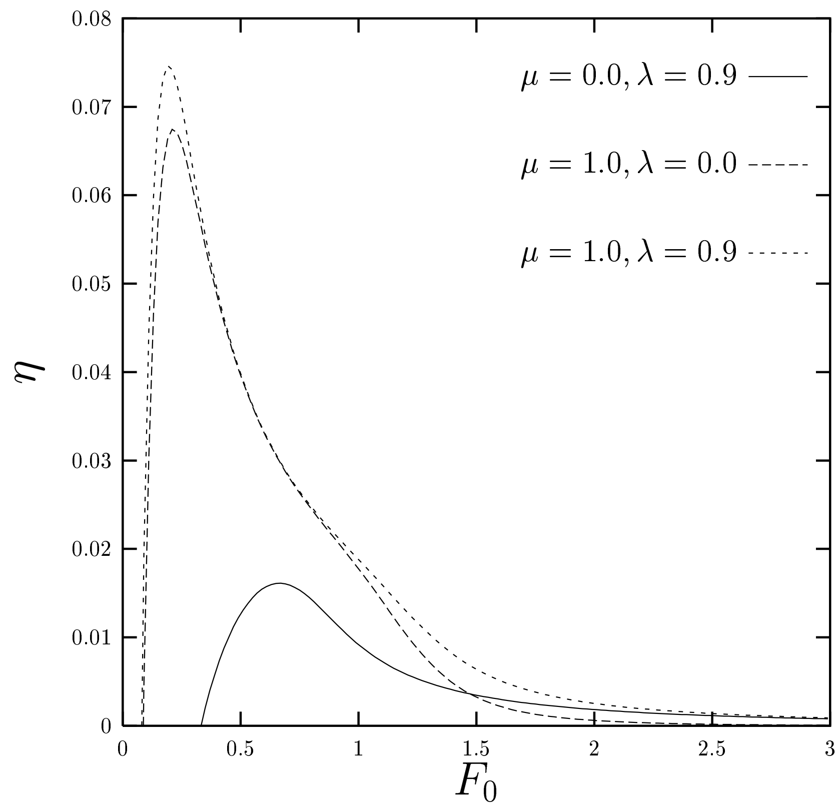

$\mu = 1.0, \lambda = 0.9$
$\mu = 1.0, \lambda = 0.0$
$\mu = 0.0, \lambda = 0.9$
η
L
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0.05
0.1
0.15
0.2

FIG. 7.



$\mu = 0.0, \lambda = 0.9$
$\mu = 1.0, \lambda = 0.0$
$\mu = 1.0, \lambda = 0.9$
η
F_0
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0.5
1
1.5
2
2.5
3

FIG. 8.

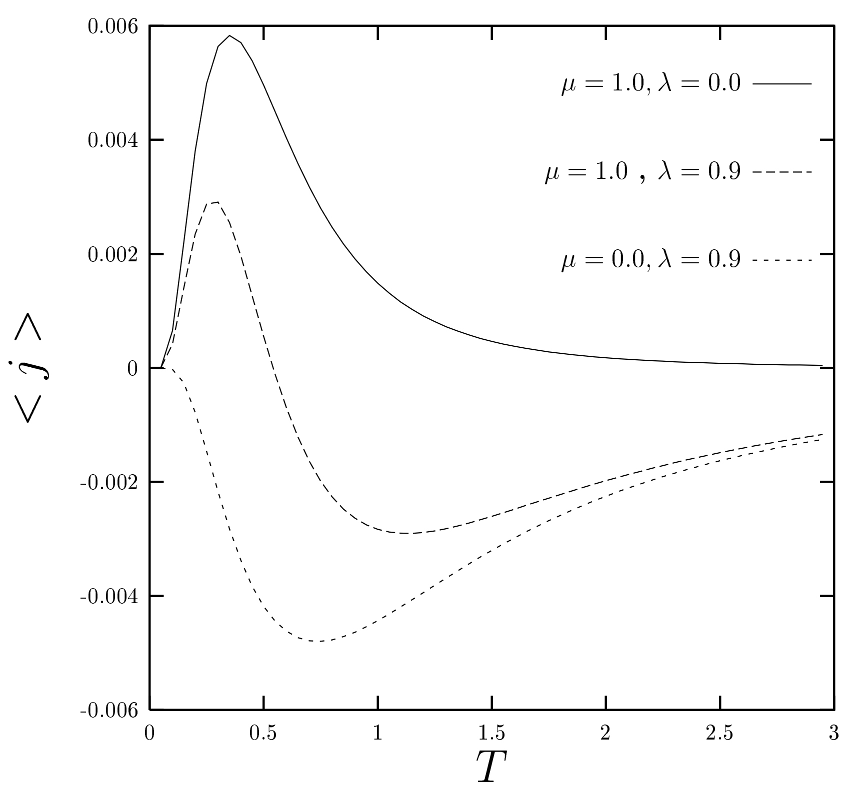
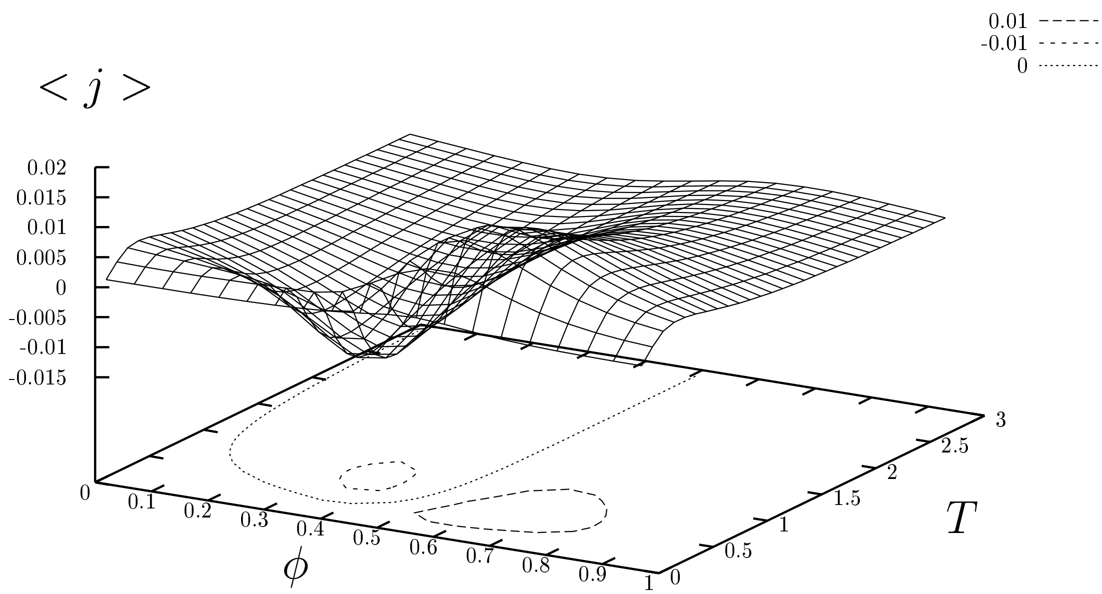

0.006
0.004
0.002
0
-0.002
-0.004
-0.006
0
0.5
1
1.5
2
2.5
3
$\mu = 1.0, \lambda = 0.0$
$\mu = 1.0$, $\lambda = 0.9$
$\mu = 0.0, \lambda = 0.9$
$< j >$
T

FIG. 9.



0.01
-0.01
0
$< j >$
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
ϕ
0
0.5
1
1.5
2
2.5
3
T

FIG. 10.

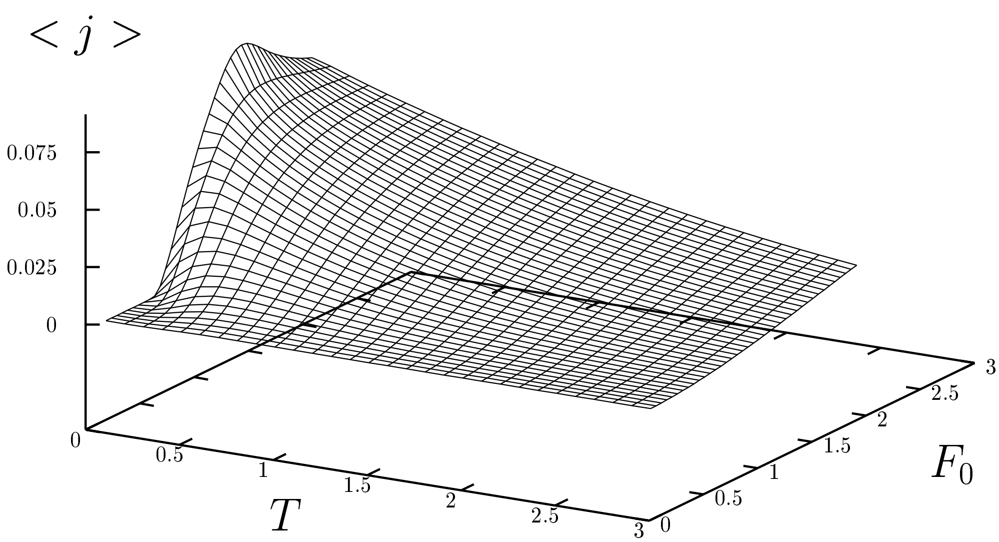


FIG. 11.

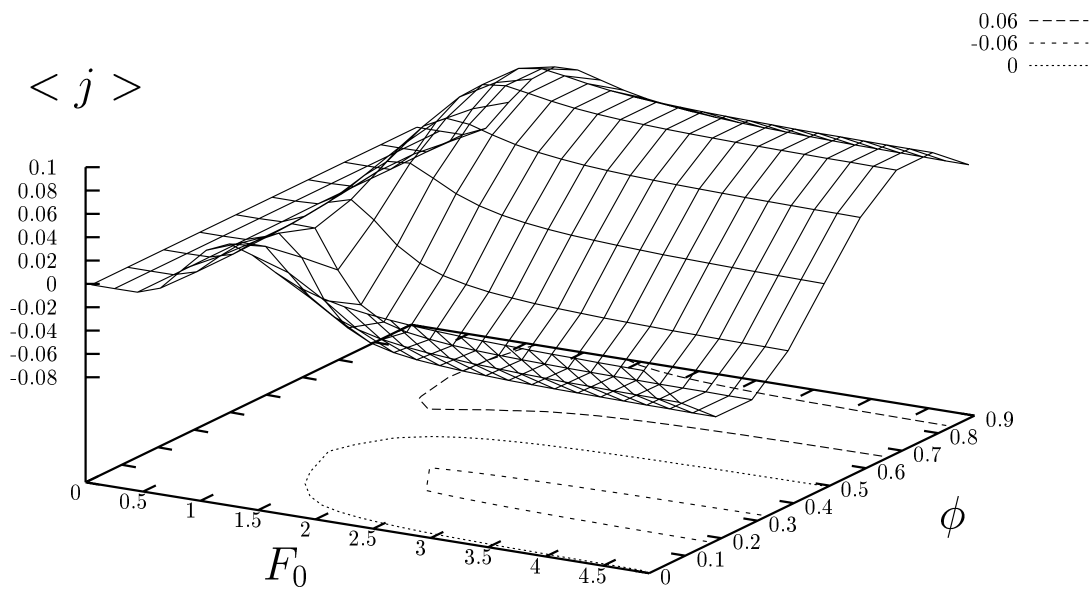

0.06
-0.06
0
< j >
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
F_0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
ϕ

FIG. 12.