

Use of transverse beam polarization to probe anomalous VVH interactions at a Linear Collider

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Abstract

We investigate use of transverse beam polarization in probing anomalous coupling of a Higgs boson to a pair of vector bosons, at the International Linear Collider (ILC). We consider the most general form of VVH ($V = W/Z$) vertex consistent with Lorentz invariance and investigate its effects on the process $e^+e^- \rightarrow f\bar{f}H$, f being a light fermion. Constructing observables with definite CP and naive time reversal (\tilde{T}) transformation properties, we find that transverse beam polarization helps us to improve on the sensitivity of one part of the anomalous ZZH coupling that is odd under CP . Even more importantly it provides the possibility of discriminating from each other, two terms in the general ZZH vertex, both of which are even under CP and \tilde{T} . Use of transverse beam polarization when combined with information from unpolarized and linearly polarized beams therefore, allows one to have completely independent probes of all the different parts of a general ZZH vertex.

1 Introduction

The Standard Model (SM) has been incredibly successful in describing all the available experimental data. However, the Higgs mechanism responsible for generating masses of all the particles in the SM still lacks direct testing. The SM predicts existence of one Higgs boson, a spin-0 particle, even under charge conjugation and parity (C, P) transformation, whereas scenarios beyond the SM usually imply more than one Higgs boson with different CP and weak isospin quantum numbers [1, 2]. For example, the Minimal Supersymmetric extension of the Standard Model (MSSM) [3] consists of five Higgs particles: two CP -even neutrals, a CP -odd neutral and a pair of charged scalars [1–3]. Therefore, search for the Higgs boson and study of its various properties is one of the major goals of all the current and future colliders [4].

Direct searches at the LEP collider have put a lower bound on the mass of the SM Higgs boson: $m_H > 114.4$ GeV [5], whereas the LEP electroweak precision measurements have been used to put an upper bound on its mass, of about 185 GeV [6] at 95% confidence level (CL). Theoretical constraints on the mass of the SM Higgs boson exist, given by demanding unitarity of scattering amplitudes, perturbativity of Higgs self-coupling, stability of the electroweak vacuum and no fine-tuning in the radiative corrections in the Higgs sector [2, 4]. The Large Hadron Collider (LHC) is expected [7] to be capable of searching for the SM Higgs boson over the mass range allowed by all the above considerations, viz. 114.4 GeV $\lesssim m_H \lesssim 800$ GeV. After the discovery of the Higgs boson, precise determination of its interactions with other particles would

be necessary to establish it as *the* SM Higgs boson. A detailed study of Higgs sector may also provide hints for new physics beyond the SM. Hence high precision measurements at the International e^+e^- Linear Collider (ILC) [8,9] and its combination with the information available from the LHC, will be required to establish the nature of Higgs boson [10] in general and its CP property in particular [11]. Part of this exercise could be achieved by determining the tensor structure of the coupling of the spin-0 state with different SM particles. The determination of the CP nature of the Higgs boson at various colliders has been a subject of many investigations [12]. Various kinematical distributions for the process $e^+e^- \rightarrow f\bar{f}H$, proceeding via vector boson fusion and Higgs-strahlung, can be used to probe the ZZH vertex [13,14]. At an e^+e^- collider, the angular and energy distribution of the Z boson in the process $e^+e^- \rightarrow ZH$ can provide information about the ZZH coupling [15,16]. Further the tensor structure of the ZZH and $t\bar{t}H$ coupling may be probed in a model independent way by looking at the shapes of the threshold excitation curve in the processes $e^+e^- \rightarrow ZH$ [17] and $e^+e^- \rightarrow t\bar{t}H$ [18] respectively. Higher order contributions in a renormalizable theory [19] or higher dimensional operators in an effective theory [20,21] may give rise to anomalous parts of VVH vertex that we discuss here. In the context of higher dimensional operators the anomalous VVH vertex has been studied in Refs. [14,22–33]. Particularly in Ref. [26] the authors have made a detailed analysis using the optimal observable technique [34], to probe ZZH and γZH couplings, whereas Refs. [27–30] use asymmetries constructed using differences in the kinematical distributions of the decay products. Similar methods can be used to probe the ZZH vertex at the LHC as well [12,35]. The anomalous VVH couplings can also be studied at an $e\gamma$ collider [36]. Previous studies [26,28–30] showed that the use of linear beam polarization and as well as measurement of the polarisation of the final state fermion for the case of the τ , improves the sensitivity of these asymmetries to some of these anomalous couplings substantially and provides independent probes of many of the parameters characterizing them. However, a few ambiguities still remain.

It has been realized that at future e^+e^- linear colliders spin rotators can be used to obtain transverse beam polarization. In the context of the ILC, use of transverse polarization in probing new physics, including anomalous VVH vertex, has been discussed in Refs. [31,37–42]. In this note we study how information obtained using transverse polarization can improve the situation and make possible completely independent determination of the different parts of the ZZH anomalous coupling. The asymmetries with unpolarized beams suffer from a suppression factor $(\ell_f^2 - r_f^2)$, where l_f (r_f) is the left-(right-)handed coupling of the fermion to the Z boson [29]. With polarized beams using the spin direction of the beam, construction of asymmetries which may not suffer from this suppression factor, becomes possible. In this note we explore use of transverse beam polarization to probe VVH vertex.

In the next section we discuss possible sources of anomalous VVH couplings and construct observables, specific to the use of transverse polarization. In sec. 3 we present numerical analysis of the same. We summarize our results in Sec. 4.

2 General structure and Probes of VVH couplings

At an e^+e^- collider the dominant Higgs production process, *viz* $e^+e^- \rightarrow f\bar{f}H$ where f is any light fermion, proceeds via the VVH interactions with $V = W, Z$. Processes with electrons and ν_e 's in the final states receive contribution from both the s- and t-channel diagrams (Figs. 1).

The interaction term involving the Higgs boson and a pair of gauge bosons arises from the Higgs kinetic term in the SM/MSSM Lagrangian. However, once we accept the SM to be only an effective low-energy theory, higher dimensional (and hence non-renormalizable) terms are allowed. The most general VVH vertex, consistent with Lorentz invariance, can be written as

$$\Gamma_{\mu\nu} = g_V \left[a_V g_{\mu\nu} + \frac{b_V}{m_V^2} (k_{1\nu} k_{2\mu} - g_{\mu\nu} k_1 \cdot k_2) + \frac{\tilde{b}_V}{m_V^2} \epsilon_{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta \right], \quad (1)$$

where k_i denote the momenta of the two W 's (Z 's). Here

$$g_W^{SM} = e \cot \theta_W M_Z, \quad g_Z^{SM} = 2 e M_Z / \sin 2\theta_W,$$

θ_W being the weak-mixing angle and $\epsilon_{\mu\nu\alpha\beta}$ the antisymmetric tensor with $\epsilon_{0123} = 1$. In the SM, at tree level $a_Z = a_W = 1$ and $b_V = \tilde{b}_V = 0$.

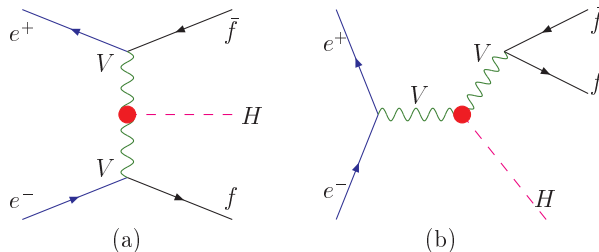


Figure 1: Feynman diagrams for the process $e^+e^- \rightarrow f\bar{f}H$; (a) is t-channel or fusion diagram, while (b) is s-channel or Bjorken diagram. For $f = e, \nu_e$ both (a) and (b) contribute whereas for all the other fermions only (b) contributes.

In general, each of the couplings of Eq. 1 (a_V , b_V and \tilde{b}_V) can be complex. However, for all the observables that we construct for the process $e^+e^- \rightarrow f\bar{f}H$, one overall phase can always be rotated away. We choose a_Z to be real and allow the others to be complex. Further, we assume a_Z and a_W to be close to their SM value i.e. $a_V = 1 + \Delta a_V$. The most general coupling of a CP -even Higgs boson with a pair of vector bosons is expressed by the terms containing a_V and b_V in Eq. 1 whereas the CP -odd one corresponds to \tilde{b}_V . Simultaneous presence of both sets of terms would indicate CP -violation. Note that a non-vanishing value for either $\Im(b_V)$ or $\Im(\tilde{b}_V)$ destroys the hermiticity of the effective theory.

In an effective theory which satisfies $SU(2)_L \otimes U(1)_Y$ symmetry, couplings b_V and \tilde{b}_V can be realized as lowest order corrections arising from dimension-six operators such as $F_{\mu\nu} F^{\mu\nu} \Phi^\dagger \Phi$ or $F_{\mu\nu} \tilde{F}^{\mu\nu} \Phi^\dagger \Phi$ where Φ is the usual Higgs doublet, $F_{\mu\nu}$ the field strength tensor and $\tilde{F}_{\mu\nu}$ its dual [21]. The coupling constants may even have non-trivial momentum dependence (form-factor behavior). However, for a theory with a cut-off scale Λ large compared to the energy scale at which the scattering experiment is to be performed, momentum dependence would be very weak and can be neglected for our study. Hence we treat Δa_V , b_V and \tilde{b}_V as phenomenological, energy-independent parameters and keep terms up to linear order in our analysis, assuming them to be small compared to the SM coupling. Table 1 shows the CP and \tilde{T} transformation

	a_V	$\Re(b_V)$	$\Im(b_V)$	$\Re(\tilde{b}_V)$	$\Im(\tilde{b}_V)$
CP	+	+	+	-	-
\tilde{T}	+	+	-	-	+

Table 1: *Transformation properties of the various operators (identified by their coefficients) in the effective Lagrangian.*

properties of various operators in the effective Lagrangian corresponding to different anomalous couplings given in the table.

In principle, the full process $e^+e^- \rightarrow Hf\bar{f}$ can receive contributions from some additional higher dimensional operators [20], which could arise (say) from a Z' exchange in an effective theory [32,33]. Under the assumption of flavour universality these additional contributions to the $e^+e^- \rightarrow f\bar{f}H$ amplitude, will have the same structure as that coming from some of the terms in our anomalous VVH vertex. Thus our present analysis includes such contributions as well, subject to the above flavour universality assumption.

As was the case with our earlier analyses [29,30] we construct various kinematical quantities \mathcal{C}_i as combinations of the available different particle momenta and their spins. Then we define observables O_i^T , as expectation values of the signs of \mathcal{C}_i , i.e. $O_i^T = \langle \text{sign}(\mathcal{C}_i) \rangle$, each of which have well-defined C , P and \tilde{T} transformation properties. More explicitly O_i^T 's can be expressed as:

$$\begin{aligned}
O_i^T &= \frac{1}{\sigma_{\text{SM}}} \int [\text{sign}(\mathcal{C}_i)] \frac{d\sigma}{d^3p_H d^3p_f} d^3p_H d^3p_f \\
&= \frac{\sigma(\mathcal{C}_i > 0) - \sigma(\mathcal{C}_i < 0)}{\sigma_{\text{SM}}}.
\end{aligned} \tag{2}$$

Within the aforementioned linear approximation, O_i^T 's may be used to probe the contribution of specific operator(s) in the effective Lagrangian with the same CP and \tilde{T} transformation properties. Of course, by construction this seems to make it impossible to have completely independent probes for those parts of the anomalous VVH vertex which have the same CP and \tilde{T} properties : viz. $\Re(b_V)$ and a_V . Such was the case with the observables we constructed earlier with unpolarized and longitudinally polarized beams [29,30]. However, we find that use of transverse beam polarization allows us to distinguish the contributions from $\Re(b_Z)$ and a_Z from each other. As a result, using information from unpolarized and transversely/linearly polarized beams, one can determine, completely independently, all the parts of the general ZZH coupling. Similar observation has also been made recently in the context of general ZZH and $Z\gamma H$ three point interactions in Ref. [31].

Table 2 lists the observables O_i^T ($i = 1-3$), specific to the case of transverse polarization, along with their transformation properties and the anomalous coupling they may constrain. A brief description of the same follows:

1. O_1^T is an observable defined as the difference in partially integrated cross sections normalized by the SM cross section and is given by

$$\begin{aligned}
O_1^T &= \frac{\sigma(\mathcal{C}_1 > 0) - \sigma(\mathcal{C}_1 < 0)}{\sigma_{\text{SM}}} \\
&= \frac{\sigma(\cos 2\phi_H > 0) - \sigma(\cos 2\phi_H < 0)}{\sigma_{\text{SM}}},
\end{aligned} \tag{3}$$

ID	\mathcal{C}_i	C	P	CP	\tilde{T}	CPT	Observable (O_i^T)	Coupling
1	$(\vec{p}_H)_x^2 - (\vec{p}_H)_y^2$	+	+	+	+	+	O_1^T	$a_V, \Re(b_V)$
2	$(\vec{P}_f)_x * (\vec{P}_f)_y * (\vec{p}_H)_z$	+	-	-	-	+	O_2^T	$\Re(\tilde{b}_Z)$
3	$(\vec{p}_H)_x * (\vec{p}_H)_y * (\vec{P}_f)_z$	-	-	+	-	-	O_3^T	$\Im(b_Z)$

Table 2: Various possible \mathcal{C}_i 's, their transformation properties, the associated observables O_i^T and the anomalous couplings on which they provide information. Here, $\vec{P}_f \equiv \vec{p}_f - \vec{p}_{\bar{f}}$ and \vec{p}_H is the momentum of Higgs boson (to be deduced from the measurement of its decay products).

where ϕ_H is the azimuthal angle of the Higgs boson with respect to the XZ-plane defined by taking Z-axis to be direction of initial state e^- momentum and choosing positive X-axis to be the direction of its transverse polarization. This observable is derived from

$$\mathcal{C}_1 \equiv [(\vec{p}_H)_x^2 - (\vec{p}_H)_y^2] (\propto \cos 2\phi_H). \quad (4)$$

One can see that this is even under CP, \tilde{T} and hence can be sensitive to a_V and $\Re(b_V)$.

2. O_2^T can be constructed using \mathcal{C}_2 , here

$$\begin{aligned} \mathcal{C}_2 &= (\vec{P}_f)_x * (\vec{P}_f)_y * (\vec{p}_H)_z \\ &\propto [\vec{S}_e \cdot \vec{P}_f] * [(\vec{S}_e \times \vec{P}_e) \cdot \vec{P}_f] * [\vec{P}_e \cdot \vec{p}_H], \end{aligned} \quad (5)$$

and $\vec{P}_e \equiv \vec{p}_{e^-} - \vec{p}_{e^+}$, $\vec{P}_f \equiv \vec{p}_f - \vec{p}_{\bar{f}}$, $\vec{S}_e \equiv \vec{s}_{e^-} - \vec{s}_{e^+}$.

Hence O_2^T is CP -odd, \tilde{T} -odd and thus would be proportional to $\Re(\tilde{b}_Z)$. It can be expressed as

$$O_2^T = \frac{\sigma(\mathcal{C}_2 > 0) - \sigma(\mathcal{C}_2 < 0)}{\sigma_{\text{SM}}}. \quad (6)$$

3. O_3^T is derived from \mathcal{C}_3 , where

$$\begin{aligned} \mathcal{C}_3 &= (\vec{p}_H)_x * (\vec{p}_H)_y * (\vec{P}_f)_z \\ &\propto [\vec{S}_e \cdot \vec{p}_H] * [(\vec{S}_e \times \vec{P}_e) \cdot \vec{p}_H] * [\vec{P}_e \cdot \vec{P}_f] \end{aligned} \quad (7)$$

is CP -even and \tilde{T} -odd. O_3^T is defined as

$$O_3^T = \frac{\sigma(\mathcal{C}_3 > 0) - \sigma(\mathcal{C}_3 < 0)}{\sigma_{\text{SM}}} \quad (8)$$

and may be used to probe $\Im(b_Z)$.

It is obvious from the structure of $\mathcal{C}_1 - \mathcal{C}_3$ that O_1^T, O_2^T and O_3^T will be nonzero only for transversely polarized beams. Out of these three observables the last two require charge measurement of the final state particles and hence cannot be used for final states with light quarks and neutrinos.

3 Numerical results

In the numerical analysis, we focus on the intermediate mass Higgs boson ($2m_b \leq m_H \leq 140\text{GeV}$), for which $H \rightarrow b\bar{b}$ is the dominant decay mode with a branching fraction $\simeq 0.68$. We consider the case of the ILC operating at a center of mass energy of 500 GeV, with a Higgs boson mass 120 GeV. Further, we take the b -tagging efficiency to be 0.7. We impose various kinematical cuts to suppress dominant backgrounds to the process under study, viz. $e^+e^- \rightarrow f\bar{f}H \rightarrow f\bar{f}b\bar{b}$. To begin with, we demand that each of the final state particles has energy above a minimum energy, as well as a minimum angular deviation from the beam pipe. Furthermore, they should be well separated from each other. For final state with neutrinos, we need to impose a cut on missing transverse momentum. The specific kinematic cuts we impose are:

$$\begin{aligned}
E_f &\geq 10\text{ GeV} && \text{for each visible outgoing fermion} \\
5^\circ &\leq \theta_0 \leq 175^\circ && \text{for each visible outgoing fermion} \\
p_T^{\text{miss}} &\geq 15\text{ GeV} && \text{for events with } \nu\text{'s} \\
\Delta R_{jj} &\geq 0.7 && \text{for each pair of jets} \\
\Delta R_{\ell\ell} &\geq 0.2 && \text{for each pair of charged leptons} \\
\Delta R_{lj} &\geq 0.4 && \text{for jet-lepton isolation ,}
\end{aligned} \tag{9}$$

where $(\Delta R)^2 \equiv (\Delta\phi)^2 + (\Delta\eta)^2$, $\Delta\phi$ and $\Delta\eta$ being the separation between the two entities in azimuthal angle and rapidity respectively.

Additional cuts may be imposed on the invariant mass of the $f\bar{f}$ system to enhance(suppress) the contributions coming from s-channel Z -exchange process, namely

$$\begin{aligned}
R1 &\equiv |m_{f\bar{f}} - M_Z| \leq 5\Gamma_Z \implies \text{select } Z\text{-pole ,} \\
R2 &\equiv |m_{f\bar{f}} - M_Z| \geq 5\Gamma_Z \implies \text{de-select } Z\text{-pole ,}
\end{aligned} \tag{10}$$

where Γ_Z is the width of the Z boson. Alternatively, for $\nu\bar{\nu}H$ final state, the same goal can be achieved by demanding

$$\begin{aligned}
R1' &\equiv E_H^- \leq E_H \leq E_H^+ , \\
R2' &\equiv E_H < E_H^- \text{ or } E_H > E_H^+ .
\end{aligned} \tag{11}$$

Here $E_H^\pm = (s + m_H^2 - (m_Z \mp 5\Gamma_Z)^2)/(2\sqrt{s})$.

We construct various $O_i^T, i = 1, 3$ as the expectation values of the sign of $C_i, i = 1, 3$ as mentioned earlier. It was pointed out in Refs. [43, 44] that in the massless limit of electron, transverse beam polarization does not affect the azimuthally integrated SM cross-section, due to the electronic chiral symmetry of the SM. The anomalous VVH vertex that we are considering does not change the chiral structure of the theory. Hence transverse beam polarization also cannot alter contributions to the total rate, coming from the anomalous parts of the VVH vertex; viz. $\Re(b_Z), \Delta a_Z$. Further to our choice of XZ plane, we assume direction of e^+ polarization to be opposite to that of e^- and take e^-, e^+ polarization ($\mathcal{P}_{e^-}^T, \mathcal{P}_{e^+}^T$) to be 80% and 60% respectively. We denote θ_H, ϕ_H as the polar and azimuthal angle of the Higgs boson and ϕ_f as the angle between production planes of the Higgs boson and final state fermion.

The statistical fluctuation in the cross section and that in an asymmetry, for a given luminosity \mathcal{L} and fractional systematic error ϵ , can be written as:

$$\Delta\sigma = \sqrt{\sigma_{SM}/\mathcal{L} + \epsilon^2\sigma_{SM}^2} \quad , \quad (12)$$

$$(\Delta A)^2 = \frac{1 - A_{SM}^2}{\sigma_{SM}\mathcal{L}} + \frac{\epsilon^2}{2}(1 - A_{SM}^2)^2, \quad (13)$$

where σ_{SM} and A_{SM} are the SM value of cross section and asymmetry respectively. Since all the anomalous couplings are kept only up to linear order, any of the observables; total rate or asymmetries, can be written as:

$$\mathcal{O}(\{\mathcal{B}_i\}) = \sum O_i \mathcal{B}_i,$$

where \mathcal{B}_i is the anomalous coupling. We calculate the possible limits of sensitivity on the anomalous couplings by demanding that $|\mathcal{O}(\{\mathcal{B}_i\}) - \mathcal{O}(\{0\})| \leq f \Delta\mathcal{O}$. Here f is the degree of statistical significance, $\mathcal{O}(\{0\})$ is the SM value of \mathcal{O} and $\Delta\mathcal{O}$ is the statistical fluctuation in \mathcal{O} . In our analysis we consider $\epsilon = 0.01$, $\mathcal{L} = 500 \text{ fb}^{-1}$ and $f = 3$. Next we present our numerical results for the case of ZZH and WWH couplings respectively.

3.1 Anomalous ZZH Couplings

We observe that transverse beam polarization will lead to additional contributions to the differential cross section only when both the beams are polarized. The additional contribution from transverse beam polarization in the squared matrix element (MESQ) is proportional to the interference between different helicity amplitudes and hence these additional terms are proportional to $\mathcal{P}_{e^-}^T \mathcal{P}_{e^+}^T$, where $\mathcal{P}_{e^\mp}^T$ is polarization of e^\mp . This proportionality factor ($\mathcal{P}_{e^-}^T \mathcal{P}_{e^+}^T$) can be understood as a consequence of electronic chiral symmetry mentioned earlier [44].

Use of transverse beam polarization to probe anomalous ZZH and $Z\gamma H$ couplings had been addressed in Refs. [14, 31]. In Ref. [14] it was pointed out that polarization does not provide qualitatively new information about the anomalous ZZH couplings if they are assumed to be real. This is no longer true once the couplings are allowed to be complex. A recent discussion [31] shows how the anomalous ZZH and $Z\gamma H$ vertex may be probed with transversely polarized beams using Z -angular distribution. However, not all the parts of the anomalous couplings are accessible in this process. For example, $\Im(b_Z)$ and $\Re(\tilde{b}_Z)$ cannot be probed using the Z -angular distribution. Our analysis goes beyond that in both of the Refs. [14, 31], in that we consider the full process $e^+e^- \rightarrow f\bar{f}H$ and with the most general form of anomalous ZZH vertex, allowing the couplings to be complex.

The structure of \mathcal{C}_1 , \mathcal{C}_2 and \mathcal{C}_3 given by Eqs. 4–7 indicates that the corresponding observables, O_1^T , O_2^T and O_3^T will be nonzero only for transversely polarized beams. This can be understood by noticing that the transverse polarization of e^- (along positive X-axis) and beam direction (along Z-axis) define a preferred, fixed XZ-plane, whereas for unpolarized or longitudinally polarized beams such a plane does not exist. Thus these three observables are unique for transversely polarized beams. Final states containing light quarks and muons receive contribution only from the s -channel Higgs-strahlung process, whereas those with electrons receive contribution from the t -channel diagram as well. O_2^T and O_3^T require charge measurement of final state particles. Hence we do not use final states with quarks while assessing their efficacy to probe the anomalous couplings.

For transverse beam polarization the contributions coming from different anomalous parts to MESQ have different azimuthal angular dependencies. Since $\mathcal{C}_1 \propto \cos 2\phi_H$ and is even under both CP - and \tilde{T} -transformation; the corresponding observable O_1^T can then in principle involve terms proportional to both Δa_Z and $\Re(b_Z)$. Fig. 2 displays differential cross section

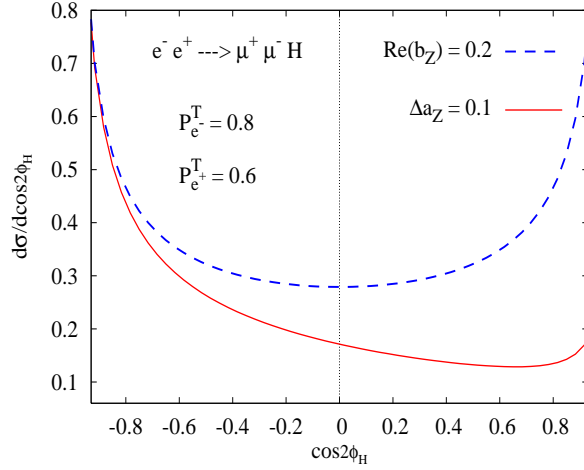


Figure 2: Azimuthal angle distribution of the Higgs boson for 80%, 60% transversely polarized e^-, e^+ beams .

($d\sigma/d\cos 2\phi_H$) as a function of $\cos 2\phi_H$ for transversely polarized e^-/e^+ beams and nonzero values of these two CP - and \tilde{T} -even anomalous ZZH couplings. We note that the observable O_1^T does not receive any nonzero contribution from $\Re(b_Z)$ and thus is an independent probe of Δa_Z alone. We will get back to this issue after discussing the possible sensitivity of O_1^T as a probe of Δa_Z .

Imposing various kinematical cuts of Eqs. 9, along-with the $R1$ -cut of Eq. 10 to select Z -pole contribution, O_1^T for muon and quark final states, with transverse polarization ($\mathcal{P}_{e^-}^T, \mathcal{P}_{e^+}^T$) of the initial beams, can be written, keeping terms up to linear order in anomalous couplings, as

$$O_1^T(R1 - \text{cut}) = \begin{cases} \frac{[\mathcal{P}_{e^-}^T \mathcal{P}_{e^+}^T] [-0.37 (1 + 2 \Delta a_Z)]}{0.86} & (\mu^+ \mu^- H) \\ \frac{[\mathcal{P}_{e^-}^T \mathcal{P}_{e^+}^T] [-5.7 (1 + 2 \Delta a_Z)]}{13.2} & (q\bar{q}H) \end{cases} \quad (14)$$

Here, each of the numerical factors are in femtobarns with the denominator being the SM cross section. It is clear from the above expression that O_1^T is nonzero only when both the beams are transversely polarized, as it should be.

Taking e^- and e^+ beam polarization to be 80% and 60% respectively, using the final state with μ and quarks, lack of deviation from the SM value of O_1^T , at 3σ level, would limit Δa_Z to:

$$|\Delta a_Z| \lesssim 0.1 \quad \text{for } \mathcal{L} = 500 \text{ fb}^{-1}. \quad (15)$$

The best possible sensitivity limit on Δa_Z obtained for the unpolarized case in our earlier analysis [29] was: $|\Delta a_Z| \leq 0.04$. However, the observable (total cross section with unpolarised beams) used to obtain this bound on Δa_Z also gets contribution from $\Re(b_Z)$ [29].

We emphasize here that though O_1^T is even under CP - and \tilde{T} -transformation, it receives contribution only from Δa_Z (and not from $\Re(b_Z)$), as shown by azimuthal angular distribution displayed in Fig 2. Construction of such an observable was not possible with unpolarized beams [29]. In fact a similar observation has been made recently in Ref. [31] in the context of the process $e^+e^- \rightarrow ZH$. The additional contribution to the MESQ for this process due to the transverse beam polarization proportional to $\Re(b_Z)$, vanishes identically as these terms are proportional to $\vec{s}_{e^-} \cdot \vec{p}_{e^-}$, $\vec{s}_{e^+} \cdot \vec{p}_{e^+}$. For the full process $e^+e^- \rightarrow f\bar{f}H$, even though the additional terms in the MESQ due to transverse polarisation are nonzero, the contribution to O_1^T proportional to $\Re(b_Z)$ constructed after integrating over the remaining phase space variables vanishes.

Furthermore, using \mathcal{C}_1 we can construct an azimuthal asymmetry defined as

$$\mathcal{A}_1^T = \frac{\sigma(\cos 2\phi_H > 0) - \sigma(\cos 2\phi_H < 0)}{\sigma(\cos 2\phi_H > 0) + \sigma(\cos 2\phi_H < 0)} \quad (16)$$

With $R1$ -cut expression for this asymmetry for muon and quark final states is given by

$$\mathcal{A}_1^T(R1 - \text{cut}) = \begin{cases} \frac{[\mathcal{P}_{e^-}^T \mathcal{P}_{e^+}^T] [-0.37 (1 + 2 \Delta a_Z)]}{[0.86 (1 + 2 \Delta a_Z) + 8.2 \Re(b_Z)]} & (\mu^+ \mu^- H) \\ \frac{[\mathcal{P}_{e^-}^T \mathcal{P}_{e^+}^T] [-0.57 (1 + 2 \Delta a_Z)]}{[1.32 (1 + 2 \Delta a_Z) + 12.5 \Re(b_Z)]} & (q\bar{q}H) \end{cases} \quad (17)$$

$$\simeq -0.43 [\mathcal{P}_{e^-}^T \mathcal{P}_{e^+}^T] [1 - 9.5 \Re(b_Z)]$$

The second expression of Eq. 17 is written to linear order in anomalous couplings. Since the numerator of Eq. 16 does not have any contribution linear in $\Re(b_Z)$, the asymmetry of Eq. 17 involves only $\Re(b_Z)$ to linear order. \mathcal{A}_1^T with final states μ 's and light quarks, for an integrated luminosity $\mathcal{L} = 500 \text{ fb}^{-1}$, leads to a 3σ constraint of

$$|\Re(b_Z)| \lesssim 0.021. \quad (18)$$

Thus we see that both the CP - and \tilde{T} -even couplings, $\Re(b_Z)$ and Δa_Z , can be probed independently using \mathcal{A}_1^T and O_1^T respectively, which was not possible earlier with unpolarized and/or linearly polarized beams [29, 30]. It is interesting to note that this sensitivity limit on $\Re(b_Z)$ obtained using \mathcal{A}_1^T is comparable to the simultaneous limit ($|\Re(b_Z)| \lesssim 0.013$) obtained in our previous analysis [29] using unpolarized cross section.

Next we discuss observables O_2^T and O_3^T constructed using momenta of final state fermions and Higgs boson. Fig. 3 shows azimuthal angular(ϕ) dependence of integrand of O_2^T and O_3^T with μ 's in the final state for partially transversely polarized ($\mathcal{P}_{e^-}^T = 0.8$, $\mathcal{P}_{e^+}^T = 0.6$) beams. The solid curve is for $\Re(\tilde{b}_Z) = 0.1$ whereas the dotted one is for $\Im(b_Z) = -0.1$. Here we have integrated over all the phase space variables of Eq. 2 other than ϕ , the azimuthal angle defined with respect to Higgs boson production plane. It is obvious from Fig. 3 that the asymmetries O_2^T and O_3^T are nonzero and indeed can probe specific parts of the anomalous ZZH vertex for transversely polarized beams.

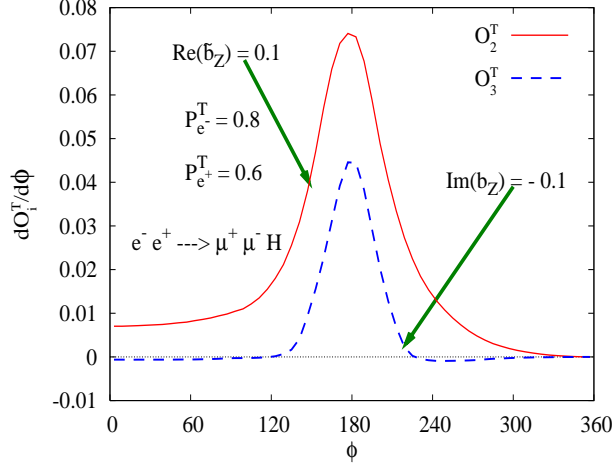


Figure 3: ϕ -angular dependence of integrand of O_2^T and O_3^T for final state μ 's with 80%, 60% transversely polarized e^-, e^+ beams. Only the values corresponding to $\Re(\tilde{b}_Z) = 0.1$ and $\Im(b_Z) = -0.1$ respectively are shown, as these are the only nonzero contributions.

O_2^T with $R1$ -cut for final state muons, electrons and for arbitrary transverse beam polarization is given by

$$O_2^T(R1 - \text{cut}) = \begin{cases} \frac{[\mathcal{P}_{e^-}^T \mathcal{P}_{e^+}^T] [1.19 \Re(\tilde{b}_Z) + 0.02 \Im(\tilde{b}_Z)]}{0.88} & (e^+e^-H) \\ \frac{[\mathcal{P}_{e^-}^T \mathcal{P}_{e^+}^T] [1.19 \Re(\tilde{b}_Z)]}{0.86} & (\mu^+\mu^-H) \end{cases} \quad (19)$$

For the e^+e^-H channel, O_2^T receives additional contribution from $\Im(\tilde{b}_Z)$ on account of the interference of the t -channel diagram with the absorptive part of the s -channel SM one. With 80%(60%) $e^-(e^+)$ transverse beam polarization O_2^T for final state μ 's put a 3σ constraint on $\Re(\tilde{b}_Z)$ as:

$$|\Re(\tilde{b}_Z)| \leq 0.22 \quad \text{for } \mathcal{L} = 500 \text{ fb}^{-1}. \quad (20)$$

This bound for $\Re(\tilde{b}_Z)$ using $R1$ -cut with transverse beam polarization is an improvement by a factor of about 4 in comparison to the limit obtained using up-down asymmetry (A_{UD}) for unpolarized states [29]. Since there is no additional contribution from the transversely polarized beams to A_{UD} , the ability of this asymmetry to probe $\Re(\tilde{b}_Z)$ is not affected with the use of transverse beam polarisation. The above mentioned improvement with transverse beam polarization is possible as O_2^T is proportional to $l_e r_e (\ell_f^2 + r_f^2)$ whereas A_{UD} is suppressed by $(\ell_e^2 - r_e^2)(\ell_f^2 - r_f^2)$. Here $r_f(l_f)$ is the right-(left-)handed coupling of the light fermion to the Z -boson. It should be mentioned that measurements for e^-e^+H final state, with $R2$ -cut (de-select Z -pole events) and unpolarized beams can offer a better sensitivity to $\Re(\tilde{b}_Z)$: $|\Re(\tilde{b}_Z)| \leq 0.067$ [29]. Nevertheless it is useful to have more than one observable determining a given coupling and also in complementary kinematic regions.

After imposing all the kinematical cuts of Eqs. 9, along-with $R1$ -cut the expression for O_3^T with final state muon for arbitrary transversely polarized beams can be written as:

$$O_3^T(R1; \mu) = \frac{[\mathcal{P}_{e^-}^T \mathcal{P}_{e^+}^T] [-0.34 \Im(b_Z)]}{0.86} \quad (21)$$

$O_3^T(R1; \mu)$ for 80%(60%) $e^-(e^+)$ beam polarization would lead to a 3σ constraint on $\Im(b_Z)$ of the form

$$|\Im(b_Z)| \leq 0.77 \quad \text{for } \mathcal{L} = 500 \text{ fb}^{-1}. \quad (22)$$

O_3^T for final state μ is proportional to $l_e r_e (l_f^2 - r_f^2)$ and is suppressed. Taking a clue from our earlier analysis for measurement of final state τ polarization along with unpolarized/longitudinally polarized beams [30], we note that the sensitivity to $\Im(b_Z)$ can be improved with simultaneous use of transverse beam polarization and measurement of polarization of the final state τ . This reduces the suppression factor present in O_3^T . In fact if isolation of events with final state τ 's in a negative helicity state with an efficiency of 40% is possible, one can constrain $\Im(b_Z)$ to $|\Im(b_Z)| \leq 0.24$ using transverse beam polarization and O_3^T . This in fact compares very favourably with the sensitivity limit $|\Im(b_Z)| < 0.35$ possible using combined polar-azimuthal asymmetry (A_{comb}) with $R1$ -cut with unpolarized beams [29].

In our previous studies [29, 30] we had observed that the observables constructed for unpolarized and longitudinally polarized beams with $R2$ -cut (de-selecting Z -pole) can put strong bounds on $\Re(\tilde{b}_Z)$, $\Re(\Delta a_W)$, $\Re(b_W)$ and $\Im(\tilde{b}_W)$. This is mostly due to t -channel enhancement in the cross section with ν 's in the final state. There is no such additional gain by using transverse beam polarization in this case. This can be understood as follows. The t -channel squared matrix element (MESQ) never includes the spin projection factors $(1 + \gamma_5 \not{s}_{e^-})$ and $(1 + \gamma_5 \not{s}_{e^+})$ in the same trace. Since the trace of an odd number of γ -matrices vanishes the MESQ for t -channel diagram does not have any transverse beam polarization dependent term. Similar observation, which is a consequence of electronic chiral symmetry of the SM [44], has been made for the case of SM t -channel diagram [43]. Presence of anomalous VVH vertex does not affect this electronic chiral structure; nor does it change the γ -matrix structure in the matrix element. Hence there is no additional contribution to the t -channel matrix element due to transverse beam polarization even when anomalous VVH vertex is included. For the s -channel processes the MESQ includes both the spin projection factors: $(1 + \gamma_5 \not{s}_{e^-})$, $(1 + \gamma_5 \not{s}_{e^+})$ in the same trace and hence it has nonzero transverse polarization dependent contribution. It should be noted here that the s -channel diagram involves two polarized e^\pm at the same vertex whereas the t -channel diagram has only one transversely polarized e at a given vertex. Hence additional contributions to the MESQ for the $R2$ cut for the transversely polarized beams come only from the interference of the s - and t -channel diagrams. Since imposition of $R2$ -cut reduces the s -channel contribution, the observables: O_1^T , O_2^T and O_3^T with $R2$ -cut are less sensitive than those with $R1$ -cut.

3.2 Anomalous WWH Couplings

As discussed before observable O_1^T can probe some parts of CP -even anomalous VVH couplings. However, note here that transverse beam polarization does not affect the squared matrix element

of the t -channel WW fusion diagram which includes the anomalous WWH couplings. As a result terms proportional to anomalous WWH couplings in O_1^T receive contribution only from the interference of t -channel diagram with the s -channel SM part. Hence O_1^T is not expected to put stronger bounds on CP - and \tilde{T} -even anomalous WWH couplings compared to the bounds obtained for unpolarized beams [29]. As a result, we have not explored numerically the reach in sensitivity of this observable with ν 's in the final state.

4 Summary

We have considered the VVH ($V = Z/W$) anomalous couplings to be complex and investigated use of transverse beam polarization in probing these couplings. This is an extension of our earlier work [29, 30] assessing the use of unpolarized or longitudinally polarized beams to probe these anomalous couplings. We have constructed observables with definite discrete transformation (CP, \tilde{T}) properties that are sensitive to a single anomalous coupling and hence can probe the corresponding operator in the effective Lagrangian with the same CP and \tilde{T} properties. We have imposed various kinematical cuts on different final state particles to reduce backgrounds and considered the events wherein the H decays into a $b\bar{b}$ pair with branching fraction ~ 0.68 , with a b -tagging efficiency of 70%, same as in our earlier analyses [29, 30].

Use of transverse beam polarization to constrain anomalous ZZH and $Z\gamma H$ interactions has been studied previously in Refs. [14, 31]. In Ref. [14] the anomalous ZZH couplings were assumed to be real and it was observed that polarization of beams does not provide any new information about the anomalous ZZH couplings. Ref. [31] had considered these couplings to be complex, but they had investigated use of Z -angular distribution in the process $e^+e^- \rightarrow ZH$, which cannot probe $\Im(b_Z)$ and $\Re(\tilde{b}_Z)$. In our analysis, we find that it is possible to have independent probes to constrain $\Re(\tilde{b}_Z)$, $\Im(b_Z)$, $\Re(b_Z)$ and Δa_Z through a study of the process $e^+e^- \rightarrow f\bar{f}H$ using transverse beam polarization. Ref. [41] contains an analysis of the four point e^-e^+ZH coupling with transverse beam polarization in the Higgs-strahlung (s -channel) process $e^+e^- \rightarrow ZH \rightarrow l^+l^-H$. The general ZZH anomalous couplings can be put into a one to one correspondence with the four point e^-e^+ZH coupling, but only with momentum dependent factors. Hence it is not possible to translate the results of Ref. [41] directly to the case of VVH interactions investigated here.

Use of transverse beam polarization allows us to construct completely independent probes of both the CP - and \tilde{T} -even couplings, $\Re(b_Z)$ and Δa_Z . This was not possible with unpolarized states [29]. In Ref. [30] we had used numerical combinations of various linearly polarized cross sections to reduce the contamination from $\Re(b_Z)$ in probing Δa_Z and vice versa, wherein the numerical coefficients depend on cuts employed as well as on the exact size of SM contribution etc. On the other hand use of transverse beam polarization can probe both these couplings individually which lead to independent determination of all the anomalous parts of the ZZH vertex. Further, it helps to improve on the sensitivity of $\Re(\tilde{b}_Z)$ by a factor of about 4 for s -channel final state muons. It is possible to get rid of the suppression factor $(\ell_f^2 - r_f^2)$ in O_3^T by isolation of events with a final state τ in definite helicity state, which can then lead to increased sensitivity.

To summarize, use of transverse beam polarization helps to construct independent probes of both the CP - and \tilde{T} -even ZZH couplings. It can also improve the sensitivity to probe one of the CP -odd anomalous part of the ZZH vertex by a factor of up to 4-5 for muons in the final

state. Use of transverse beam polarization along with measurement of final state τ polarization can increase the sensitivity to $\Im(b_Z)$ also by 30% compared to the unpolarized case.

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