Modelling of Stochastic Absorption in a Random Medium

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We report a detailed and systematic study of wave propagation through a stochastic absorbing random medium. Stochastic absorption is modeled by introducing an attenuation constant per unit length α in the free propagation region of the one-dimensional disordered chain of delta function scatterers. The average value of the logarithm of transmission coefficient decreases linearly with the length of the sample. The localization length is given by $\xi = \xi_w \xi_\alpha/(\xi_w + \xi_\alpha)$, where ξ_w and ξ_α are the localization lengths in the presence of only disorder and of only absorption respectively. Absorption does not introduce any additional reflection in the limit of large α, i.e., reflection shows a monotonic decrease with α and tends to zero in the limit of $\alpha \to \infty$, in contrast to the behavior observed in case of coherent absorption. The stationary distribution of reflection coefficient agrees well with the analytical results obtained within random phase approximation (RPA) in a larger parameter space. We also emphasize the major differences between the results of stochastic and coherent absorption.

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I. INTRODUCTION

Wave propagation in an active random medium has attracted much attention during the past decade. Recently, many experiments have reported lasing action of light in optically active strongly scattering media [1]. These systems exhibit interesting physical properties due to the combined effects of static disorder-induced multiple scattering and of coherent amplification/absorption [2–15]. In the extensively studied case of an electron motion in a random medium it is well established that quantum interference effects arising from a serial disorder in one-dimensional systems lead to Anderson localization [16,17]. Studies on different types of wave propagation such as quantum electron transport in disordered conductors and light propagation in random dielectric media or sound propagation in inhomogenous elastic media, etc. complement each inspite of the fact that treatment is quantum and classical respectively. These qualitatively different types of waves in an appropriate limit follow the same mathematical equation, namely the Helmholtz equation. It is basically the wave character leading to interference and diffraction which is the common operative feature. Light can be absorbed or amplified retaining the phase coherence. In most of the theoretical studies amplification or absorption is modeled phenomenologically by introducing an imaginary potential (optical potential) in the Hamiltonian. In the case of light (electro-magnetic waves) this corresponds to a medium with a complex dielectric constant. Several interesting effects have been predicted which include statistics of super-reflection and transmission [2–13] and the dual symmetry between absorption and amplification [14,15]. Media thus modeled are referred to as coherently absorbing or amplifying.

In the case of electron transport, inelastic scattering (due to phonons) leads to loss of phase memory of the wave function. Thus the motion of electrons becomes phase incoherent and sample to sample fluctuations become self-averaging in the high temperature limit leading to a classical behavior. There has been much interest in the effect of inelastic scattering on the coherent tunneling through potential barriers. To allow for the possibility of inelastic decay on the otherwise coherent tunneling through potential barriers, several studies invoke absorption [18,19]. To study the above phenomenon, one resorts to the optical potential models (coherent absorption models).

In the optical potential model the potential is made complex $V(x) = V_r(x) - iV_i$. The Hamiltonian becomes non-Hermitian resulting in absorption or amplification of probability current depending on the sign of V_i. The presence of imaginary potential (absorption/amplification) leads to many counter-intuitive features. In the scattering case, in the vicinity of the absorber, the particle

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experiences a mismatch in the potential (being complex) and therefore it tries to avoid this region by enhanced back reflection. Imaginary potential plays a dual role of an absorber and a reflector [3,20]. In other words, in such models absorption without reflection is not possible. Naively one expects the absorption to increase monotonically as a function of V_i. However, the observed behavior is non-monotonic [3,20]. At first absorption increases and after exhibiting a maximum decreases to zero as $V_i \rightarrow \infty$. The absorber, in this limit acts as a perfect reflector. During each scattering event an electron picks up an additional scattering phase shift due to V_i which along with multiple interference leads to additional coherence or resonances in the system [21]. Thus, due to the presence of imaginary potentials, we have additional reflection and resonances in the system. In the presence of coherent absorption and quenched disorder, the stationary distribution for reflection coefficient has been calculated [2]. This has been done within random phase approximation (RPA) using the invariant imbedding method [22]. The stationary distribution is given by:

$$P_s(r) = \frac{|D|exp(|D|)exp(-\frac{|D|}{1-r})}{(1-r)^2} \quad \text{for} \quad r \leq 1 \qquad (1)$$
$$= 0 \qquad\qquad \text{for} \quad r > 1.$$

Here D is proportional to V_i/W, W being the strength of disorder. Notice that the distribution has a single peak which shifts towards $r = 0$ with increasing absorption strength V_i. However, the exact distribution obtained numerically for strong disorder and strong absorption shows significant qualitative departure from this analytical distribution [3,13]. For sufficiently strong absorption, the numerically obtained stationary distribution shows a double peak structure. In the limit $V_i \rightarrow \infty$ the distribution becomes a delta function at $r = 1$. This corresponds to the limit where the absorber acts as a perfect reflector.

To this end we would like to develop a model where absorption does not lead to concomitant reflection and additional resonances as discussed above. Recently, such a stochastic absorption model was developed by Pradhan [23,24] based on the work of Büttiker [25,26]. In his treatment several absorptive side-channels are added to the purely elastic channels of interest. A particle that once enters the absorbing or the side-channel never returns back and is physically lost. He has obtained the Langevin equation for the reflection amplitude $R(L)$ for a random medium of length L by enlarging the S-matrix to include side-channels. In continuum limit the equation for $R(L)$ is [23,24]:

$$\frac{dR}{dL} = -\alpha R(L) + 2ikR(L) + ikV(L)[1 + R(L)]^2, \qquad (2)$$

where α is the absorption parameter and $V(L)$ is the random potential representing the static disorder. Interestingly, within the random phase approximation (RPA), the stationary probability distribution for the reflection coefficient $P_s(r)$ (for $L \rightarrow \infty$) is again given [23,24] by Eq.1. In our present work we develop another simple model for absorption which can be readily used to study the case of amplifying medium as well. The medium comprises of random strength delta function scatterers at regular spatial intervals a. To model absorption (leaking out) of electrons, an attenuation constant per unit length α is introduced. Every time the electron traverses the free region between the delta scatterers, we insert a factor $exp(-\alpha a)$ in the free propagator following Ref. [27]. We find that this method of modeling absorption does not lead to additional reflection and resonances as in the case of optical potential models. We obtain the localization length and study the statistics of reflection and transmission coefficients. The stationary distribution of reflection coefficient agrees with Eq.1 in a larger parameter space. Following earlier method [23], the continuum limit of our model leads to the same Langevin equation (Eq. 2) for $R(L)$ where α is replaced by 2α. Naturally, agreement of our result with Eq.1 follows. In Sec.II we give the details of our model and the numerical procedure. The section after that is devoted to results and discussion.

II. THE MODEL

We carry out calculations on the wave propagation in an absorbing medium characterized by an attenuation constant α and interspersed by a chain of uniformly spaced independent delta-function scatterers of random strengths. The i^{th} delta-function scattering center is described by a transfer matrix [28]

$$\mathsf{M_i} = \begin{pmatrix} 1 - iq_i/2k & -iq_i/2k \\ iq_i/2k & 1 + iq_i/2k \end{pmatrix}$$

where q_i is the strength of the i^{th} delta-function. The q_i's are uniformly distributed over the range $-W/2 \leq q_i \leq W/2$, i.e., $P(q_i) = 1/W$. Here W is the disorder strength. We set units of $\hbar$ and $2m$ to be unity. The energy of the incident wave is $E = k^2$. For further analysis, W and α are scaled with respect to a and are made dimensionless. Propagation of the wave in-between two consecutive delta-function scatterers separated by a unit spacing ($a = 1$) can be described by the matrix

$$\mathsf{X} = \begin{pmatrix} e^{ik-\alpha} & 0 \\ 0 & e^{-ik+\alpha} \end{pmatrix}.$$

The total transfer matrix for the L-site system is constructed by repeated application of $\mathsf{M_i}$ and X [28]:

$$M = \mathsf{M_L X....X M_2 X M_1}.$$

From M the reflection and transmission amplitudes are calculated using

$$R = -\frac{M(2,1)}{M(2,2)}$$

and

$$T \;=\; -\frac{\mathsf{det}M}{M(2,2)}.$$

The reflection and transmission coefficients are $r = |R|^2$ and $t = |T|^2$ respectively and the absorption is given by $\sigma = 1 - r - t$. Thus, due to absorption the total flux is not conserved and we have $r + t \neq 1$.

III. RESULTS AND DISCUSSION

In our studies we consider at least 10,000 realizations for calculating various distributions and averages. In the case of stationary distributions, the length of the samples considered were about 5 to 10 times the localization length. We also verified that the corresponding distributions or averages do not evolve any further with increasing sample length L. All results are shown for incident energy $E = k^2 = 1.0$ unless specified otherwise.

We first consider the behavior of $\langle lnt \rangle$. The angular bracket denotes the ensemble average. In Fig.1 we plot $\langle lnt \rangle$ as a function of length L for ordered absorptive medium ($W = 0.0$, $\alpha = 0.05$), ensemble averaged disordered non-absorptive medium ($W = 1.0$, $\alpha = 0.0$) and disordered absorptive medium ($W = 1.0$, $\alpha = 0.05, 0.1, 0.15$). In all the cases transmission decays exponentially with the length. The absorption-induced length scale ξ in random medium associated with the decay of transmission coefficient is always less than both ξ_a and ξ_w. The localization length for disordered nonabsorptive medium scales as [29] $\xi_w = 96k^2/W^2$ and for ordered absorptive medium, as $\xi_a = 1/\alpha$. In Fig.2 we show the plot of $1/\xi$ versus $1/\xi_w + 1/\xi_a$ obtained by changing α for various values of disorder strength W. We have numerically calculated $1/\xi$ for the different cases. All the points fall on a straight line with unit slope indicating the relation $1/\xi \;=\; 1/\xi_w + 1/\xi_a$. Such a relation exists for the case of coherently absorbing and amplifying media [5,13]. The decay of $\langle lnt \rangle$ with sample length L follows from the general theory of random matrices also [30–32]. In our approach scattering properties are described in the framework of 2x2 transfer matrices. Total tranfer matrix of the medium is the product of individual transfer matrices $\mathsf{M_i}$ of the individual scatterers. The limit $L \to \infty$ corresponds to multiplication of infinite number of such random matrices drawn independently from the same ensemble. In this limit the two random eigenvalues $exp(\pm x)$ of $MM^\dagger$ tend to the nonrandom values $exp(\pm L/\xi)$ with ξ independent of L. This follows from both the Furstenberg's theorem [30] as well as the multiplicative ergodic theorem [32]. The inverse localization length $1/xi$ is referred to as the Lyapunov exponent of the random matrix product in the literature. It should be noted that for large but finite L, the x has a small gaussian fluctuation [32] around the asymptotic value L/ξ. This fact has an important bearing on the nature of fluctuations in the finite size sample.

To study the nature of fluctuations in the transmission coefficient, in Fig.3 we plot, on log-scale, average t ($\langle t \rangle$), root-mean-squared variance $t_v = \sqrt{\langle t^2 \rangle - \langle t \rangle^2}$ and root-mean-squared relative variance $t_{rv} = t_v / \langle t \rangle$ as a function of length L/ξ for $W = 1.0$ and $\alpha = 0.01$. We see from the figure that t_v is less than $\langle t \rangle$ and t_{rv} is less than unity upto $L/\xi \approx 3$. But, beyond that t_v becomes greater than $\langle t \rangle$ and t_{rv} crosses unity. We find that in the asymptotic limit $ln(t_{rv})$ is positive given by the numerical value $0.18\ L/\xi$. The value $0.18/\xi$ is the generalized Lyapunov exponent characterizing the relative variance of transmission coefficient. Whenever the root-mean-squared variance of a physical quantity exceeds the average value, i.e. when relative fluctuations become larger than unity, the physical quantity is said to be non-self-averaging [33]. For example, it has been recognized long since that the resistance (which is related to the transmission coefficient [33,34]) of a one-dimensional random sample exhibits large statistical fluctuations when the sample size exceeds the localization length in absence of absorption or inelastic scattering [33,35]. The resistance fluctuations over the ensemble of macroscopically identical samples dominate the ensemble average. As a consequence the relative variance of resistance grows exponentially with the sample length L. In our study we find that in spite of the presence of absorption the relative variance of transmission coefficient grows exponentially with the sample length as mentioned earlier. Thus the transmission coefficient is a non-self-averaging quantity for samples of length $L \gg \xi$. The transmission coefficient becomes very sensitive to spatial realizations of the impurity configurations for finite size samples. This follows from the non-commutative nature of the $\mathsf{M_i}$ transfer matrices.

In Fig.4 we show the distribution $P(t)$ at different sample lengths for $\alpha = 0.01$ and $W = 1.0$. For small lengths L, resonant transmission dominates and $P(t)$ peaks at a large value of t. In fact for $L \to 0$, $P(t) \to \delta(t-1)$. As the length becomes comparable to the localization length $L \sim \xi$, multiple reflections start dominating. Consequently, the time spent inside the medium increases leading to more absorption. Thus, the peak of the distribution shifts to smaller values of t and the distribution broadens due to randomization by disorder. In the long length limit $L >> \xi$, the distribution develops a long tail and its peak shifts towards $t = 0$. The transmittance shows large sample-to-sample fluctuations and becomes a non-self-averaging quantity. Finally, as expected, for $L \to \infty$, $P(t) \to \delta(t)$.

From the previous discussion it is clear that the transmission becomes non-self-averaging. The large fluctuations in the transmission coefficient of a non-absorbing random medium owe their existence to the presence of resonant realizations (Azbel resonances) [36,37]. For a strongly localized one-dimensional system in the absence of absorption at particular energies the transmission coefficient decays exponentially as a function of length L with a well-defined localization length. However, for some rare realizations there exists a localized state close to the center of the sample for which incident electron can resonantly tunnel through the sample via this localized state with probablility approaching unity [33,36,37]. Such rare realizations play an important role in determining the fluctuations. Therefore, it is worthwhile to investigate the nature of resonances and the effect of absorption on them. Specifically, we would like to understand if the presence of absorption would give rise to any new resonances. It is well known from the studies in passive disordered media that the ensemble fluctuation and the fluctuations for a given sample as a function of chemical potential or energy are expected to be related by some sort of ergodicity [33,38], i.e., the measured fluctuations as a function of the control parameter are identical to the fluctuations observable by changing the impurity configurations. In Fig.5(a) we show the plot of t versus k for $W = 1.0$ and $\alpha = 0$ at $L = 100$ for a given realization of the random potential. Figure 5(b) shows a plot of t versus k for the same realization but with $\alpha = 0.01$. By mere visual inspection one can see that the only effect of absorption, apart from reducing the value of transmission, is to increase the width of resonance peaks for the passive case. Thus the presence of absorption does not introduce any new resonances. This can be seen from Fig.5(c) and (d) which emphasize Fig.5(a) and (b) respectively by enlarging a narrow region between $k = 1.5$ to $k = 2.0$. We do not see any new peaks in the transmission spectrum for absorptive case. Similar effect is observed in case of reflection also.

We now turn our attention to the statistics of reflection coefficient. In Fig.6 we plot $\langle lnr \rangle$ as a function of length L for a fixed value of disorder strength $W = 1.0$ and different values of absorption strength α as indicated in the figure. It increases with L initially and for $L \gg \xi$, it saturates. At any L, $\langle lnr \rangle|_{W,\alpha} < \langle lnr \rangle|_{W,0}$. This is in contrast to the behavior observed for the case of coherent absorption. As we know, in the case of coherent absorption, the reflection coefficient tends to unity for absorption strength becoming very large. In this regime, the predominantly reflecting nature of optical potential makes reflection larger than that in the corresponding passive case.

In Fig.7 we have shown $P_s(r)$ for various values of α. In the small α range, i.e., for $\alpha = 0.001$, the distribution has a peak at large r. As we increase α the peak shifts to smaller values of r. The thick line shows the fit obtained using the analytical expression given in Eq.1. In the limit of large α, the distribution tends to become a delta function at $r = 0$. This is in sharp contrast to the behavior observed for coherent absorption [3]. The distribution is always single peaked. For all non-zero values of α the medium acts as an absorber only and there is no additional reflection due to absorption. Figure 8 shows a monotonic decrease of saturated value of average reflection coefficient $\langle r \rangle_s$ as a function of α. The average absorption, defined as $\langle \sigma \rangle = 1 - \langle r \rangle - \langle t \rangle$, increases monotonically with increasing α and in the limit of $\alpha \to \infty$ saturates to unity in contrast to the optical model wherein it tends to zero. In the case of the optical model, the absorption coefficient is a non-monotonic function of absorption strength V_i and for values of V_i near the peak the stationary distribution of reflection coefficient displays a double peak [3,13]. In fact our model exhibits the properties in agreement with physical expectations of an absorbing medium, i.e., stronger the absorption lesser are the reflection and transmission across the medium.

Finally, we discuss the phase distribution. Figure 9 shows the stationary distribution of phase of the reflected wave for a fixed disorder strength $W = 1.0$ and various for values of α. For small values of disorder one generally expects the phase distribution to be uniform if the system size is around the localization length. This is seen in Fig.9(a) for the case of weak absorption. As we increase α the phase distribution develops two distinct peaks – a feature observed for coherent absorption also. This is related to the fact that the localization length decreases with α. We would like to point out that the stationary distribution $P_s(r)$ is same within the RPA for the case of coherently as well as stochastically absorbing media. Unlike in the case of coherently absorbing medium, Eq.1 seems to be valid in a larger parameter space for the stochastic absorbing medium where the RPA may not be valid. The parameters for validity of the RPA are determined by the observation of uniform phase distribution. However, beyond the RPA, $P_s(r)$ for the case of stochastic and coherent absorbing media are qualitatively distinct from each other.

In conclusion, we have studied a new phenomenological model of stochastic absorption to understand the statistics of quantum transport in random systems. The behavior observed for transmission and reflection coefficients is in accordance with physical expectations of an absorbing medium. This model can be extended to the case of stochastically amplifying medium. It exhibits duality between absorption and amplification which has received much attention recently. Results for this will be reported elsewhere [39]. It is to be noted that the treatment is a phenomenological one. A better treatment based on first principles like density matrix involving system and its coupling with environment is called for.

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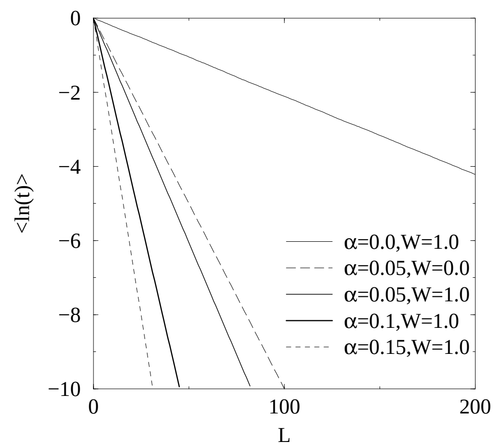


FIG. 1. Average of logarithm of transmission coefficient t versus length L for different values of absorption strength α and disorder strength W as indicated in the figure.

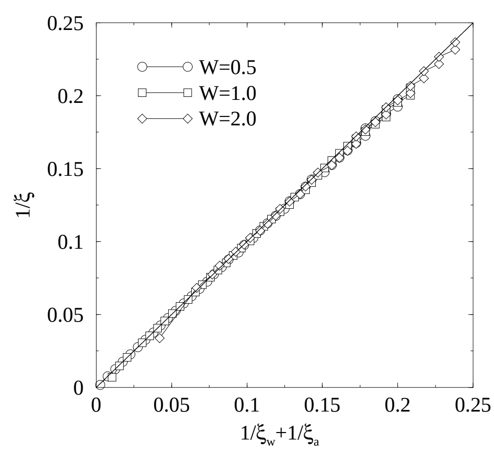


FIG. 2. $1/\xi$ versus $1/\xi_w + 1/\xi_a$, where ξ_w is the localization length for non-absorptive disordered system, ξ_a is the localization length for ordered absorptive system and ξ is the localization length for absorptive disordered system.

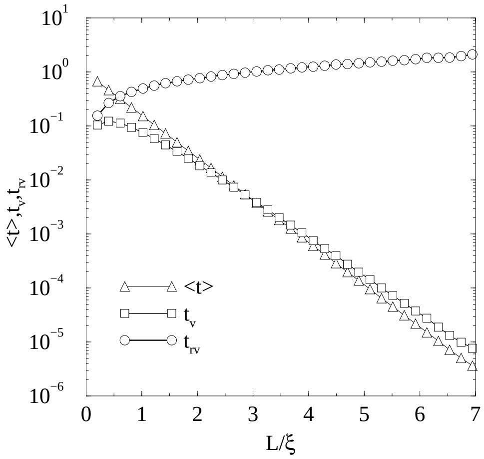


FIG. 3. Average transmission coefficient $\langle t\rangle$, variance of t and relative variance of t as a function of L/ξ for fixed disorder $W = 1.0$ and absorption $\alpha = 0.01$.

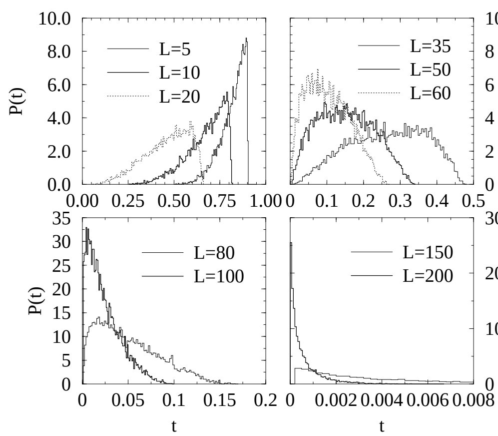
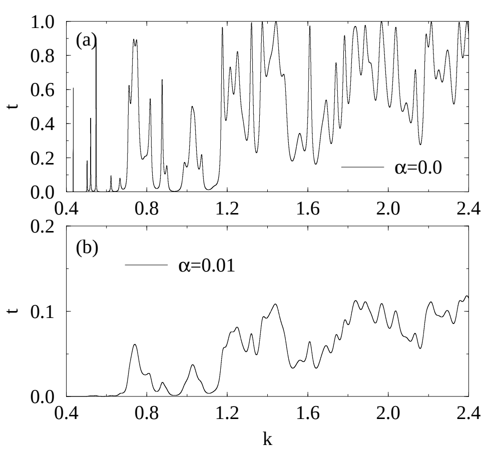


FIG. 4. Distribution of transmission coefficient t from a disordered absorptive system with $W = 1.0$ and $\alpha = 0.01$ at different lengths L.



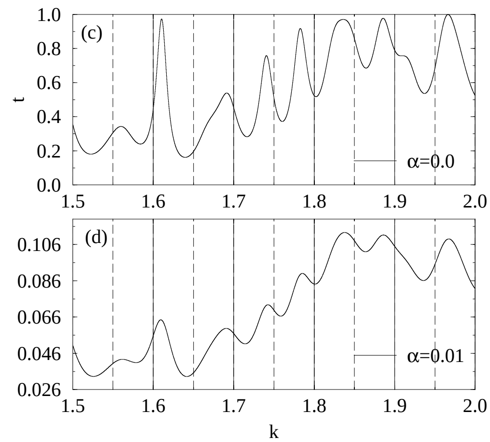


FIG. 5. Transmission coefficient as a function of incident wavenumber k for (a),(c) disordered non-absorptive sample $W = 1.0$ of length $L = 100$ and (b),(d) disordered absorptive sample $W = 1.0, \alpha = 0.01$ of length $L = 100$.

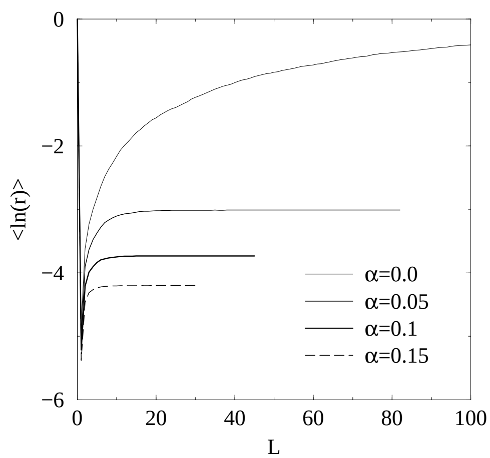


FIG. 6. Average of logarithm of reflection coefficient versus sample length for a fixed value of disorder $W = 1.0$ and different values of absorption α indicated in the figure.

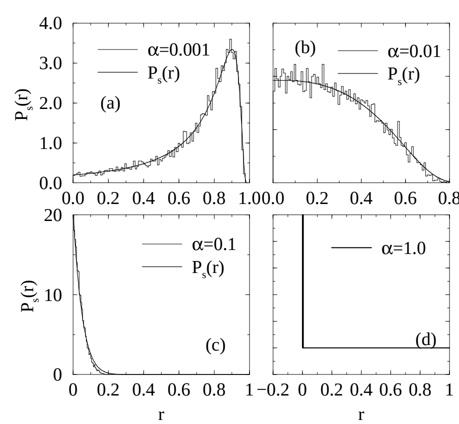


FIG. 7. Stationary distribution of reflection coefficient for a fixed value of disorder strength $W = 1.0$ and different values of absorption α. The thick line shows the single parameter fit of analytical expression Eqn.1 with (a) $D = 0.197$ for $\alpha = 0.001$,(b) $D = 1.92$ for $\alpha = 0.01$ and (c) $D = 20.53$ for $\alpha = 0.1$.

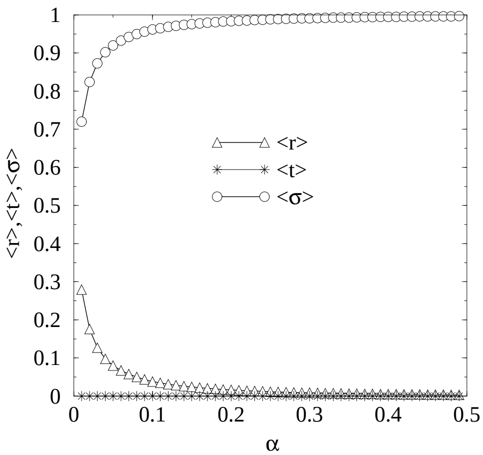


FIG. 8. Average value of reflection coefficient ($\langle r \rangle$), transmission coefficient ($\langle t \rangle$) and absorption ($\langle \sigma \rangle$) versus absorption strength α for $W = 1.0$ and $L/\xi = 10$.

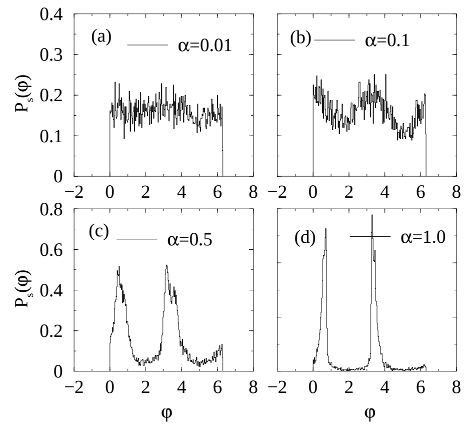


FIG. 9. Stationary distribution of phase of reflected wave for fixed disorder strength $W = 1.0$ and different values of absorption α.