Triplet higgs bosons at $e^+e^-$ colliders

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Whereas all the charged fermions must get their masses via Yukawa couplings with Higgs doublets[1], the vacuum expectation values (vev) of triplets can give Majorana masses to left-handed neutrinos. It is thus natural to consider other phenomenological implications of triplet scalars[2]. However, the vev of the neutral member of the triplet gives additional contributions to the parameter $\rho = \frac{m^2_W}{m^2_H \cos^2 \theta_W}$ (where $\theta_W$ is the Weinberg angle) at the tree-level, tending to deviate its value from unity. Since the present experimental value of $\rho$ is $(1.0004 \pm 0.0022 \pm 0.002)$ [3], any scenario with scalar triplets must be constrained accordingly.

One way to avoid this problem is to postulate that the vev of the neutral member of the triplet is small enough so that its contribution to $\rho$ is within the experimental limits. The other option is to assume [4], [5], that there are in fact more than one triplets, arranged in such a manner that their contributions to $\rho$ cancel each other. For this the minimum requirement is one complex ($Y=2$) and one real ($Y=0$) triplet, in addition to the $Y=1$ complex doublet of the minimal standard model. Furthermore the vev's of these two triplet fields must be equal to guarantee $\rho = 1$.

We have examined some testable consequences of triplets in both these situations, using the fact that with triplets, there exists a tree-level coupling involving a charged Higgs, $W$ and a $Z$. This interaction vertex was studied earlier in the context of LEP-I [6]. We find that in LEP-II and higher energy $e^+e^-$ colliders also, it leads to new types of signals.

In the first of the two cases the complex triplet $\Delta$ and a doublet $\phi$ are of the form

$$\Delta = \begin{pmatrix} \Delta^+ \\ \Delta^0 \end{pmatrix}, \quad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

(1)

Taking the vacuum expectation values to be $< \phi^0 > = \nu$ and $< \Delta^0 > = w$ the $\rho$ parameter in this model is $\rho = (1 + \frac{2w^2}{\nu^2})/(1 + \frac{4w^2}{\nu^2})$. The current experimental constraint translates (at 99% confidence level) to

$$\frac{w}{\nu} \leq 0.066$$

(2)
After obtaining the mass eigenstates, the doublet-triplet mixing angle turns out to be \( s_{H'} \equiv \sin \theta_{H'} \) etc. \[
 s_{H'} = \frac{\sqrt{2w}}{\sqrt{v^2 + 2w^2}}, \quad c_{H'} = \frac{v}{\sqrt{v^2 + 2w^2}} \tag{3}
\]
It is worth noting here that in this case the \( H'^+ \) couples to fermions through the doublet component.

The above scenario causes hierarchy problems because of the large splitting of the two vacuum expectation values. A solution is to add a real triplet field \( \chi \) \((Y = 0)\)
\[
 \chi = \begin{pmatrix}
 \chi^+ \\
 \chi^0 \\
 \chi^-
 \end{pmatrix}
\tag{4}
\]
and impose on the Higgs potential a \( SU(2)_L \otimes SU(2)_R \) symmetry which at tree level forces the two vacuum expectation values, \( < \Delta^0 > \) and \( < \chi^0 > \) to be equal. As a result the \( \rho \) parameter is one at tree level.

Upon diagonalisation of the mass matrix of the scalar sector, in general one finds, after duly absorbing the Goldstone bosons, a 5-plet, \( H_5^{++,-,+,0,-} \), a 3-plet, \( H_3^{++,0,-} \) and two singlets, \( H_1^0 \) and \( H_1^{*0} \), as the physical states. The different multiplets characterise their respective transformation properties under the custodial \( SU(2) \). The members of the \( H_5 \)-plet in this case do not have any overlap with the doublet \( \phi \), and as such they cannot interact with fermion-antifermion pairs.

Another constraint on the triplet scenario arises from the lepton-lepton couplings of the complex \( Y=2 \) triplet. The mass thus acquired by the electron neutrino will be given by
\[
 M_{\nu_e} = h_{\nu e} s_H \frac{M_W}{2} \frac{s_H}{g} \tag{5}
\]
with
\[
 s_H = \frac{2w}{\sqrt{v^2 + 4w^2}} \tag{6}
\]
where \( h_{\nu e} \) is the \( \Delta L = 2 \) coupling strenght of the first generation. The experimental constraints from neutrinoless double beta-decay imply that \( M_{\nu_e} < 1eV \). This means that either the doublet-triplet mixing angle or the \( \Delta L = 2 \) Yukawa coupling is restricted to very small values. Since in this model \( \rho = 1 \) at tree level a large mixing angle \( s_H \) is \textit{a priori} not excluded. Though it still requires fine-tuning to maintain the equality of the triplet vev’s at higher orders, the degree of such fine-tuning is not higher than that involved in connection with the naturalness problem within the standard model itself [7],[8].

As has been mentioned before, in both the models discussed above the \( H^+ W^- Z \) coupling exists at the tree level. The lagrangians are
\[
 L_{H W Z}^{(1)} = - \frac{g M_W}{\cos \theta_W} s_{H'} c_{H'} H'^+ W^- Z^\mu + \text{h.c.} \\
 L_{H W Z}^{(2)} = - \frac{g M_W}{\cos \theta_W} s_H H_5^+ W^- Z^\mu + \text{h.c.} \tag{7}
\]

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We now focus on the production of $H^\pm (H'^\pm$ or $H_5^\pm$) via the above interactions. First, there is the $s$-channel process $e^+e^- \rightarrow Z^\ast \rightarrow WH^\pm$ which will dominate at lower energies. Here we concentrate upon the final states consisting of the leptonic decay products of the $W$, i.e., on $e^+e^- \rightarrow H^\pm \nu\bar{\nu}$. In such cases, the $\nu\bar{\nu}$ final state also receives contributions from a $t$-channel diagram with the $H^\pm$ emitted from the propagator. For large values of the centre-of-mass energy, this latter diagram is dominant. Fig. 1 shows the cross-sections for $H^\pm \nu\bar{\nu}$-production plotted against $M_{H^\pm}$ for different values of $\sqrt{s}$. The contributions due to the electronic and muonic final states are added. The curves correspond to $s_{H} = 1$ and $c_{H'}s_{H'} = 1$ for the two cases of $H_5^\pm$, $H'^\pm$ respectively. The cross-sections for various values of the mixing angles can be read off by multiplying by the appropriate factor. It is clear from the graphs that in the LEP-II case (thin solid line), assuming an integrated luminosity of $10^{39} cm^{-2}$ per year there will be a few hundreds of events for $s_{H} = 1$ up to at least $M_{H_5} = 110 GeV$. If now one has only the $Y = 2$ triplet, the restriction from the $\rho$-parameter allows a maximum $s_{H}^{\ast}$ of 0.009. This leaves us with about 2 events per year. In such a case, the only reasonable chance of observing this process exists in a higher-energy $e^+e^-$ machine. As can be seen from the plot (dotted line), such a machine with a luminosity of $50 fb^{-1}/year$ [9] can lead to few tens of events up to $M_{H'} \simeq 300 GeV$ even with a value of $s_{H'}$ well within the limits imposed by $\rho$. On the other hand, since this restriction gets relaxed if one presupposes a complex and a real triplet, at LEP-II itself one can investigate a large area of the $\rho - s_{H}$ parameter space in the latter scenario.

The signals of the triplet scalars thus produced also depend on their decay channels. For the (complex+real) case, as has been mentioned, $H_5^\pm$ do not have
tree-level interactions with fermions. Possibilities of observing them through loop-induced decays into fermion pairs have already been studied [7]. However, there are also the tree-level decays into four-fermions mediated by (real or virtual) W and Z coupling with $H_5$. We have explicitly calculated such decay widths using methods described in ref. [10] modified appropriately. The decay widths into this channel for leptonic final states (with $s_H = 1$) are as a function of $M_{H_5}$ by the dashed line in

![Graph showing decay widths](image)

**Figure 2.** The different possible two body decay widths ($H_5^+ \rightarrow e\bar{s}$) at loop level (dotted line), tree level (solid line) and the tree level leptonic four body decay width $\Gamma(H_5^+ \rightarrow W^+Z^+ \rightarrow 4 \text{ leptons})$ (dashed line) for the charged Higgs. Again the mixing angles are put equal to 1 as in fig. 1. The $\Gamma(H_5^+ \rightarrow e\bar{s})$ (loop) has been taken from ref. [7]. The four body decay width is summed over electrons and $\mu$'s in the final state. For four quark final state the numbers are obtained by multiplying the dashed figure by $\sim 35$.

fig. 2. Here we have summed over the muons and the electrons in the final state. For purposes of comparison we also show by the dotted line one of the representative results for the loop induced partial width $\Gamma(H_5^+ \rightarrow e\bar{s})$ taken from ref. [7] (also drawn there for $s_H = 1$). The figure clearly shows that the partial decay width for our four-fermion modes is of similar order for $M_{H_5} \sim 50\text{GeV}$, and completely dominate for higher values of $M_{H_5}$. Consequently, the branching ratio for decays into a pair of weak gauge-bosons (WZ) remains close to 100 per cent over most of the parameter space. The 4-lepton ($l^+l^-'l^+l^-$) channel then has a healthy branching ratio of $\sim 1.4\%$. The corresponding signal is practically free from standard model backgrounds, and thus it should be considered as the principal technique in looking for triplet Higgs bosons of this type, produced in $e^+e^-$ collider experiments. In addition, with appropriate invariant mass cuts, the signal with four jets in the final states can be used increasing the useful branching ratio to $\sim 50\%$. This would increase the discovery range of $H_5^\pm$ even further since the width into four quarks final state will be a factor $\sim 35$ higher than the four lepton channel. Hence, the discovery range at LEP-200 (NLC) gets extended to 125 (425) GeV, for $s_H = 1$.

For the case of model 1 (i.e. $H^+$) the two-fermion decay-modes can occur at
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tree level. As can be seen from $\Gamma(H'^+ \rightarrow c\bar{s})$ given by the solid line in fig. 2 (and
dotted line in fig. 3) this decay mode will dominate over the four-lepton decay mode upto $M_{H'^+} = 150$ GeV. However, the width for the 4-quark decay mode, shown by dashed line in fig. 3, will be comparable to the two-quark decay mode for $M_{H'^+} \geq 120$ GeV.

Another way of detecting a triplet scenario in $e^+e^-$ machines is to look for production of a doubly charged scalar via processes such as $e^+e^- \rightarrow l^-l^-H'^++$. As mentioned before, these involve the $\Delta L = 2$ interactions which are not very tightly constrained when the doublet-triplet mixing angle is small. A detailed investigation is currently under way.

References


