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Soft Gluons and the Energy Dependence of Total Cross-Sections[1]

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Abstract

We discuss the high energy behaviour of total cross-sections for protons and photons, in a QCD based framework with particular emphasis on the role played by soft gluons.

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Abstract. We discuss the high energy behaviour of total cross-sections for protons and photons, in a QCD based framework with particular emphasis on the role played by soft gluons.

INTRODUCTION

Energy dependence of hadronic total cross-sections has fascinated particle physicists for decades now. In this talk we address a number of questions which arise when studying total hadronic cross-sections, namely

- Is it possible to study the energy dependence of the cross-sections for pp, $p\bar{p}$, γp and $\gamma\gamma \to hadrons$ in the same phenomenological/theoretical framework?
- What governs the energy dependence of these total cross-sections?
- What is the role played by the electromagnetic form factors in the description of the total cross-section?

The first question about treating *together* the $pp, p\bar{p}$ case on the one hand and the $\gamma p, \gamma\gamma$ case on the other, arises naturally as the 'hadronic' structure [1] of the photon has now been established in both e^+e^- and ep experiments conclusively [2]. Further, the photonic partons seem to have nontrivial effects on the photon-induced processes at high energies [3]. Equally importantly, along with the data already available for the $pp, p\bar{p}$ case [4], data have become available on total cross-sections for photon-induced processes reaching up to high γ energies, γp and $\gamma\gamma$ processes being studied in ep[5, 6, 7, 8], and e^+e^- [9, 10] collisions respectively. In Fig.1 we show a compilation of these proton and photon total cross sections, including cosmic ray data as well [11]. In order to put all the data on the same scale[12, 13], we have used a multiplication factor suggested by quark counting and Vector Meson Dominance[14], namely a factor $2/3\sum_{V=\rho,\omega,\pi}\left(4\pi\alpha_{QED}/f_V^2\right)$. Using a running α_{QED}, the VMD factor ranges from $1/250$ at low energy to $1/240$ at HERA energy. Square of this factor enters the photon-photon cross-sections.

At first glance, these data raise two questions: (i) whether the $\gamma\gamma$ total cross-section rises faster than the others and (ii) whether these various sets of data are mutually con-

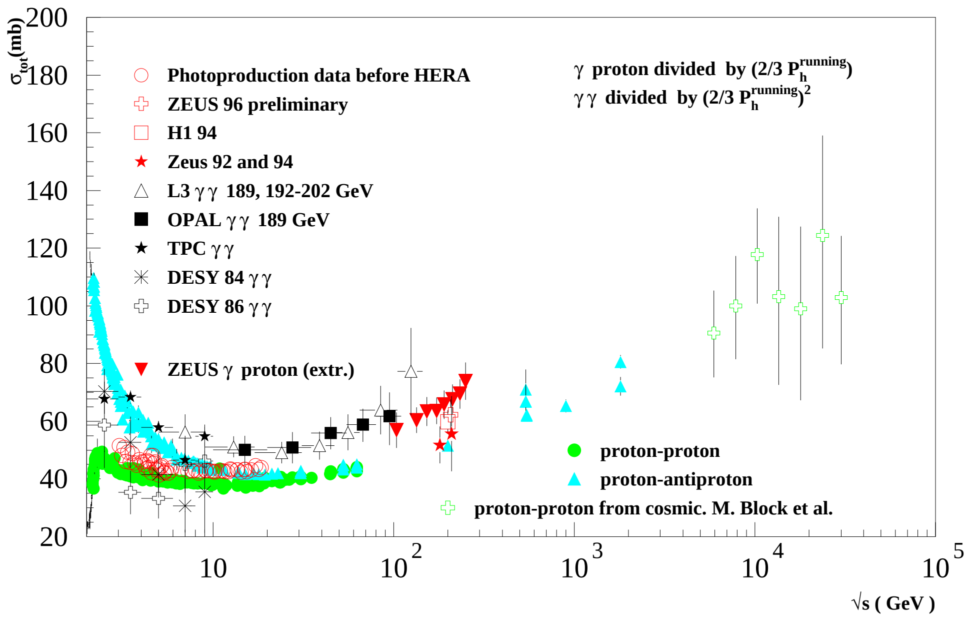

FIGURE 1. A compilation of $pp, p\bar{p}, \gamma p$ and $\gamma\gamma$ total cross sections with scaling factors described in the text.

sistent (at least at low energies) with the factorization hypothesis [15]. The uncertainty in the normalization of photon processes does not yet allow for a definite answer, but the photon-photon cross-sections do seem to be rather different, both from the point of view of the normalization [15] as well as the rise [13, 16, 17].

The next question is whether and how can we understand these data with our present means to deal with QCD. It appears that not all but many of the observed features are quantitatively obtainable from QCD. Our present goal is to obtain a QCD description of the initial decrease and the final increase of total cross-sections through soft gluon summation (via Bloch-Nordsieck Model) and mini-jets. Thus, our physical picture includes multiple parton collisions and soft gluons dressing each collision. We shall describe in the following sections details of the theoretical model proposed.

A QCD APPROACH

The task of describing the energy behaviour of total cross-sections can be broken down into three parts:

- the rise
- the initial decrease
- the normalization

The rise [18] can be obtained using the QCD calculable contribution from the parton-parton cross-section, whose total yield increases with energy, as shown in Fig.(2), where the jet cross-sections for proton-proton, γp and $\gamma\gamma$ are scaled by a common factor α.

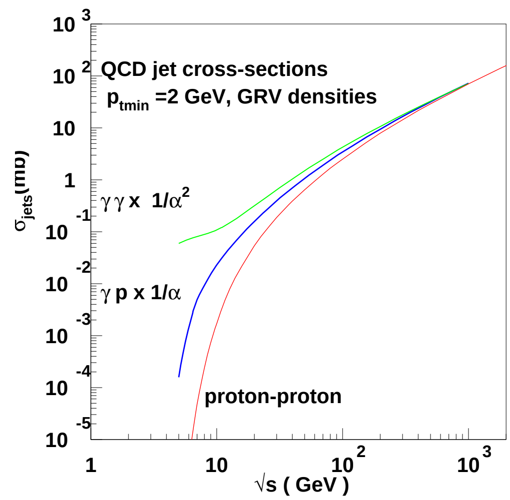

FIGURE 2. Minijets: Integrated jet cross-sections

In all cases, in particular for the proton case (where there are no direct scattering terms), one observes that σ_{jet} rises too fast for the observed values of σ_{tot} (less than 100 mb at the Tevatron) and that other terms, due to soft interactions, are missing. For a unitary description, the jet cross-sections are embedded into the eikonal formalism [19], namely one writes

$$\sigma^{\rm tot}_{pp(\bar{p})} = 2\int d^2\vec{b}[1-e^{-\chi_I(b,s)}cos(\chi_R)] \tag{1}$$

where the eikonal function $\chi = \chi_R + i\chi_I$ contains both the energy and the transverse momentum dependence of matter distribution in the colliding particles, through the impact parameter distribution in b-space[20]. The simplest formulation with minijets to drive the rise, in conjunction with eikonalization to ensure unitarity, is:

$$2\chi_I(b,s) \equiv n(b,s) = A(b)[\sigma_{soft} + \sigma_{jet}] \tag{2}$$

The normalization depends both upon σ_{soft} and the b-distribution. A very first working hypothesis is that the impact parameter distribution follows the matter distribution inside hadrons, namely that it is given by the Fourier transform of the electromagnetic form factors of the colliding particles, i.e.

$$A_{ab}(b) \equiv A(b;k_a,k_b) = \frac{1}{(2\pi)^2}\int d^2\vec{q}e^{iq\cdot b}\mathcal{F}_a(q,k_a)\mathcal{F}_b(q,k_b) \tag{3}$$

With such hypothesis, it is possible to describe the early rise, which takes place around $10-50$ GeV for proton-proton and proton-antiproton scattering, using GRV [21] densities for the protons and a transverse momentum cut-off in the jet cross-sections,

$p_{tmin} \simeq 1$ GeV, but then the cross-sections begin to rise too rapidly. One needs a $p_{tmin} \approx 2$ GeV in order to reproduce the Tevatron data, with the drawback, however, that one misses the early rise. In Fig.(3) we show a straightforward application of the Eikonal Minijet Model (EMM), with different values of p_{tmin}, to illustrate this feature.

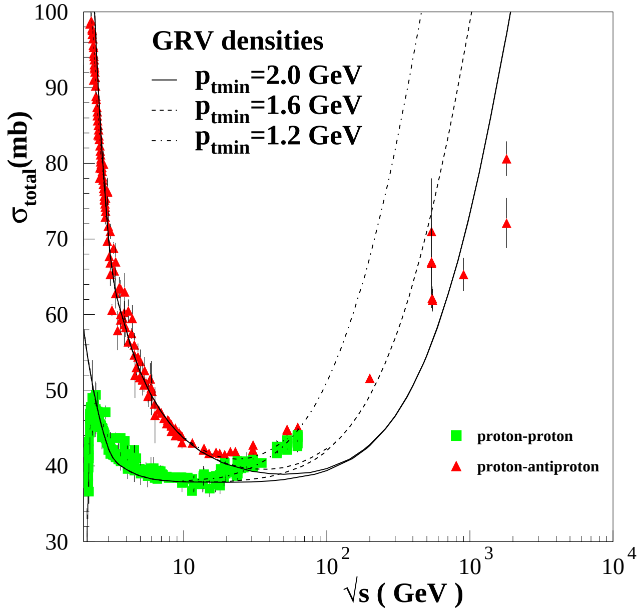

FIGURE 3. Total cross sections for pp and $p\bar{p}$ from EMM for various p_{tmin}.

A possible way to circumvent this problem lies in the use of soft gluons instead of form factors, but before turning to the issue of how to reproduce the early rise in proton-proton as well as the further Tevatron data points, we discuss the question of the photon cross-sections.

PHOTON PROCESSES AND MINIJETS

Photo-production and extrapolated data from Deep Inelastic Scattering (DIS) can be described through the same simple eikonal minijet model, with the relevant parton densities for the jet cross-sections, scaling [22] the non perturbative part given by σ_{soft} with the VMD and quark counting factor discussed above. The minijet cross-sections are then embedded into the eikonal formalism, with proper choice of impact parameter distribution. One needs a b-distribution of partons in the photon, which can be chosen to be a meson-like form factor.

The result is shown in Fig.(4), where the band corresponds to different sets of model parameters, with both GRV [23] and GRS [24] densities for the photon, and the dotted line corresponds to the predictions of the so-called Aspen Model[15]. The low energy region is obtained using quark counting and VMD from the proton data, while the high energy part is obtained from the QCD minijet cross-section and the impact parameter distribution from proton and pion-like form factors. As discussed in [12], the scale parameter k_0 in the photon form factor is allowed to vary in the range $0.4 - 0.66$ GeV.

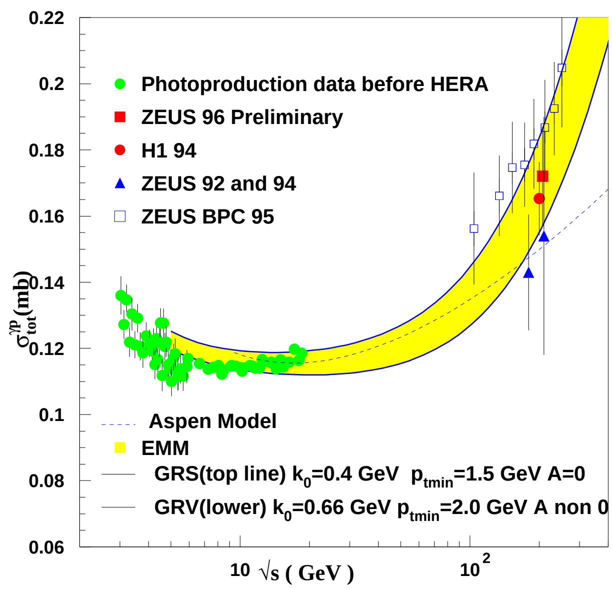

FIGURE 4. Description of photoproduction data with the EMM (band) and Aspen model (dotted line)

One encounters the same problem as in proton-proton case, albeit in a less severe form. When the parameters of the EMM are chosen so as to reproduce the low as well as the high energy data, the early rise is not well described. Modelling of $\gamma\, p$ data is further complicated, however, by the existence of data extrapolated from DIS [7] which lie above, but within 1σ, from recent photoproduction measurements [8]. Using a set of parameters consistent with those used to obtain the band of Fig.4, one can now attempt a description of photon-photon collisions and make predictions for future linear and photon colliders.

As before, one starts with the mini-jet cross-sections, for various parton densities and different values of p_{tmin}, as shown in Fig.(5). Note that the set of curves which lie higher at higher energies correspond to the GRV densities. These minijets are then embedded into the eikonal, with parameters consistent [25] with the $\gamma\, p$ band shown in Fig.(4). Present LEP data are shown in Fig.(6) where EMM predictions [12, 13, 25] are compared with those from various models [15, 26, 27, 28, 29, 30] which have been proposed to describe $\gamma\gamma$ total cross-sections. The uncertainty in the predictions of photon-photon collisions is reflected in the uncertainty in $e^+e^- \to hadrons$, albeit, in such case, the difference between the predictions of different models for $\gamma\gamma$ total cross-section is, at the end, at most a factor 2, even at TESLA energies[25, 31].

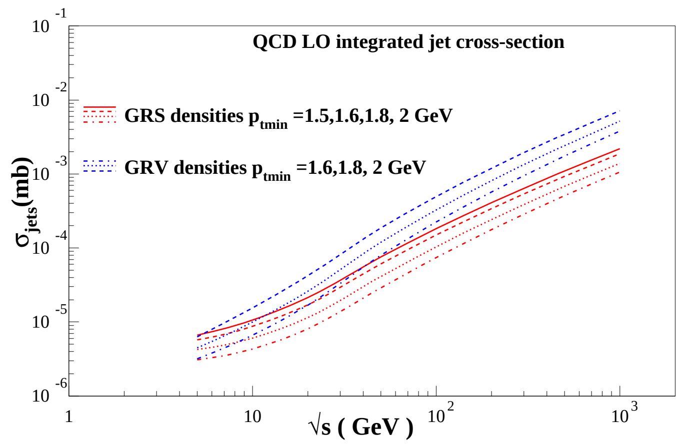

FIGURE 5. Minijets in photon-photon collisions

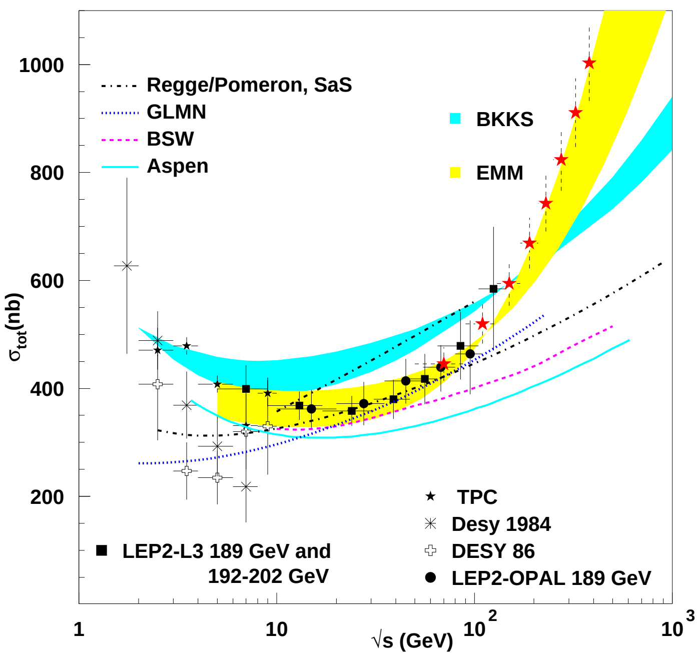

FIGURE 6. Photon photon total cross section data compared with various models.The stars at high photon-photon energies correspond to pseudo-data points extrapolated [31] from EMM predictions

THE TAMING OF THE RISE THROUGH SOFT GLUON SUMMATION

The fast rise due to mini-jets and the increasing number of gluon-gluon collisions as the energy increases, can be reduced if one takes into account that soft gluons, emitted mostly by the initial state valence quarks, give rise to an acollinearity between the partons which reduces the overall parton-parton luminosity. That is, as the energy increases, the larger phase space available for soft gluon emission implies more and more acollinearity and thus a reduced collision probability. This is the physical picture underlying the eikonal minijet model with Bloch-Nordsieck resummation[20]. In this model, the impact parameter distribution of partons is the (normalized) Fourier transform of the total transverse momentum distribution of valence quaks, obtained through soft gluon resummation, i.e.

$$A(b,s) = \frac{e^{-h(b,s)}}{\int d^2\vec{b}\; e^{-h(b,s)}} \tag{4}$$

with

$$h(b,s) = \int_{k_{min}}^{k_{max}} d^3\bar{n}(k)[1 - e^{-i\vec{k}_\perp \cdot \vec{b}}] \tag{5}$$

where $d^3\bar{n}(k)$ is the single soft gluon differential distribution and the integral runs, in principle, from zero to the maximum kinematic limit. Phenomenological applications of this expression encounter two main problems, one of theoretical origin, the other more of a phenomenological nature, namely, on the one side, a lack of our knowledge of the infrared behaviour of α_s , and, on the other, the unavailabily of reliable unintegrated parton distributions, i.e. parton distributions before the integration of their initial transverse momentum. The second difficulty can be phenomenologically overcome by averaging the function $A(b,s)$ over the parton densities to obtain the total number of collisions as

$$n(b,s) = A_{soft}(b)\sigma_{soft} + A_{PQCD}(b,s)\sigma^{LO}_{jet} \tag{6}$$

with $A_{soft}(b)$ as in the simpler EMM (form factors), and $A_{PQCD}(b,s)$ given by eqs.(4,5). The maximum energy for single soft gluon emission is obtained by averaging over the valence parton densities, i.e,

$$M \equiv < k_{max}(s) > = \frac{\sqrt{s}}{2} \frac{\sum_{i,j} \int \frac{dx_1}{x_1} f_{i/a}(x_1) \int \frac{dx_2}{x_2} f_{j/b}(x_2) \sqrt{x_1 x_2} \int dz(1-z)}{\sum_{i,j} \int \frac{dx_1}{x_1} f_{i/a}(x_1) \int \frac{dx_2}{x_2} f_{j/b}(x_2) \int (dz)}$$

with $z_{min} = 4p^2_{tmin}/(sx_1x_2)$. The quantity M can be calculated as a function of s for different values of p_{tmin}. For p_{tmin} values between 1 and 2 GeV, it ranges between 700 MeV and 3 GeV as $\sqrt{s}$ goes from 20 GeV to 10 TeV.

To proceed further, one also needs to specify the lower limit of integration, or, if the value zero is assumed, the behaviour of $\alpha_s(k_t)$ as $k_t \to 0$. Our model assumes $k_{min} = 0$ and two different trial behaviours are utilized for the above limit, a frozen α_s model i.e. $\alpha_s(0) = constant$ and a model in which α_s is singular, but integrable[32, 33]. Since a single soft gluon is never observed, one only needs integrated quantities and, at least phenomenologically, this model seems adequate. As discussed elsewhere [20], the effect of soft gluon summation is mostly to introduce an energy dependence in the large b-behavior. In the frozen α_s case, the large b-behaviour is not depressed enough, compared to the form factor case, thus indicating the need to introduce an intrinsic transverse momentum cut off, namely a gaussian decrease in the b-variable. Different is the singular α_s case, where the expression [32] $\alpha_s(k_\perp) = \frac{12\pi}{(33-2N_f)} \frac{p}{\ln[1+p(\frac{k_\perp}{\Lambda})^{2p}]}$, produces an increasingly faster falloff in the b-distribution as the energy increases. The s-dependence of the b-distribution modifies strongly the energy behaviour of the average number of collisions, as one can see from Figs.(7,8).

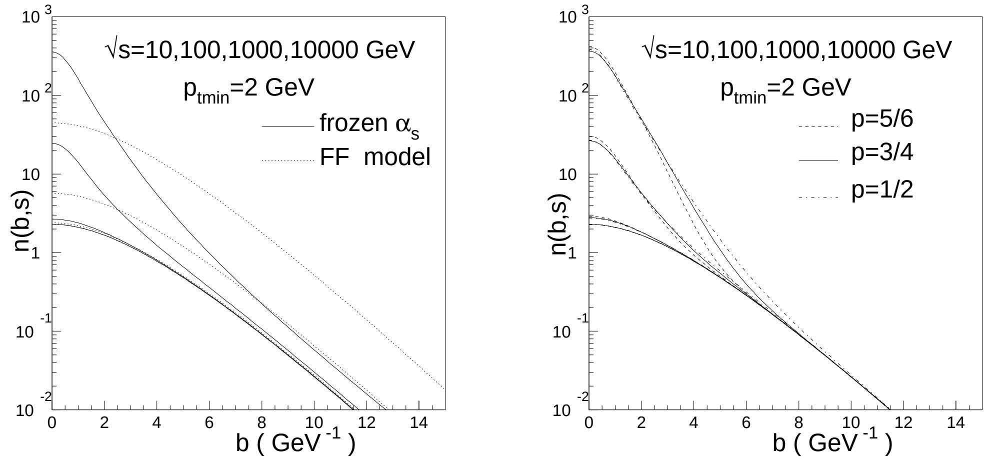

FIGURE 7. The average number of collisions for the frozen α_s case in comparison with the form factor (FF) model (left) and the singular α_s case (right) for different values of the singularity parameter p.

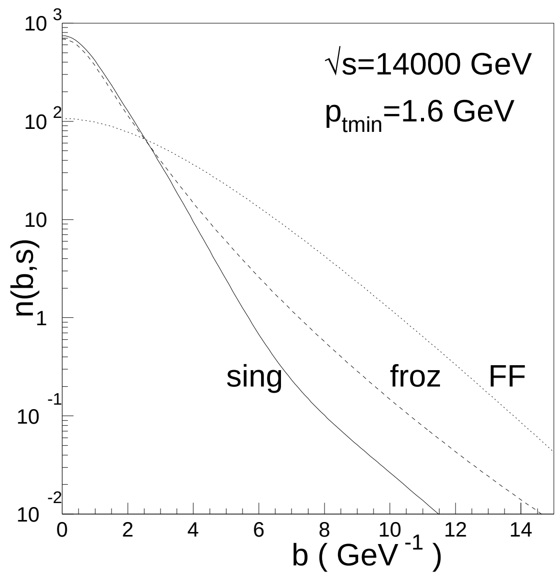

FIGURE 8. The average number of collisions in the form factor model and the Bloch Nordsieck model, at LHC energy

As the energy increases, the average number of collisions, relative to the form factor model, is strongly depressed at large b, thus smaller b-values contribute to the total cross-section, and the cross-section remains in general smaller than in the form factor case. In Fig.(9), we show how the integrand of eq.(1) behaves as a function of b, for $\sqrt{s} = 100, 1000$ and $10,000$, in the three models examined here. Note that we take $\cos\chi_R = 1$. The peak position shifts with increasing energy to higher b values and the area under the curve rises. The integrand is peaked at different b-values as the energy

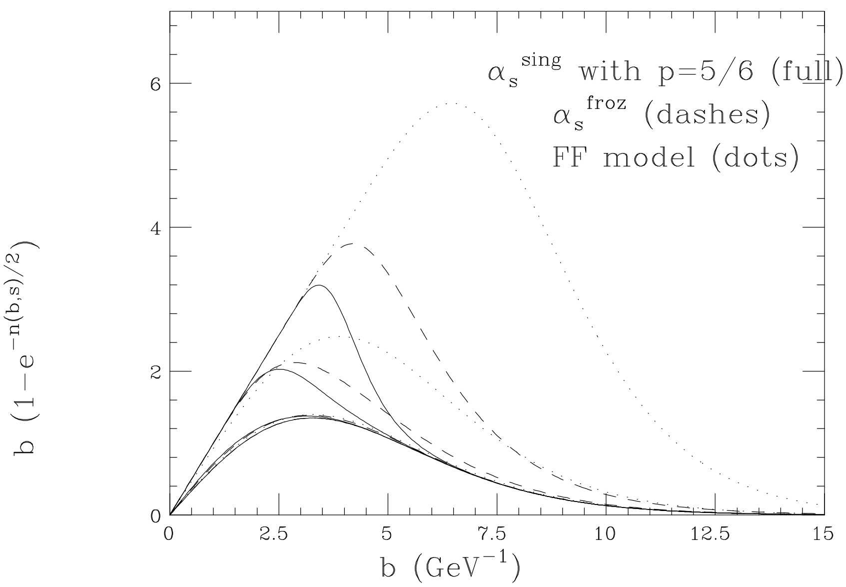

FIGURE 9. Integrand of the eikonal function for σ_{tot} in the three different models

increases, but also as the model for $A(b)$ changes. The rise with energy of the area under the curve, i.e. the cross-section, at the same energy, shrinks for the more singular α_s case. All the above features are illustrated in the plots given in the left panel of Fig.(10). We see that the effect of soft gluon summation in the singular α_s model reproduces quite well the early rise and the asymptotic softening. In comparison, the frozen α_s model appears almost as bad as the form factor model. The Bloch-Nordsieck model is practically indistinguishable from more conventional curves obtained through the Regge-Pomeron exchange [26] or the QCD inspired Aspen model[15], labelled BGHP in Fig.(10).

The analysis of proton collisions implies that straightforward applications of the minijet model through form factors are unable to describe correctly the large energy rise of total cross-sections. On the other hand, we have seen that the EMM can reproduce well the rise observed in $\gamma\gamma$ collisions in the present energy range, $\sqrt{s_{\gamma\gamma}} \approx 50-100$ GeV. But is the trend predicted by the EMM for photon-photon scattering correct at larger c.m. energies? It is quite possible that photon-photon data are only showing the early rapid rise, and that the rise at higher energies needs further corrections of the type we have described. Soft gluons are probably necessary in order to extrapolate to the higher energies of future electron-positron colliders such as TESLA, CLIC, NLC or Photon Colliders. An application of the Bloch-Nordsieck method to the case of photon-photon collisions is shown in the right panel of Fig.(10).

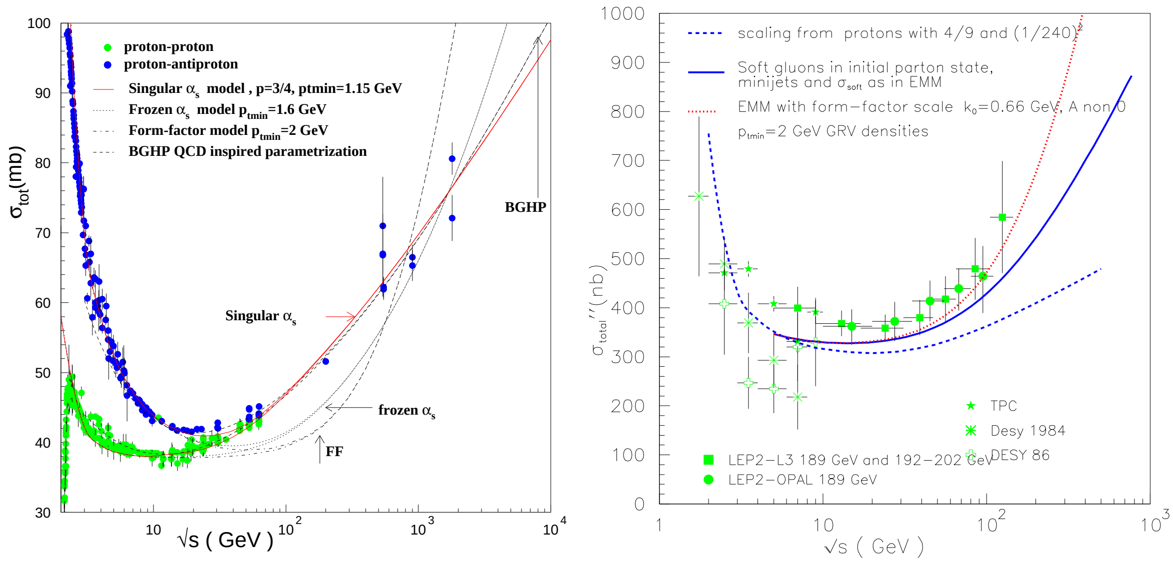

FIGURE 10. Total cross sections for pp and $p\bar{p}$ with frozen and singular α_s and form factor (FF) models (left panel). LEP data are compared with EMM with and without soft gluons and with a curve scaled from protons (right panel).

CONCLUSIONS

We have described a unified approach to the calculation of total cross-sections for protons and photons. In all cases, the driving cause for the rise of total cross-sections is the energy dependent perturbative QCD parton-parton cross-section. For photon induced processes the model seems to describe the rise adequately. However for all proton processes it gives a rise which appears way too strong. Taming of the rise can be accomplished by an energy dependent impact parameter distribution, and different models for the infrared behaviour of α_s in the soft gluon summation have been explored. Our phenomenological analysis indicates a distinct preference for a singular but integrable α_s which automatically produces the desired effect of an initial intrinsic tranverse momentum of partons in the hadrons. The resulting physical picture is that of multiple scattering between partons, implemented by initial state soft gluon bremmstrahlung.

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