Energetics of rocked inhomogeneous ratchets

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We study the efficiency of frictional thermal ratchets driven by finite frequency driving force and in contact with a heat bath. The efficiency exhibits varied behavior with driving frequency. Both nonmonotonic and monotonic behavior have been observed. In particular the magnitude of efficiency in finite frequency regime may be more than the efficiency in the adiabatic regime. This is our central result for rocked ratchets. We also show that for the simple potential we have chosen, the presence of only spatial asymmetry (homogeneous system) or only frictional ratchet (symmetric potential profile), the adiabatic efficiency is always more than in the nonadiabatic case.

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Much has been studied in ratchet models (Brownian motors) to determine how directed motion appears out of nonequilibrium fluctuations in the absence of any net macroscopic force. Here athermal fluctuation combined with spatial or temporal anisotropy conspire to generate systematic motion even in the absence of net bias [1]. These studies have been inspired by the observations on molecular motors in biological systems [2]. To this effect several physical models have been proposed under the name of rocking ratchets, flashing ratchets, diffusion ratchets, correlation ratchets, frictional ratchets [1] etc. In most of these systems, focus was mainly on the behavior of probability current with change in system parameter like temperature, amplitude of external force, correlation time, etc. The efficiency with which these ratchets convert fluctuation to useful work is a subject of much recent interest [3,4]. New questions regarding the nature of heat engines (reversible or irreversible) at molecular scales are being investigated. Especially the source of irreversibility and whether the irreversibility can be suppressed such that efficiency can approach that of Carnot cycle [2,5] and generalization of thermodynamics principles to nonequilibrium steady state are being investigated [6]. We use the method of stochastic energetics developed by Sekimoto [3]. In this scheme, quantities like heat, work done and input energy can be calculated within the framework of Langevin equation. Using this approach efficiency has been studied mainly as a function of temperature and load in rocking, oscillating and frictional ratchets. In some cases it has been shown that efficiency can be maximized at finite temperature [4,7]. The efficiency in these systems are rather small, the reason being inherent irreversibility of these engines due to finite current.

In our present work we mainly explore the nature of efficiency in frictional rocking ratchets as function of frequency of external drive. The systematic study of efficiency as a function of frequency in rocked ratchets has not been studied so far. We show in the following that a rocking ratchet with inhomogeneous friction coefficient can have efficiency which is a nonmonotonic function of frequency. In some parameter range, **the efficiency in the nonadiabatic regime can even be larger than**

in the adiabatic regime. This is solely due to the interplay between the asymmetry in the potential and the space dependent friction coefficient. In absence of frictional inhomogeneity our system reduces to a conventional rocked ratchet. It may also happen that inspite of this nonmonotonic behavior with frequency the adiabatic efficiency is larger than the nonadiabatic efficiency. This shows that in the nonequilibrium regime efficiency exhibits complex behavior some which are against the established tenets of equilibrium phenomena, like efficiency in quasi-static processes is maximum. It is difficult to find any systematic principle or procedure that can optimize the efficiency.

Transport properties in overdamped inhomogeneous systems have been dealt with great detail previously [8–10]. Occurrence of multiple current reversals [11], current reversal under adiabatic or deterministic conditions, unidirectional motion in the absence of potential [10], stochastic resonance in the absence of periodic forcing have also been observed [12]. Most of these phenomena arise solely due to the presence of frictional inhomogeneity. It is to be noted that systems with space dependent friction are not uncommon. Brownian motion in confined geometries show space dependent friction. Particles diffusing close to surface have space dependent friction coefficient [11]. It is also believed that molecular motors move close along the periodic structure of microtubules and will therefore experience a position dependent mobility [10]. Frictional inhomogeneities are common in super lattice structures and semiconductor systems [8].

We consider an overdamped Brownian particle moving in an inhomogeneous 1D ratchet like potential, rocked by a finite frequency driving force. We consider an asymmetric potential of the form $V(x) = -1/(2\pi)(\sin(2\pi x) + \mu/4 \sin(4\pi x)) + Lx$, where L is the external load against which the Brownian particle moves on average. μ is the asymmetry parameter and is in between the range 0 and 1. The direction of load is chosen against the mean drift of the Brownian particle so that the work done by the particle is positive. The system is rocked by a zero mean external force of the form $F(t) = A\sin(\omega t)$. The correct Langevin equation for such a motion has been derived using microscopic treatment of system bath coupling [9,13].

$$\dot{x} = -\frac{(V'(x) - F(t))}{\eta(x)} - k_B T \frac{\eta'(x)}{(\eta(x))^2} + \sqrt{\frac{k_B T}{\eta(x)}}\xi(t), \tag{1}$$

The quantity x represents the spatial position of the system. It should be noted that the above equation involves a multiplicative noise with an additional temperature dependent drift term which turns out to be essential for the system to approach correct thermal equilibrium state in absence of external drive $F(t)$ and load L [9,13]. The Gaussian white noise $\xi(t)$ is delta correlated with mean zero, i.e., $< \xi(t)\xi(t') >= 2D\delta(t-t')$. The friction coefficient $\eta(x) = \eta_0(1 - \lambda \sin(2\pi x + \phi))$, $|\lambda| < 1$ and ϕ determines the relative phase shift between friction coefficient and potential. The Fokker Planck equation corresponding to eqn. (1) is given by [14]

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial J(x,t)}{\partial x} = \frac{\partial}{\partial x}\frac{1}{\eta(x)}[k_B T \frac{\partial}{\partial x} + (V'(x) - F(t))]P(x,t), \tag{2}$$

where $J(x,t)$ and $P(x,t)$ are the current density and probability density respectively. The mean current

$$J = \lim_{t\to\infty} \frac{1}{\tau}\int_t^{t+\tau} dt \int_0^1 J(x,t)dx, \tag{3}$$

which is obtained numerically by solving eqn. (2) by the method of finite difference. The work done against the load, given by $W = LJ$. The average input energy E is given by $E = \lim_{t\to\infty} \frac{1}{\tau}\int_t^{t+\tau} dt \int_0^1 F(t)J(x,t)dx$. The efficiency of the system to transform the external force to useful work (storing potential energy) is [4]

$$\eta = \frac{W}{E} = \frac{LJ}{\lim_{t\to\infty} \frac{1}{\tau}\int_t^{t+\tau} dt \int_0^1 F(t)J(x,t)dx}, \tag{4}$$

where J is calculated from eqn. (3).

We now discuss the effect of finite frequency drive, spatial asymmetry and inhomogeneous friction coefficient on the efficiency of energy transduction. It is observed that spatial asymmetry or space dependent friction coefficient alone cannot enhance the nonadiabatic efficiency as compared to the adiabatic one in a rocked thermal ratchet. However, the interplay of both can enhance the efficiency in the nonadiabatic regime.

First we discuss the nature of efficiency in an asymmetric ratchet in the absence of spatial frictional inhomogeneity ($\lambda = 0, \mu = 1$). This ratchet in contact with thermal bath produces directed motion when rocked by a finite force. The direction of current being dependent both on the direction of asymmetry as well as the frequency of the driving force [15]. When rocked adiabatically, the current shows maxima at some nonzero value of temperature. Even though the current shows a maxima the efficiency monotonically decreases with temperature [4,7]. The situation changes in the nonadiabatic regime as shown in the fig. 1. Throughout this work temperature and frequency have been scaled appropriately to make them dimensionless [14]. In fig. 1 we have plotted η vs T for various values of ω. It can be seen that unlike the adiabatic case, η shows a maxima at a nonzero value of temperature. The value at which η peaks, decreases with decreasing frequency, as it should be. We have observed that even though the efficiency peaks at nonzero value of temperature in the nonadiabatic regime, efficiency in the adiabatic regime (at $T = 0$)is much larger than the peak nonadiabatic efficiency.

In fig. 2 we have plotted efficiency as function of rocking frequency for various values of T (and A, in the inset). Current reversal as function of frequency is a common phenomena in a driven asymmetric ratchet. Since beyond a critical frequency, the current reverses its direction [15], the load has been applied in the opposite direction in that regime so that work is done against the load. As shown in the fig. 2, for low frequencies, efficiency shows a monotonic decrease with frequency. The rate of decrease of η with ω being critically dependent on temperature T and amplitude A. In the current reversed regime, the efficiency shows a maxima with ω, though its value is much less than the adiabatic efficiency. We have verified this fact by exhaustive numerical work with our given potential.

We now consider a system in which friction is space dependent with a *symmetric* potential profile. Unidirectional current results whenever $\phi \neq 0, \pi$ or 2π as discussed in ref. ([7]). In these models inversion symmetry is broken dynamically by space dependent friction. This system does not exhibit current reversal with any of the variables like T, A or ω (in the absence of L) and hence we keep the load fixed in one direction for comparison of efficiency. In fig. 3 we have plotted η *vs* ω for $A = 0.5, \phi = 0.6\pi$ and $L = -0.012$. For all values of T, λ and ϕ, the efficiency monotonically decreases with ω, i.e., for a given T the adiabatic efficiency is always maximum. In the two cases considered above ($\lambda = 0$ with asymmetric potential and $\lambda \neq 0$ with symmetric potential) adiabatic current is always more than the absolute value of the peak current in the nonadiabatic regime. The efficiency in our present case is mainly determined by the nature of currents and hence the result follows.

We now concentrate on frictional ratchets with asymmetric potential profile ($\lambda \neq 0, \mu \neq 0$). The efficiency characteristics of these ratchets have many novel and counterintuitive features. In fig. 4 we plot efficiency as function of ω for two values of forcing amplitude A with $T = 0.08, \lambda = 0.9, \phi = 0.2\pi$ and $L = 0.015$. It can be clearly seen that the nonadiabatic efficiency is higher than the adiabatic efficiency, which is contrary to common belief that a rocked Brownian ratchet is inefficient in the nonadiabatic domain. This enhanced efficiency basically results from the *increase in the current with increase of frequency in current reversed regime.* The increase of current in this regime can be ascribed due to mutual interplay of spatial asymmetry and space dependent friction coefficient [7]. The phase difference ϕ is chosen in such a manner so that the steeper side has lower friction coefficient than the slanted side. The inset shows a different qualitative behavior of η with increase of ω. Here $A = 1.5, T = 0.4, \lambda = 0.1, \phi = 0.2\pi$ and $L = -0.001$. In this parameter regime there is no current reversal. Here as we increase frequency from adiabatic regime, η increases till it exhibits a maxima at very high frequency and decreases on further increasing the frequency.At high

T and in the adiabatic regime, particles get sufficient kicks and enough time to cross the barrier on both the side, but the frictional drag on the steeper side is less. Hence the current flow is in the negative direction. On increasing the frequency, the Brownian particles get less time to cross the right barrier as it has to travel larger distance to reach the basin of attraction of the next well than from the left side. Hence the net current increases. From the above argument it can be easily seen that the efficiency increases with increasing asymmetry (increasing μ) and vice-versa which we have checked in our work. For too high frequencies the particles do not get sufficient time to cross either of the barriers and the current decreases which reflects in the decrease of efficiency as shown in the fig. 4. Hence efficiency optimization at high frequency is not only a phenomenon in current reversed regime but other wise also. On decreasing temperature the asymmetric ratchet effect becomes more pronounced and efficiency increases along with the shift of the peak efficiency to lower frequency regime.

Like other previous cases depending on the parameter values the efficiency (as a function of T) may or may not be maximized at finite temperature [4,7]. This optimum value (if maxima exists) increases initially with increasing frequency and then for too high frequencies it decreases as discussed earlier. The temperature at which η peaks increases with increasing frequency as shown in the fig. 5. This shows that unlike conventional wisdom where we associate high driving frequency and temperature with inefficient energy conversion, here both *high frequency* (nonadiabatic regime) and *temperature enhances efficiency.*

In conclusion we have studied the efficiency of energy transduction in a forced frictional ratchet as function of rocking frequency. Both nonmonotonic and monotonic behavior have been observed. In particular the magnitude of efficiency in finite frequency regime may be more than the efficiency in the adiabatic regime. This implies that in these rocked ratchet systems quasi-static operation may not be efficient for conversion of input energy into mechanical work. Observation of peak in the efficiency as function of system parameters can be qualitatively attributed to the peak in the current and not to the behavior of input energy, though the occurrence of peak in current may not guarantee a peak in efficiency as observed earlier [4,7]. Here we have taken a simple ratchet type potential with space dependent friction coefficient to illustrate the above results. We do not rule out the fact that similar result can also be obtained in homogeneous systems for different ratchet potentials provided they exhibit larger absolute peak current in the nonadiabatic regime. It is interesting to explore this possibility. It should be noted that in flashing ratchets unlike rocking ratchets, efficiency can show peaking behaviour as a function of frequency. This is because both in zero frequency and in high frequency limit flashing ratchet does not exhibit current [2]. Detailed study of input energy, work done and dependence of efficiency on other system parameters will be reported in future.

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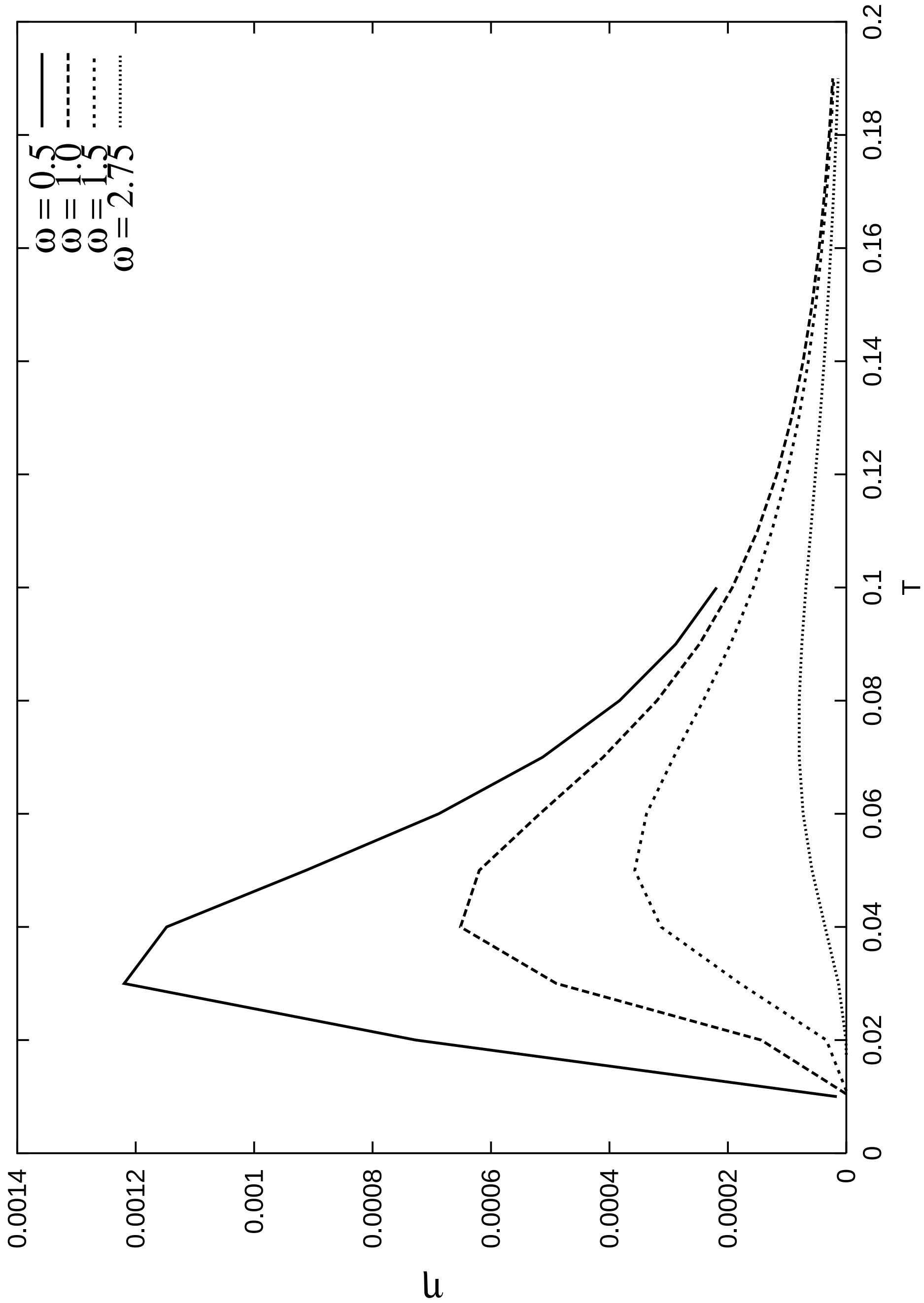

FIG. 1. Efficiency η vs temperature T for $A = 0.5, \mu = 1.0, \lambda = 0, L = 0.001$ and various values of ω. The curve for $\omega = 0.5$ follows the same trend as other curves beyond $T = 0.1$

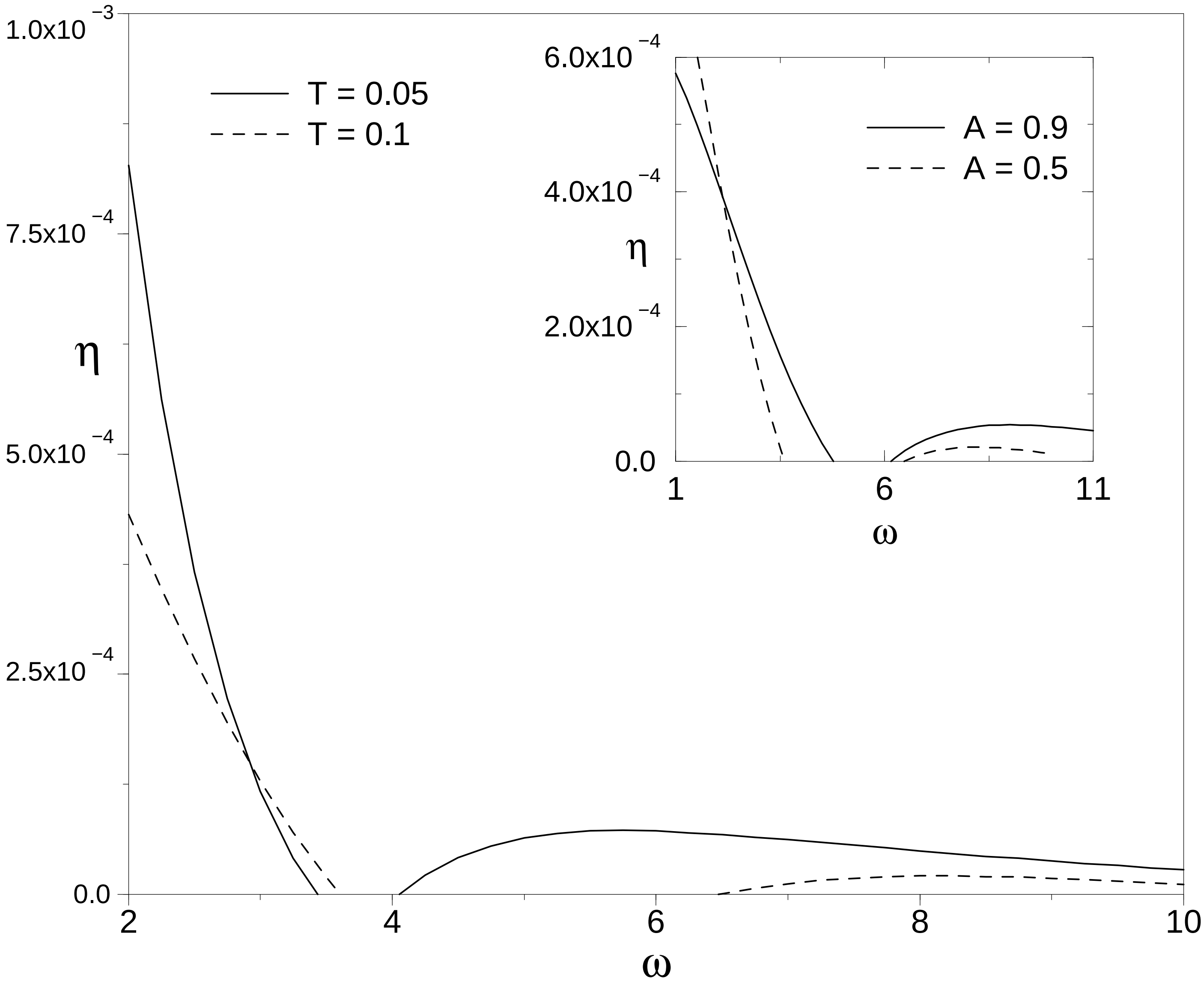

FIG. 2. Efficiency vs ω for two values T at $A = 0.5, \mu = 1.0, \lambda = 0.0$ and $|L| = 0.005$. The inset shows variation of η with ω for two values of A at $T = 0.1$, all other parameter values remaining same.

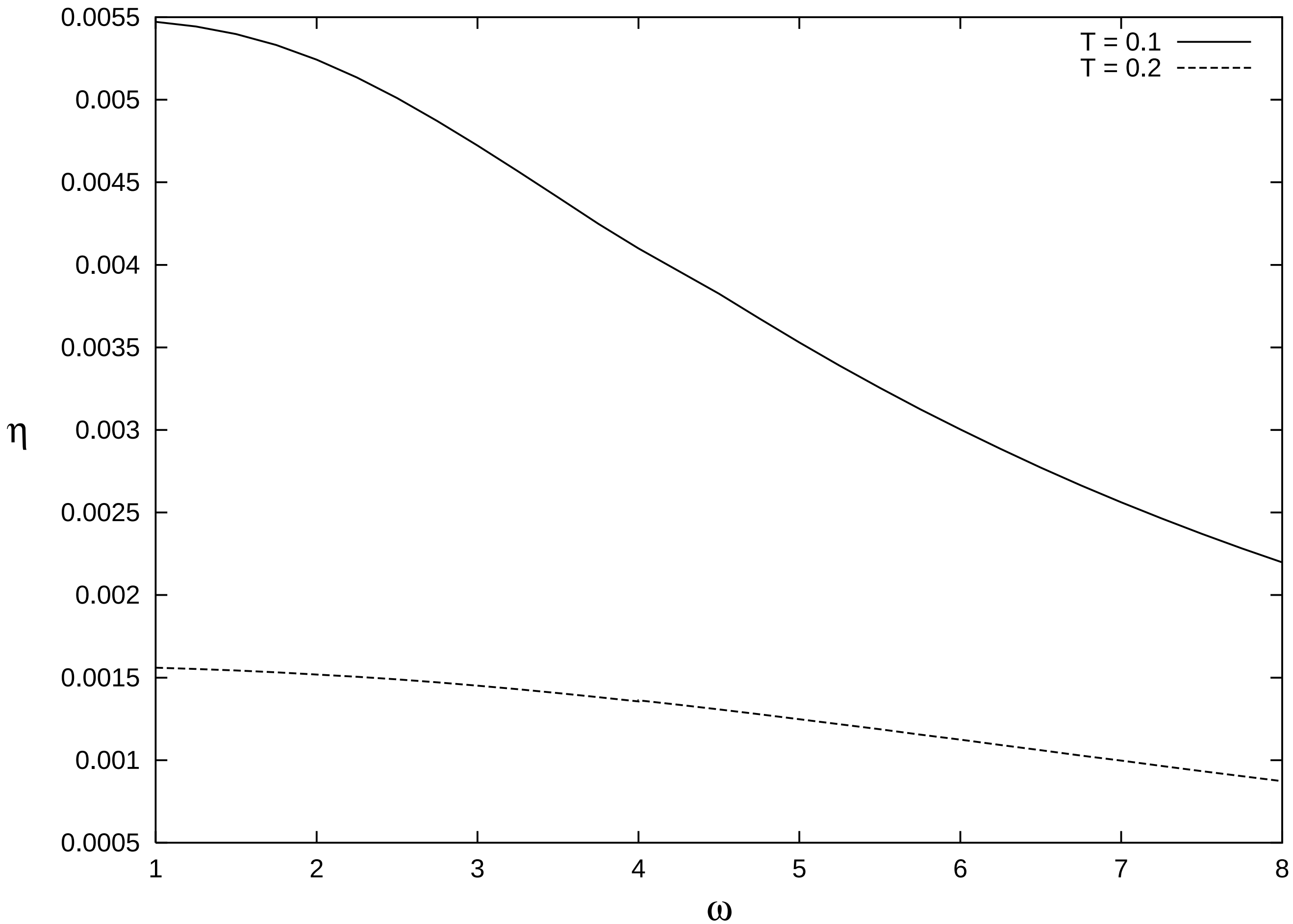

FIG. 3. Efficiency vs ω for $\mu = 0, \lambda = 0.9, \phi = 0.6\pi, L = -0.012$ and for two values of T

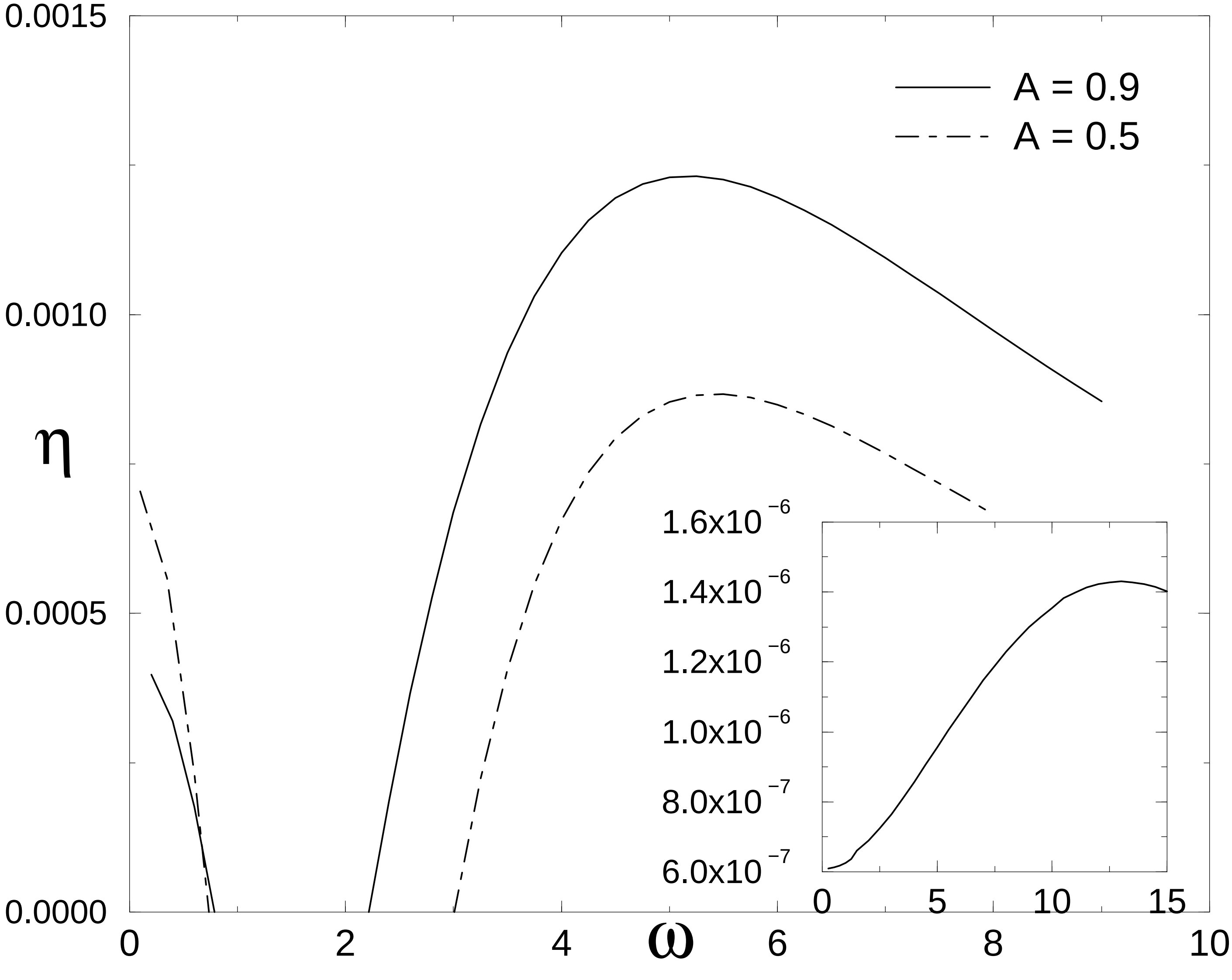

FIG. 4. Efficiency vs ω for two values of A. For other parameter values see text. The inset shows the variation of η vs ω for $A = 1.5, T = 0.4$.

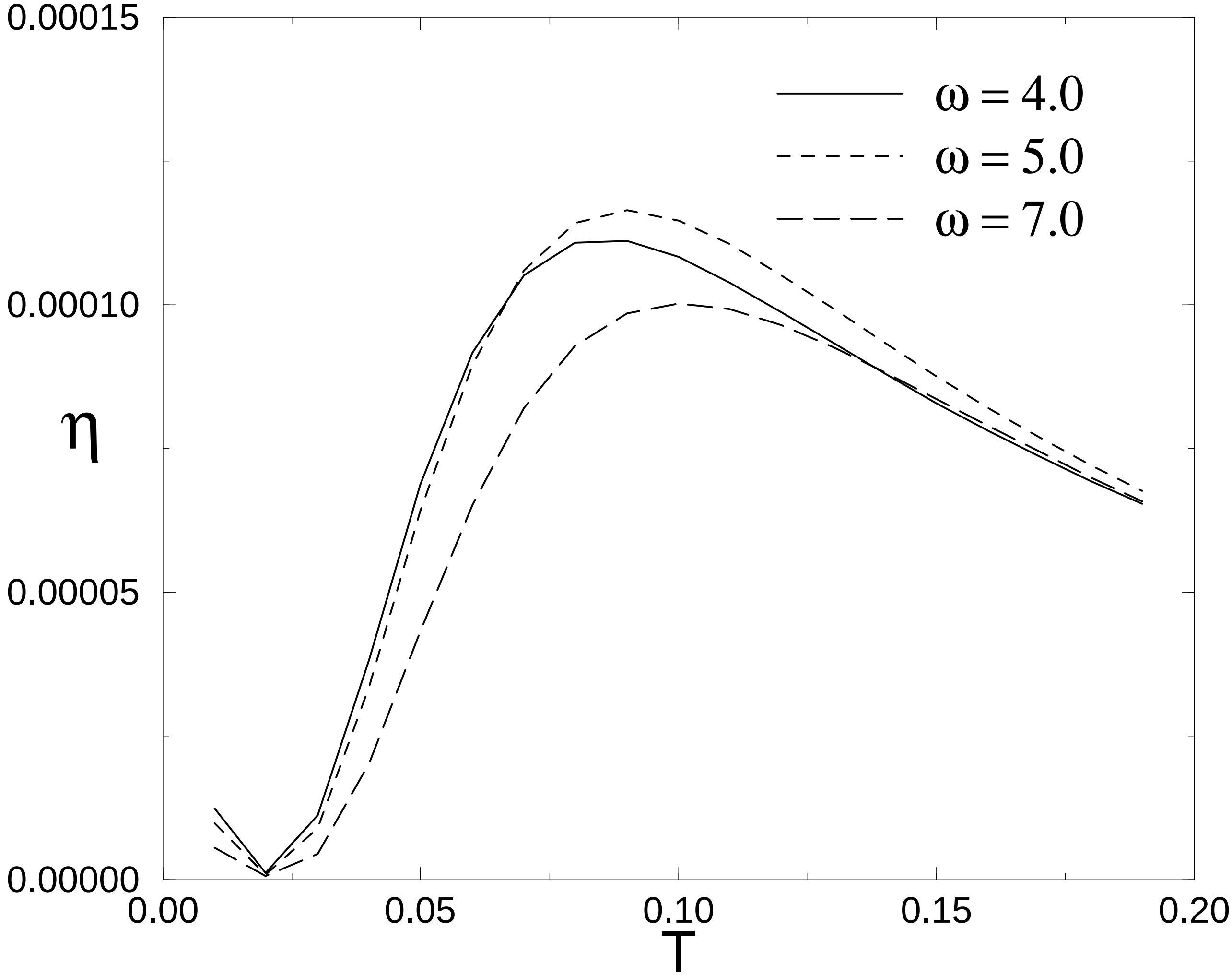

FIG. 5. Efficiency vs temperature for different values of ω. Here $\mu = 1, \lambda = 0.9, A = 0.5, \phi = 0.2\pi$ and $L = -0.001$