Multiple current reversals in forced inhomogeneous ratchets

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Transport properties of overdamped Brownian paricles in a rocked thermal ratchet with space dependent friction coefficient is studied. By tuning the parameters, the direction of current exhibit multiple reversals, both as a function of the thermal noise strength as well as the amplitude of rocking force. Current reversals also occur under deterministic conditions and exhibits intriguing structure. All these results arise due to mutual interplay between potential asymmetry, noise, driving frequency and inhomogeneous friction.

Fluctuation induced transport in ratchet systems has been an active field of research over the last decade. In these systems in the absence of any net microscopic forces, asymmetric potential can be used to induce a unidirectional particle flow when subjected to an external nonthermal fluctuations. These studies have been motivated in part by the attempt to understand the mechanism of movement of protein motors in biological systems [1]. To this effect several physical models have been proposed under the name of rocking ratchets [2–4], flashing ratchets [5,6], diffusion ratchets [7], correlation ratchets [8], etc. In all these studies the potential is taken to be asymmetric in space. It has also been shown that one can obtain unidirectional current in the presence of spatially symmetric potentials. For these nonequilibrium systems external random force should be time asymmetric [9] or the presence of space dependent mobility is required [12,10,11,13–16]. By suitably tuning the system parameters such as temperature, friction coefficient, mass, etc, one can even change the direction of the current. Indeed, the study of current reversal phenomena has given rise to a research activity on its own. The motivation being the possibility of new particle separation devices superior to existing methods such as electrophoretic method for particles of micrometer scale [17].

Bartussek *et. al.* [4] showed the occurrence of current reversal in rocked thermal ratchet with both amplitude of rocking force as well as the temperature of thermal bath. They attributed this current reversal to the "mutual interplay between noise and finite-frequency driving". Multiple current reversals have also been shown in the deterministic limit of these ratchets when the inertial term is taken into account [3,20]. However these multiple current reversals in inertial ratchets are not robust in the presence of noise. Beside rocking ratchets current reversals have also been observed in flashing ratchets [21,22,6]. We in this work report that that multiple reversals can be achieved even in rocked *overdamped* ratchet in the presence of space dependent mobility, as a function of the noise strength and amplitude of the rocking force. More over these systems show current reversals when rocked adiabatically. In the deterministic overdamped case we get current reversal as a function of the amplitude of rocking force. Most of our results are attributed to the presence of space dependent mobility. We have studied the same system as due to Bartussek *et al* [4], except the presence of space dependent friction term. In earlier work the spatial asymmetry of the potential is responsible for unidirectional currents and their reversal as function of frequency. As mentioned previouly one can get unidirectional currents in the presence of symmetric potentials, but for this space dependent friction is required [12,10,11,13–16]. In these systems transport direction is given by the phase shift between mobility and periodic potential. This phase shift induces spatial symmetry breaking as required for the directed motion. Appropriately choosing the phase shift leads to current reversal. We would like to emphasize that space dependent friction does not alter the equilibrium property of the system, however when the system is driven out of equilibrium not trivial dynamical effects arise due to space dependent friction [16,18,19]. Naturally in our present system we expect additional effects arising due to combination of spatial asymmetry and position dependence of friction coefficient. It is to be noted that systems with space dependent friction are not uncommon. Brownian motion in confined geometries show space dependent friction [23]. Particles diffusing close to surface have space dependent friction coefficient [23,24]. It is believed that molecular motor proteins move close along the periodic structure of microtubules and will therefore experience a position dependent mobility [15]. Frictional inhomogeneities are common in super lattice structures and Josephson junctions [25] also.

We consider an overdamped Brownian particle moving in a asymmetric potential V(x) with space dependent friction coefficient η(x) under the influence of external force field F(t) at temperature T. Throughout our analysis we take the ratchet potential V(x) $= -\frac{1}{2\pi}(\sin(2\pi x) + \frac{\mu}{4}\sin(4\pi x))$. Here μ is the asymmetry parameter with values taken in the range $0 < \mu < 1$, friction coefficient $\eta(x) = \eta_0(1 - \lambda\sin(2\pi x + \phi))$, $|\lambda| < 1$. ϕ determines the relative phase shift between friction coefficient and potential . The forcing term is taken to be F(t) = Asin$(wt + \theta)$, ($w = \frac{2\pi}{\tau}$, where τ is period of force) . Without any loss of generality θ is taken to be zero. The correct Langevin equation for this system in the overdamped limit is given by [12,26,27]

$$\dot{x} = -\frac{(V'(x) - F(t))}{\eta(x)} - k_BT\frac{\eta'(x)}{(\eta(x))^2} + \sqrt{\frac{k_BT}{\eta(x)}}\xi(t), \tag{1}$$

where $\xi(t)$ is a zero mean thermal Gaussian noise with correlation $< \xi(t)\xi(t') >= 2\delta(t - t')$ The equation of motion is equivalently described by the Fokker Planck Equation (FPE) [12]

$$\frac{\partial P(x,t)}{\partial t} = \frac{\partial}{\partial x}\frac{1}{\eta(x)}[k_BT\frac{\partial}{\partial x} + (V'(x) - F(t))]P(x,t). \tag{2}$$

where $P(x,t)$ is the probability density at position x at time t. Equation (2), in the form of a continuity equation

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial J(x,t)}{\partial x}, \tag{3}$$

where

$$J(x,t) = -\frac{1}{\eta(x)}[(V'(x) - F(t)) + k_BT\frac{\partial}{\partial x}]P(x,t), \tag{4}$$

is the probability current. Since the potential and the driving force have spatial and temporal periodicity respectively, therefore $J(x,t) = J(x+1, t+\tau)$, [28,4]. The average current j in the system is given by

$$j = \lim_{t\to\infty}\frac{1}{\tau}\int_t^{t+\tau}dt\int_0^1 J(x,t)dx. \tag{5}$$

It should be noted that for symmetric potential and $\lambda = 0$, $j = 0$. Rectification of current is possible if the potential is either asymmetric or $\lambda \neq 0$ with $\phi \neq 0, \pi$ [16]. j is independent of the initial phase θ of the driving force.

We explore various parameter regimes of the problem extensively by solving the FPE, Eq. (2) numerically with finite difference method. Throughout we have set current j and all other physical quantities such as T, A, w in dimensionless form .

In the Fig. 1, the average current j is plotted *vs* T for various values of w. Here $\lambda = 0.1, \phi = 0.2\pi$, $A = 0.5$ and $\mu = 1$. Unlike Bartussek et al. [4], where they show that current reversal is not possible under adiabatic conditions, however current reversal even in adiabatic condition can be obtained in the present case for some values of ϕ [16]. For moderately high frequency $w = 4.0$ the current reverses its sign *twice* at $T = 0.1$ and $T = 0.22$ and asymptotically goes to zero for higher values of T as shown in the figure. This phenomena of twice current reversal with temperature T is the foremost feature of our system, previously unseen in any overdamped system. The inset shows the zero contour plot of the current j versus T and ϕ for three values of w. Crossing this zero contour line implies current reversal. It can be seen that twice current reversal occurs only for a very narrow range of ϕ. For frequencies higher than certain critical frequency $w_c(\phi)$ current flows in only one direction for all temperatures, i.e no current reversal occurs. The plot w_c *vs* ϕ is shown in Fig. 2. The adiabatic curve $w < 1$ is not shown in the figure as it goes much beyond the scale of the graph. However it has a similar qualitative shape as for $w = 3.0$ curve. As mentioned before, currents are due to the combined effect of phase shift ϕ coming from space dependent friction and asymmetry parameter μ. In the regime $\phi = 0.2\pi$ and $A = 0.5$ and in the absence of asymmetry current flows in the negative direction for all values of T. The asymmetric case ($\mu = 1.0$) in the absence of space dependent friction give current in the positive direction only as a function of temperature. Separately in both these cases absolute value of current exhibits a maxima as a function of T, reminiscent of stochastic resonance phenomena. In a purely asymmetric case ($\lambda = 0$) current vanishes rapidly when T exceeds the temperature associated with the barrier height. Whereas, in the symmetric case due to space dependent friction absolute values of currents are significantly higher and decay slowly to zero in the large temperature regime. Naturally in the presence of both asymmetry and space dependent friction for the case under study the low temperature regime is dominated by the effect of asymmetry while the high temperature regime is dominated by space dependent friction. From this , one can qualitatively explain the current reversals from positive to negative side as a function of temperature even in the adiabatic limit. In the regime $\phi > \pi$ current due to space dependent friction and potential asymmetry flows in same direction and hence no reversal is possible as a function of T. For frequencies higher than the interwell frequency w_0, the low temperature scenario changes. The direction of current in this regime is more of a interplay between potential asymmetry and w than that of λ. Due to higher frequency the Brownian particles do not get enough time to cross the right barrier which is at a larger distance from the minima. Number of particles moving about the potential minima increases. This fact is amply reflected in Fig. 3, where the time averaged probability curve $P_{av}(x) = \frac{1}{\tau}\int_0^\tau P(x,t)dt$ is plotted as function of x for various values of w with $T = 0.05$. It is to be noted that the distribution is independent of $\eta(x)$. Figure 3 shows that the probability of finding the Brownian particles near the minima of the potential well increases with increasing frequency, consequently the probable number of particles near the potential barrier decreases. Since the distance from a potential minima to the basin of attraction of next minima is less from the steeper side than from the slanted side, hence in one period the particles get enough time to climb the potential barrier from the steeper side than from the slanted side, resulting in a negative current. On increasing the temperature , the particles get kicks of larger intensity and hence they easily cross the slanted barrier, resulting in a current reversal and positive current. On further increasing the temperature, the effect of λ dominates as mentioned previously and as a result the current again flows in the negative direction, implying a second current reversal as shown in dotted curve of Fig 1. It is obvious from the above argument that for higher frequency the first reversal temperature will be higher as shown in the Fig. 4. The dotted line in the base of Fig. 4 shows the contour of zero current. But as mentioned previously, since the effect of λ dominates for higher temperature, therefore the second reversal temperature decreases with increasing frequency. Beyond the critical frequency w_c there is no reversal of current as shown in Fig.(4, 2). In the high frequency regime the effect of space dependent friction dominates the nature of current. We would like to emphasize here that in absence of asymmetry current reversals does not take place.

Multiple current reversals can also be seen when the amplitude (A) of the forcing term is varied in a suitable parameter regime of our system. In Fig. 5, the plot of j versus A is shown for different values of w, keeping λ, ϕ and T fixed at 0.1, 0.88π and 0.05 respectively. For $w = 4.0$ curve, we can see as many as four current reversals. For very large value of A, the current asymptotically goes to a constant value depending on the value of ϕ, as was previously shown for the adiabatic case [16]. This value was shown to be $-\frac{\lambda}{2}\sin(\phi)$. This is special to the space dependent friction ratchet where the currents saturate to a finite value in the large A limit. In the absence of space dependent friction it is to be noted that currents decay to zero in the same asymptotic regime. As the asymptotic value depends on the phase ϕ, so we can choose it appropriately to make the asymptotic current positive or negative. In the present case ϕ is chosen such that the asymptotic current is negative which guarantees at least one current reversal irrespective of frequency. The oscillatory behavior in the $j - A$ characteristics is the reminiscent of the deterministic dynamics [4,29] which will be discussed later. The inset in Fig. 5 shows the zero contour of j versus ϕ and A for $w = 4.0$. For $\phi > \pi$ only two current reversals can be seen. For $\pi > \phi > \pi - \epsilon$ (where ϵ is a small number) or $0 < \phi < \epsilon$, four or more current reversals can occur. The value of ϵ depends critically on T, large for small T and vice-versa. It is also to be noted that we have even number of reversals for finite frequency driving. These oscillatory features along with their associated current reversals disappear in the high temperature regime as expected. Figure 6 is for $w = 0.25$ and shows that current reversal is also possible in the deterministic regime of the overdamped system. This deterministic reversal of current cannot be attributed to the chaotic motion of the system like in an forced underdamped oscillator [20]. It solely arises due the presence of space dependent friction. As shown in Ref. [4] here too the deterministic current shows quantization and phase locking behaviour. However all these complex features are not robust in the presence of noise as discussed in earlier literature.

In conclusion, we have studied the transport properties of overdamped Brownian particles moving in an asymmetric potential with space dependent friction coefficient and rocked by periodic force. We observe several novel and complex features arising due to the interplay between asymmetry and inhomogeneous friction. Currents in the low temperature regime is mostly influenced by the asymmetry of the potential. At higher temperatures it is controlled by the modulation parameter λ of the friction coefficient. We find current reversal with temperature even when the forcing is adiabatic. In the presence of finite frequency, twice current reversals are seen. As function of amplitude of the forcing term we observe multiple current reversals. Current even reverses its sign in the adiabatic deterministic regime. All the above results can be understood in a qualitative manner. We expect that our analysis should be applicable for the motion of particle in porous media and for molecular motors where space dependent friction can arise due to the confinement of particles.

MCM acknowledges partial financial support and hospitality from the Institute of Physics, Bhubaneswar. MCM and AMJ acknowledgepartial financial support from the Board of Research in Nuclear Sciences, DAE, India.

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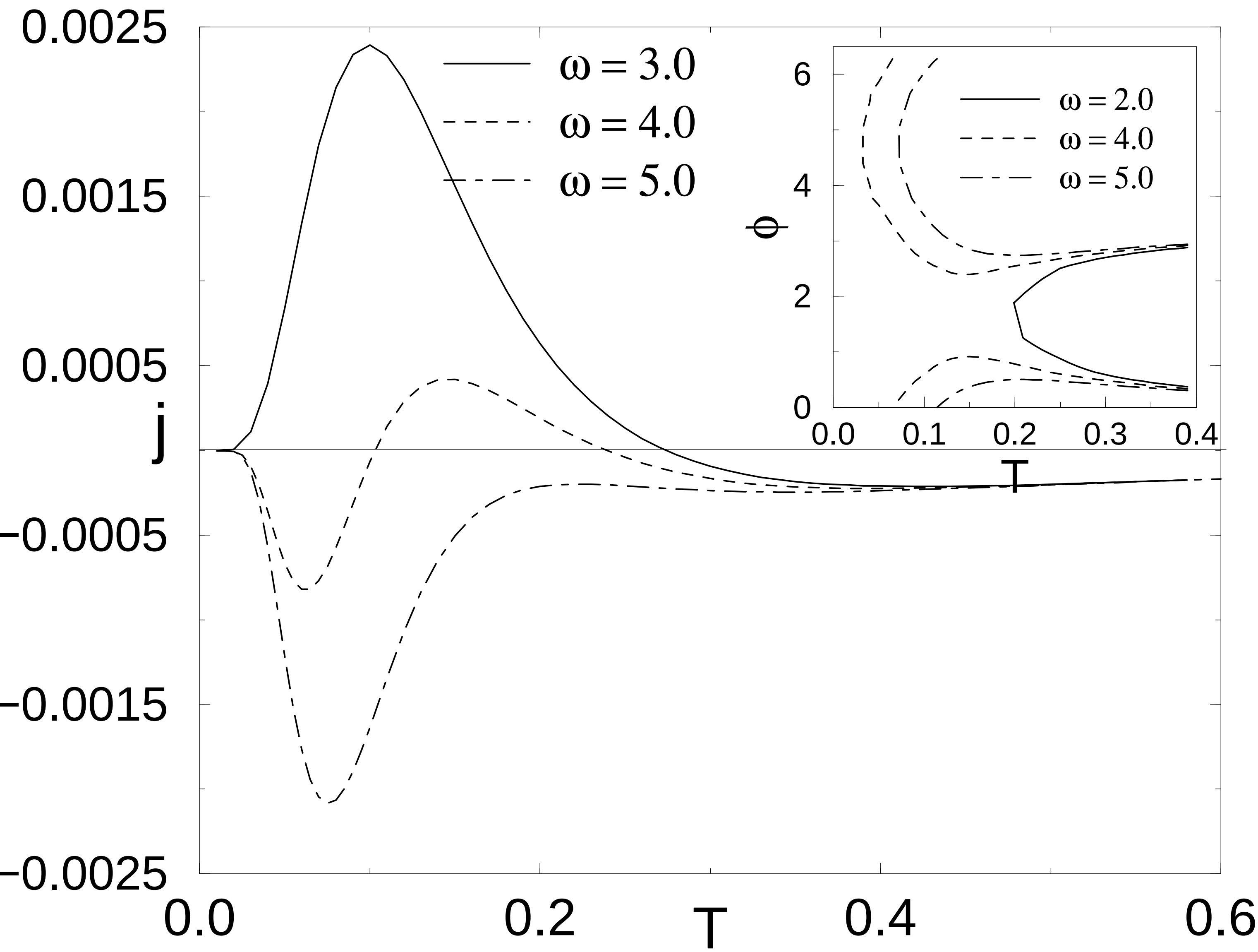

FIG. 1. The mean current j *vs* temperature T for $\phi = 0.2\pi$, $A = 0.5$ and $\lambda = 0.1$. The driving frequencies are $w = 3.0, 4.0$ and 5.0. The inset shows the contour of zero current for same values of T, λ. Regions enclosed on the right hand side of a given contour is the negative current region and vice versa. Note that we have only one reversal for all values of $\phi > \pi$.

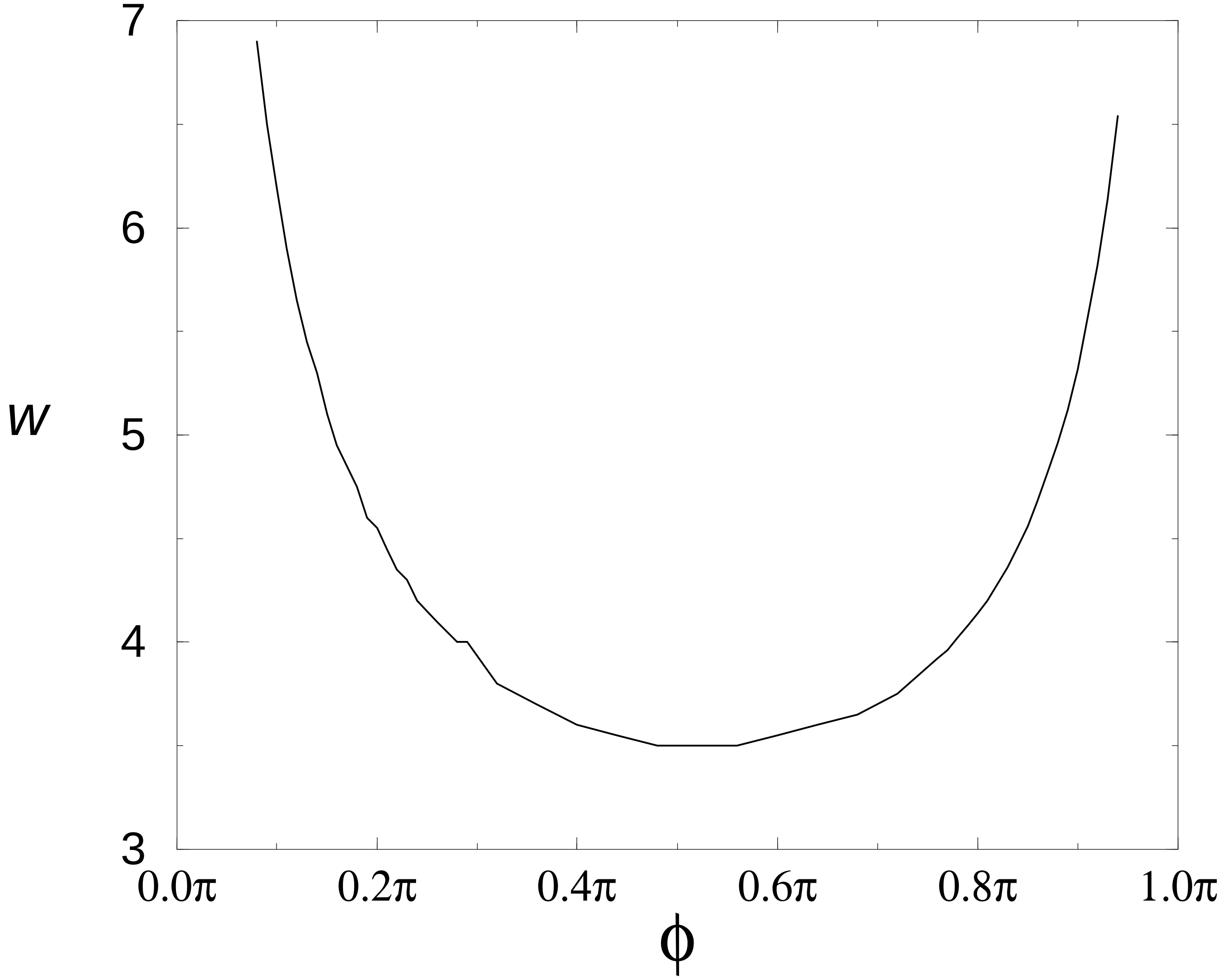

FIG. 2. The critical w above which no current reversal with T occurs *vs* ϕ for $A = 0.5$ and $\lambda = 0.1$.

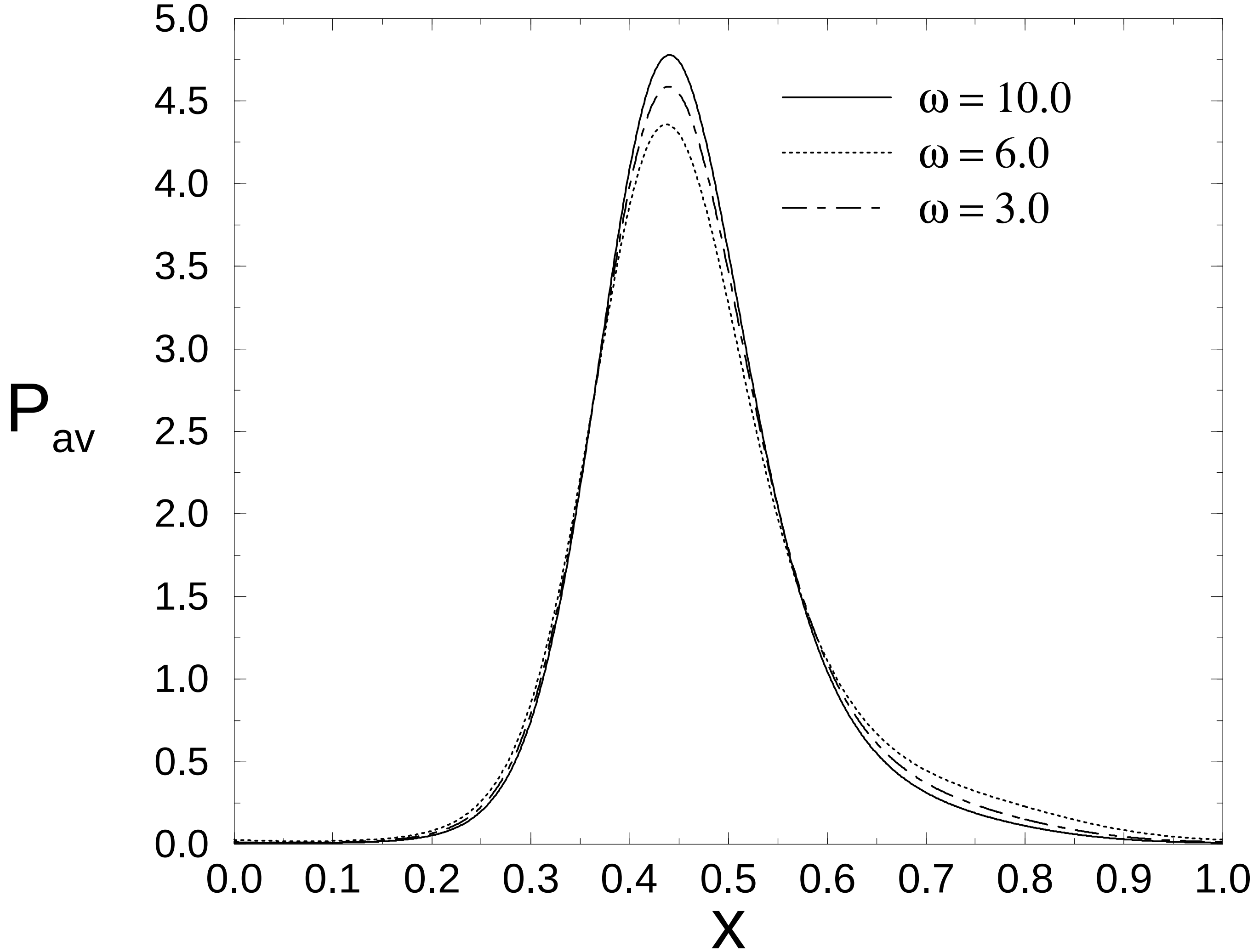

FIG. 3. The time averaged probability P_{av} *vs* x for various values of w and $T = 0.05$ and $A = 0.5$. The distribution is independent of ϕ and λ.

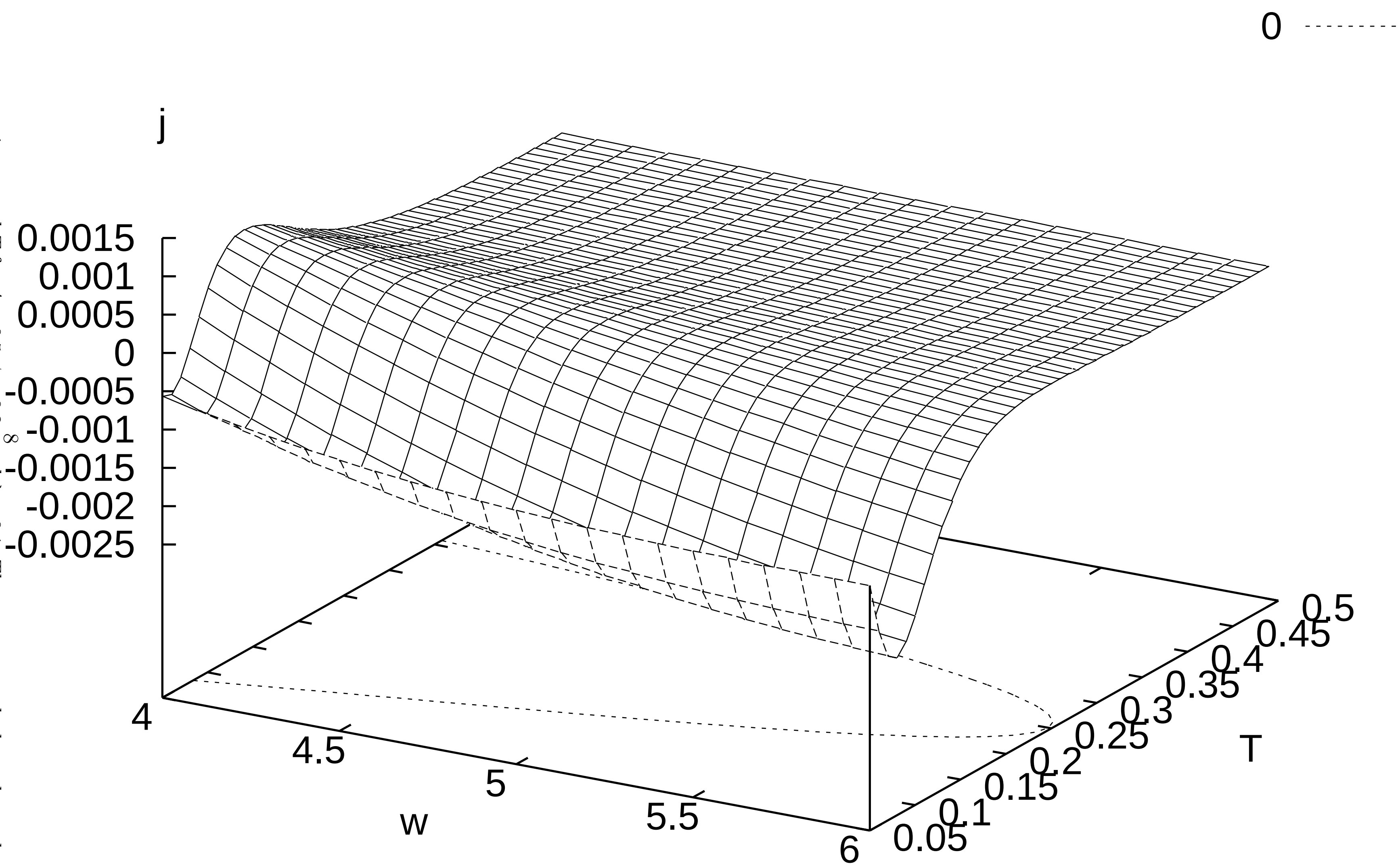

FIG. 4. The mean current j vs w and T for $A = 0.5, \phi = 0.2\pi$ and $\lambda = 0.1$. The contour on the base shows the zero current.

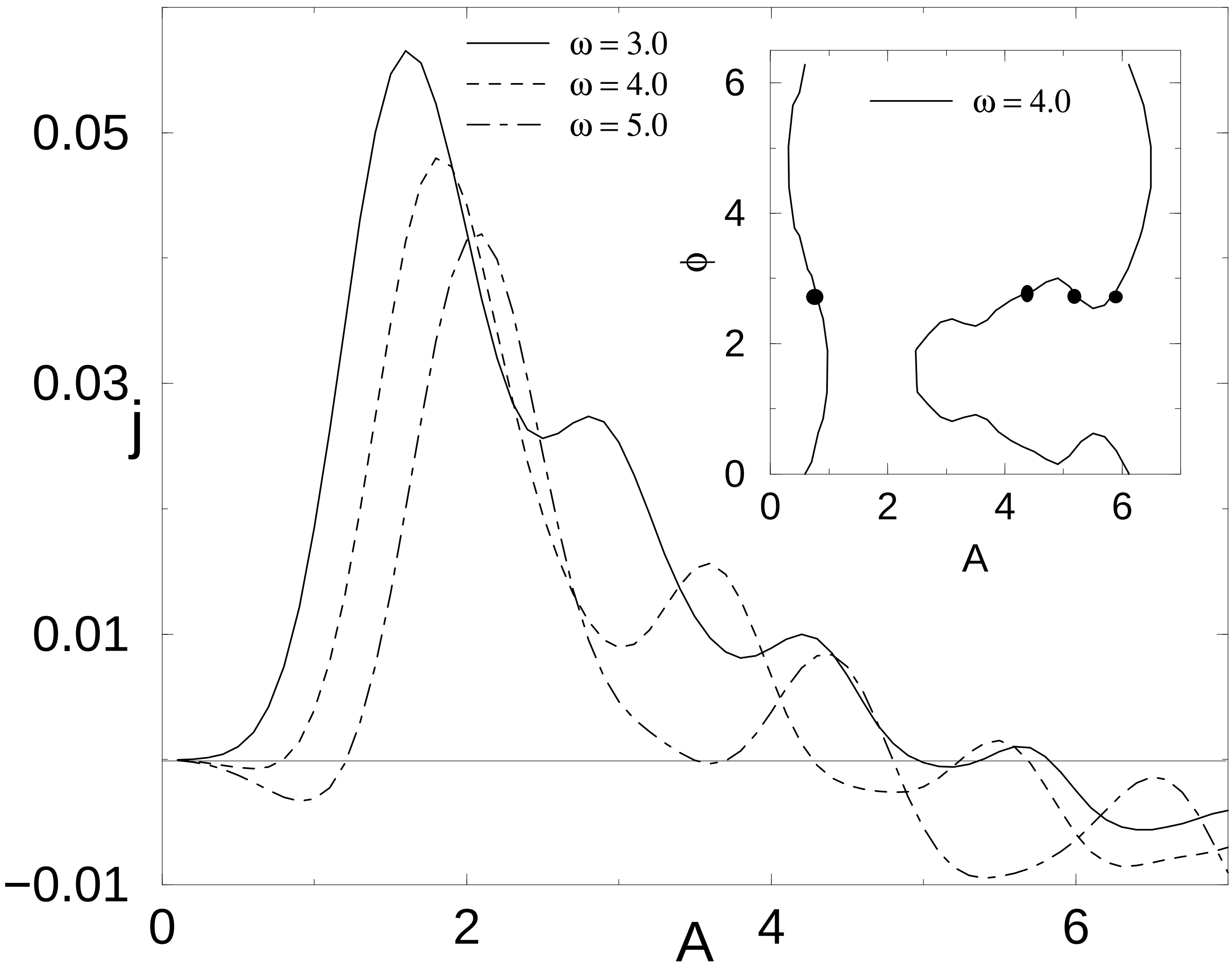

FIG. 5. The mean current j with amplitude A of the forcing term for $\phi = 0.88\pi, T = 0.05$ and $\lambda = 0.1$ with $w = 3.0, 4.0, 5.0$. The inset shows the contour of zero current for $T = 0.05$ for $w = 4.0$. The dots in the inset shows the four values of A for $\phi = 0.9\pi$ where the current reversal occurs.

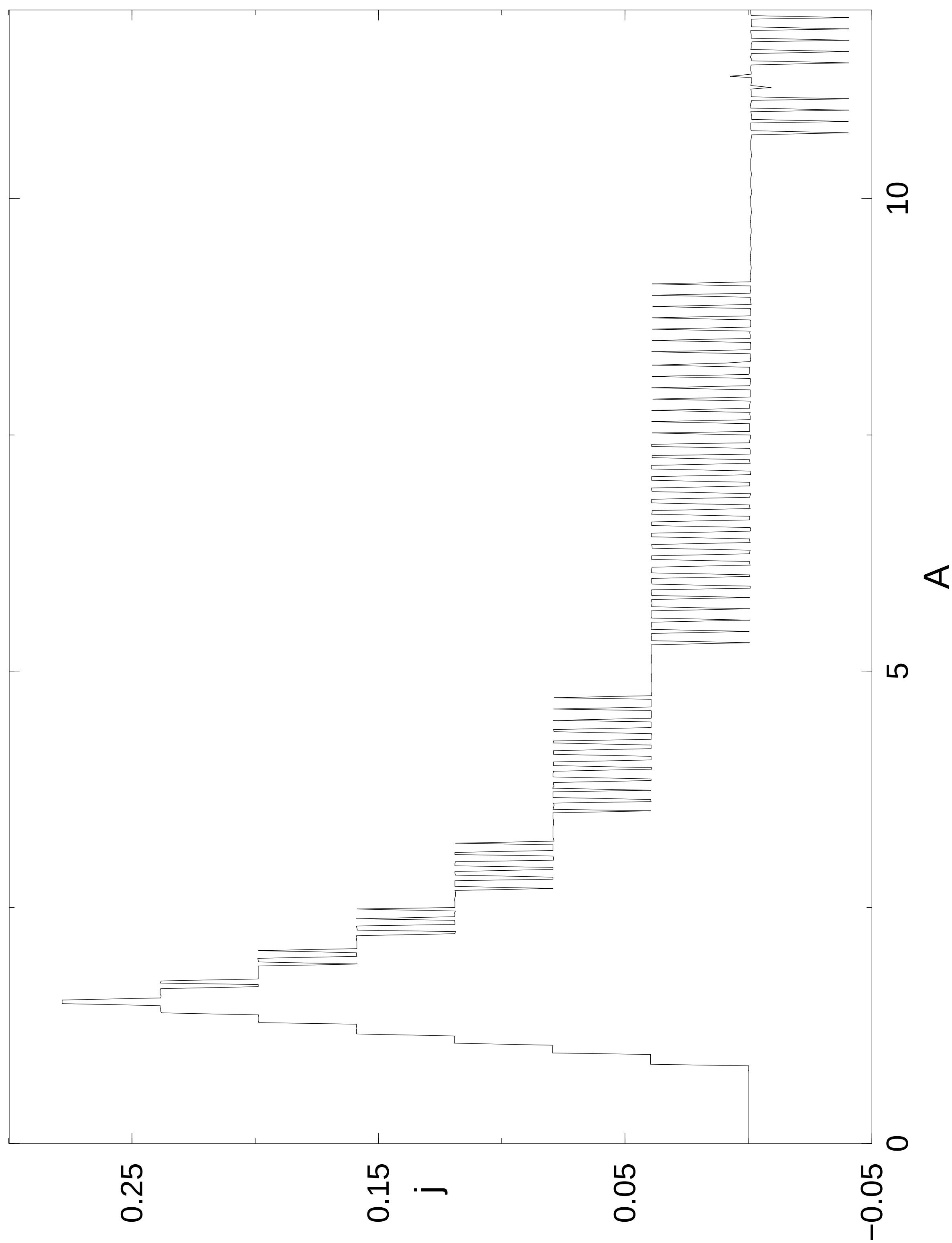

FIG. 6. The deterministic current j *vs* amplitude of forcing A for $w = 0.25$ and $\phi = 0.2\pi$ and $\lambda = 0.1$.