

Soft gluon radiation and energy dependence of total hadronic cross-sections

R.M. Godbole*

Center for High Energy Physics, Indian Institute of Science, Bangalore, 560 012, India

A. Grau†

Departamento de Fisica Teorica y del Cosmos, Universidad de Granada, Spain

G. Pancheri‡

INFN, Laboratori Nazionali di Frascati,

P.O. Box 13, I-00044 Frascati, Italy

Y. N. Srivastava§

Physics Department and INFN, University of Perugia, Perugia, Italy

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Abstract

An impact parameter representation for soft gluon radiation is applied to obtain both the initial decrease of the total cross-section (σ_{tot}) for proton-proton collisions as well as the later rise of σ_{tot} with energy for both pp and $p\bar{p}$. The non-perturbative soft part of the eikonal includes only limited low energy gluon emission and leads to the initial decrease in the proton-proton cross-section. On the other hand, the rapid rise in the hard, perturbative jet part of the eikonal is tamed into the experimentally observed mild increase by soft gluon radiation whose maximum energy rises slowly with energy.

*Electronic address: rohini@cts.iisc.ernet.in

†Electronic address: igräu@ugr.es

‡Electronic address: pancheri@lnf.infn.it

§Electronic address: srivastava@pg.infn.it

I. INTRODUCTION

In this paper, we extend our approach to the role played by soft gluon radiation in determining the energy dependence of total cross-sections [1], by including new low energy features. The experimental information on the total cross-section is now sufficient to allow for definite progress towards its description through QCD. Thanks to the recent measurements at HERA [2, 3, 4, 5] and LEP [6, 7] providing knowledge of total hadronic cross-sections involving photons in the energy interval $\sqrt{s} = 1 \div 100 \text{ GeV}$, we now possess a complete set of processes to study in detail and in depth, namely pp , $p\bar{p}$, γp and $\gamma\gamma$. The purely hadronic processes are well measured over an extended range, up to cosmic ray energies [8], leading to quite precise parametrizations [9], while the other two allow for probes of the hadronic content of the photon versus that of the proton. They also allow for checks of the Gribov-Froissart factorization hypothesis [10, 11].

The three basic features which the data exhibit and which require a theoretical explanation and understanding are: (i) the normalization of the cross section, (ii) an initial decrease and (iii) a subsequent rise with energy. All three are reasonably well described in the Regge trajectory language, which suggests a parametrization of all total cross-sections[12] as a sum of powers of the square of the c.m. energy s . The oldest and simplest of these parametrizations is

$$\sigma_{tot}(s) = X s^\epsilon + Y s^{-\eta}. \quad (1)$$

In this model, the initial decrease reflects the disappearance (with increasing energy) of a Regge trajectory exchanged in the t -channel, with $\eta = 1 - \alpha_\rho(0)$ where the intercept (at $t = 0$) of the leading Regge trajectory is $\alpha_\rho(0) \approx 0.5$. At the same time, the rise in the cross section is attributed to the exchange of a trajectory (the Pomeron) with the quantum numbers of the vacuum, such that $\alpha_P(0) = 1 + \epsilon$. ϵ is expected to be a small number so as not to defy too much the Froissart bound, and, phenomenologically, $\epsilon = 0.08 \div 0.12$ [13, 14].

The two terms in which the cross-sections are split in Eq.(1) describe well the main features of the observed total cross-section data, with an apparently constant behaviour at $\sqrt{s} \approx \frac{\eta}{\epsilon} \frac{Y}{X}$. For proton-proton and proton-antiproton scattering, this occurs between 15 and 25 GeV, where the cross-section is about 40 mb. At large energies, the Regge term disappears, while the first term is important everywhere, and dominates asymptotically. Thus the decrease is due to the Regge term, the rise described by the Pomeron with some

taming due to the Regge term and the normalisation is determined by both the Regge and the Pomeron terms. More recent analyses add further terms such as $\log^2(s/s_0)$ terms[15].

In the following we shall discuss these features one by one within the context of QCD, using the simple but useful parametrization of Eq.(1) as reference. We shall modify it and through it develop further the model of ref.[1] to increase our understanding of the energy dependence within QCD. In section II, we discuss the QCD origin of the two component model of Eq. (1). In section III, we summarise how the Born term of the QCD contribution, the mini-jets, is embedded in the eikonal formalism which provides a unitarised description of the total cross-section. In section IV, we discuss the analyticity requirements upon the impact parameter amplitudes of the eikonal formalism. In section V, we review the soft gluon transverse momentum distribution on which our model for the b-distribution of partons is based. In section VI we discuss the main features of the model[1] for the energy dependence of the impact parameter distribution, induced by soft gluon emission. In section VII, through soft gluon emission, we obtain the transverse parton overlap function for the non-perturbative part of the cross-section. In section VIII, we introduce the normalisation of the cross-section. We present our model for σ_{tot} for pp and $p\bar{p}$ and compare them with currently available data. In section IX, we present an interesting feature of our model: in a proton or antiproton collision, the (average) distance between the scattering centres (i.e. the constituents) in the transverse space decreases as the energy increases. Similar results have been previously obtained in an analysis of the hadronic events at the Tevatron, through an impact parameter picture. We note again that our model of the energy dependent impact parameter distribution offers a reasonable understanding of both the initial fall and later rise with energy of the σ_{tot} , whereas for the normalization there is still further work to do.

Before beginning the detailed discussion, we indicate below briefly and qualitatively, why we believe that it is both important and necessary to include effects of soft-gluon emission in the mini-jet formalism [16, 17, 18]. We recall here that in the mini-jet picture the rise in cross-sections is driven by the increasing number of low-x gluon-gluon collisions and that the predicted rate of the rise is generally found to be uncomfortably large. In section VI we give the details of the mechanism by which the effect of soft gluon emissions can reduce the rate of this rise, at any given c.m. energy. Below we summarise the effect qualitatively.

In a calculation in the QCD improved parton model, the effect of gluon radiation on the longitudinal momentum carried by the partons is included, at least in part, in the Dok-

shitzer, Gribov, Lipatov, Altarelli and Parisi (DGLAP) [19] evolved parton distribution. Here we crucially look at the effect of initial state gluon radiation on the transverse momentum distribution of partons. In our model the soft gluon resummed transverse momentum distribution of partons in the hadrons and the parton distribution in impact parameter space are Fourier Transforms of each other. The larger the transverse momentum the larger is the acollinearity of the two colliding partons, leading to a reduction in parton luminosity and hence to a reduction of the cross-section. The higher the c.m. energy of the parent hadron, the more energetic are the parent valence quarks emitting gluons and the more is the acollinearity of the two partons involved in the parton subprocess. Hence, we expect the effect to be dependent on the c.m. energy.

II. THE TWO COMPONENT MODEL

Before entering into the technical details about unitarization, it is well to ask (i) where the “two component” structure of Eq.(1) comes from and (ii) why the difference in the two powers (in s) is approximately a half.

Let us first remember that the two terms of Eq.(1) reflect the well known duality between resonance and Regge pole exchange on the one hand and background and Pomeron exchange on the other, established in the late 60’s through FESR [20]. This correspondance meant that, while at low energy the cross-section could be written as due to a background term and a sum of resonances, at higher energy it could be written as a sum of Regge trajectory exchanges and a Pomeron exchange.

Our present knowledge of QCD description of hadronic phenomena gives further insight into the nature of these two terms. We shall start answering the above two questions through considerations about the bound state nature of hadrons which necessarily transcends perturbative QCD. For hadrons made of light quarks(q) and glue(g), the two terms arise from $q\bar{q}$ and gg excitations. For these, the energy is given by a sum of three terms: (i) the rotational energy, (ii) the Coulomb energy and (iii) the “confining” energy. If we accept the Wilson area conjecture in QCD, (iii) reduces to the linear potential[21, 22]. Explicitly, in the CM frame of two massless particles, either a $q\bar{q}$ or a gg pair separated by a relative distance r with relative angular momentum J , the energy is given by

$$E_i(J, r) = \frac{2J}{r} - \frac{C_i \bar{\alpha}}{r} + C_i \tau r, \quad (2)$$

where $i = 1$ refers to $q\bar{q}$, $i = 2$ refers to gg , τ is the “string tension” and the Casimir’s are $C_1 = C_F = 4/3$, $C_2 = C_G = 3$. $\bar{\alpha}$ is the QCD coupling constant evaluated at a some average value of r and whose value will disappear in the ratio to be considered. The hadronic rest mass for a state of angular momentum J is then computed through minimising the above energy

$$M_i(J) = \text{Min}_r \left[\frac{2J}{r} - \frac{C_i \bar{\alpha}}{r} + C_i \tau r \right], \quad (3)$$

which gives

$$M_i(J) = 2\sqrt{(C_i \tau)[2J - C_i \bar{\alpha}]}. \quad (4)$$

The result may then be inverted to obtain the two sets of linear Regge trajectories $\alpha_i(s)$

$$\alpha_i(s) = \frac{C_i \bar{\alpha}}{2} + \left(\frac{1}{8C_i \tau} \right) s = \alpha_i(0) + \alpha'_i s. \quad (5)$$

Thus, the ratio of the intercepts is given by

$$\frac{\alpha_{gg}(0)}{\alpha_{q\bar{q}}(0)} = C_G/C_F = \frac{9}{4}. \quad (6)$$

Using our present understanding that resonances are $q\bar{q}$ bound states while the background, dual to the Pomeron, is provided by gluon-gluon exchanges[23], the above equation can be rewritten as

$$\frac{\alpha_P(0)}{\alpha_R(0)} = C_G/C_F = \frac{9}{4}. \quad (7)$$

If we restrict our attention to the leading Regge trajectory, namely the degenerate $\rho - \omega - \phi$ trajectory, then $\alpha_R(0) = \eta \approx 0.48 - 0.5$, and we obtain for $\epsilon \approx 0.08 - 0.12$, a rather satisfactory value. The same argument for the slopes gives

$$\frac{\alpha'_{gg}}{\alpha'_{q\bar{q}}} = C_F/C_G = \frac{4}{9}. \quad (8)$$

so that if we take for the Regge slope $\alpha'_R \approx 0.88 - 0.90$, we get for $\alpha'_P \approx 0.39 - 0.40$, in fair agreement with lattice estimates [24].

We now have good reasons for a break up of the amplitude into two components. To proceed further, it is necessary to realize that precisely because massless hadrons do not exist, Eq.(1) violates the Froissart bound and thus must be unitarized. To begin this task, let us first rewrite Eq.(1) by putting in the “correct” dimensions

$$\bar{\sigma}_{tot}(s) = \sigma_1(s/\bar{s})^\epsilon + \sigma_2(\bar{s}/s)^{1/2}, \quad (9)$$

where we have imposed the nominal value $\eta = 1/2$. In the following, we shall obtain rough estimates for the size of the parameters in Eq.(9).

A minimum occurs in $\bar{\sigma}_{tot}(s)$ at $s = \bar{s}$, for $\sigma_2 = 2\epsilon\sigma_1$. If we make this choice, then Eq.(9) has one less parameter and it reduces to

$$\bar{\sigma}_{tot}(s) = \sigma_1[(s/\bar{s})^\epsilon + 2\epsilon(\bar{s}/s)^{1/2}]. \quad (10)$$

We can isolate the rising part of the cross-section by rewriting the above as

$$\bar{\sigma}_{tot}(s) = \sigma_1[1 + 2\epsilon(\bar{s}/s)^{1/2}] + \sigma_1[(s/\bar{s})^\epsilon - 1]. \quad (11)$$

Eq.(11) separates cleanly the cross-section into two parts: the first part is a “soft” piece which shows a saturation to a constant value (but which contains no rise) and the second a “hard” piece which has all the rise. Moreover, \bar{s} naturally provides the scale beyond which the cross-sections would begin to rise. Thus, our “Born” term assumes the generic form

$$\sigma_{tot}^B(s) = \sigma_{soft}(s) + \vartheta(s - \bar{s})\sigma_{hard}(s). \quad (12)$$

with σ_{soft} containing a constant (the “old” Pomeron with $\alpha_P(0) = 1$) plus a (Regge) term decreasing as $1/\sqrt{s}$ and with an estimate for their relative magnitudes ($\sigma_2/\sigma_1 \sim 2\epsilon$). We shall assume that the rising part of the cross-section σ_{hard} is provided by jets which are calculable by perturbative QCD, obviating (atleast in principle) the need of an arbitrary parameter ϵ .

An estimate of σ_1 may also be obtained through the hadronic string picture. Eq.(3) gives us the mean distance between quarks or the “size” of a hadronic excitation of angular momentum J in terms of the string tension

$$\bar{r}(J)^2 = \frac{2J - C_F\bar{\alpha}}{\tau}. \quad (13)$$

So the size R_1 of the lowest hadron (which in this Regge string picture has $J = 1$, since $\alpha_R(0) = 1/2$) is given by

$$R_1^2 = \frac{1}{\tau} = 8\alpha' \quad (14)$$

If two hadrons each of size R_1 collide, their effective radius for scattering would be given by

$$R_{eff} = \sqrt{R_1^2 + R_1^2} = \sqrt{2}R_1, \quad (15)$$

and the constant cross-section may be estimated (semi-classically) to be roughly

$$\sigma_1 = 2\pi R_{eff}^2 = 4\pi R_1^2 \approx \frac{4\pi}{\tau} = 32\pi\alpha', \quad (16)$$

which is about 40 *mb*, a reasonable value. In the later sections, for the “soft” cross-section we shall take a value of this order of magnitude as the nominal value.

The unitarization now proceeds very simply by eikonal exponentiation in impact parameter space, as described in detail in the sections to follow.

III. THE RISE WITH ENERGY

QCD offers an elegant explanation of the rise in the minijet formalism as has been pointed out by several authors in the past [16, 17, 18]. The original suggestion was that the rise of σ_{tot} with energy is driven by the rapid rise with energy of the inclusive jet cross-section

$$\sigma_{jet}^{ab}(s) = \int_{p_{tmin}}^{\sqrt{s}/2} dp_t \int_{4p_t^2/s}^1 dx_1 \int_{4p_t^2/(x_1 s)}^1 dx_2 \sum_{i,j,k,l} f_{i|a}(x_1) f_{j|b}(x_2) \frac{d\hat{\sigma}_{ij \rightarrow kl}(\hat{s})}{dp_t}, \quad (17)$$

where subscripts a and b denote particles (γ, p, \dots), i, j, k, l are partons and x_1, x_2 the fractions of the parent particle momentum carried by the parton. $\hat{s} = x_1 x_2 s$ and $\hat{\sigma}$ are hard partonic scattering cross-sections. Note that $d\hat{\sigma}/dp_t \propto p_t^{-3}$; the cross-section defined in Eq.(17) therefore depends very sensitively on p_{tmin} , which is supposed to parametrize the transition from perturbative to nonperturbative QCD. The rise of the inclusive jet cross-section with energy is understood in terms of the increasing number of hard partons which gives rise to an increasing probability for the occurrence of hard scattering processes. Quantitatively, factorisation of QCD allows us to use the currently available parametrisations of the scale dependent parton densities and calculate the energy dependence of the resulting jet cross-section, by convoluting the parton densities with the subprocess cross-section determined by perturbative QCD.

For any fixed p_{tmin} , typically 1 ÷ 2 GeV, one finds that this cross-section is a steeply rising function of energy. If $\sqrt{s} \gg p_{tmin}$, the integral in Eq.(17) receives its dominant contribution from $x_{1,2} \ll 1$. The relevant parton densities can then be approximated by a simple power law, $f \propto x^{-J}$. In case of pp or $\bar{p}p$ scattering, $a = b$ and the cross-section asymptotically scales like [25]

$$\sigma_{jet} \propto \frac{1}{p_{tmin}^2} \left(\frac{s}{4p_{tmin}^2} \right)^{J-1} \log \frac{s}{4p_{tmin}^2}, \quad (18)$$

if $J > 1$. For $J \simeq 1.3$, as measured by HERA, the jet cross-section will therefore grow much faster than the total $\bar{p}p$ cross-section, which only grows $\propto \log^2 s$ (Froissart bound [26]), or, phenomenologically [12] for $\sqrt{s} \leq 2$ TeV, $\propto s^{0.08}$. Eventually the jet cross-section (17) will therefore exceed the total $\bar{p}p$ cross-section. In fact, this rise is far more violent than the experimentally observed gentle rise of the total cross-section [18, 27]. This has led to various phenomenological strategies (among them the eikonal formalism [28]) directed towards softening this rise. The apparent paradox is solved by the observation that, by definition, inclusive cross-sections include a multiplicity factor. Since a hard partonic scattering always produces a pair of (mini-)jets, we can write

$$\sigma_{\text{jet}}^{ab} = \langle n_{\text{jet pair}} \rangle \sigma_{\text{inel}}^{ab}, \quad (19)$$

where $\langle n_{\text{jet pair}} \rangle$ is the average number of (mini-)jet pairs per inelastic collision. $\sigma_{ab}^{\text{jet}} > \sigma_{\text{inel}}^{ab}$ then implies $\langle n_{\text{jet pair}} \rangle > 1$, which means that, on average, each inelastic event contains more than one hard partonic scatter. The simplest possible assumption about these multiple partonic interactions is that they occur completely independently of each other, in which case $n_{\text{jet pair}}$ obeys a Poisson distribution. At a slightly higher level of sophistication, one assumes these interactions to be independent only at fixed impact parameter b ; indeed, it seems natural to assume that events with small b usually have larger $n_{\text{jet pair}}$. This leads to the eikonal formalism mentioned above.

Convenient and elegant, the eikonal method reduces the rise of this cross-section, and allows one to enforce the requirement of s-channel unitarity. Here one obtains the total cross-section through the eikonal formula

$$\sigma_{\text{tot}} = 2 \int d^2\vec{b} [1 - e^{-\Im m \chi(b,s)} \cos \Re \chi(b,s)] \quad (20)$$

and one introduces the jet cross-section as the term which drives the rise in the eikonal function. This can be done unambiguously by defining the inelastic cross-section [29] given in the eikonal formulation by

$$\sigma_{\text{inel}} = \int d^2\vec{b} [1 - e^{-2\Im m \chi(b,s)}]. \quad (21)$$

This expression can also be obtained upon summing multiple collisions which are Poisson distributed with an average number $n(b,s) = 2 \Im m \chi(b,s)$. Making the approximation $\Re \chi = 0$, one obtains a very simple expression

$$\sigma_{\text{tot}} = 2 \int d^2\vec{b} [1 - e^{-n(b,s)/2}] \quad (22)$$

To proceed further, one needs to introduce the soft processes, which cannot be described by perturbative QCD. Following the separation shown in Sect.II, one can approximate $n(b, s)$ by introducing a separation between soft and hard processes as:

$$n(b, s) = n_{soft} + n_{hard} = A_{soft}(b)\sigma_{soft}(s) + A_{jet}(b)\sigma_{jet}(s) \quad (23)$$

The separation between hard and soft processes is of course approximate and so is the factorization into energy and transverse dimension dependence. However, it is useful as it allows one to break down the calculation into building blocks, which can be separately understood and put together again later as part of the whole structure.

The procedure of Eq.(20) reduces the fast rise due to the mini-jet cross-section, but the extent to which this softens the rise, is highly dependent on the impact parameter (b)-dependence of $n(b, s)$. The simplest [28] ansatz, which introduces a minimum number of parameters is to assume that the b -dependence is the same for both the soft and the jet component, and further that it is given by the Fourier transform of the electromagnetic form factor of the colliding hadron, $\mathcal{F}(q)$ [28]. Thus for protons and antiprotons one will have

$$A_{soft}(b) = A_{jet}(b) = \frac{1}{(2\pi)^2} \int d^2\vec{b} e^{iq \cdot b} [\mathcal{F}_p(q)]^2 = \frac{1}{(2\pi)^2} \int d^2\vec{b} e^{iq \cdot b} \left[\frac{\nu^2}{q^2 + \nu^2} \right]^4 \quad (24)$$

It is well known [30] that these Eikonal Minijet Models (EMM) are unable to properly reproduce the experimentally observed, complete rise of the cross-section from the beginning up to the asymptotia, without introducing further parameters. As an example, results obtained using Eq.(24) with $\nu = 0.71 \text{ GeV}^2$, and current Glück, Reya and Vogt (GRV) parton densities for the proton [31] to calculate the jet cross-sections in Eq.(17), are shown in FIG. (1). One can see from the figure that a $p_{tmin} \approx 2 \text{ GeV}$ is needed, in order to obtain a numerical value of the total proton cross-sections in the 80 mb range, at Tevatron energies. However, for such $p_{tmin} = 2 \text{ GeV}$, the rise does not begin until \sqrt{s} is already in the 100 GeV region. On the other hand, a smaller value for the regulator p_{tmin} , typically just above 1 GeV, would allow for the beginning of the rise around $20 \div 30 \text{ GeV}$, as the data indicate, but then the cross-section rises too rapidly in comparison with the Tevatron data. In FIG. (1), in all jet cross-sections computed using Eq. (17), we use the strong coupling constant α_s at scale p_t . Even at $p_t = p_{tmin}$, this value for the scale, albeit low, is still in a range where the asymptotic freedom expression is approximately valid. For the low energy behaviour, all the curves shown in FIG. 1 are obtained with the same phenomenological fit as in ref.[1].

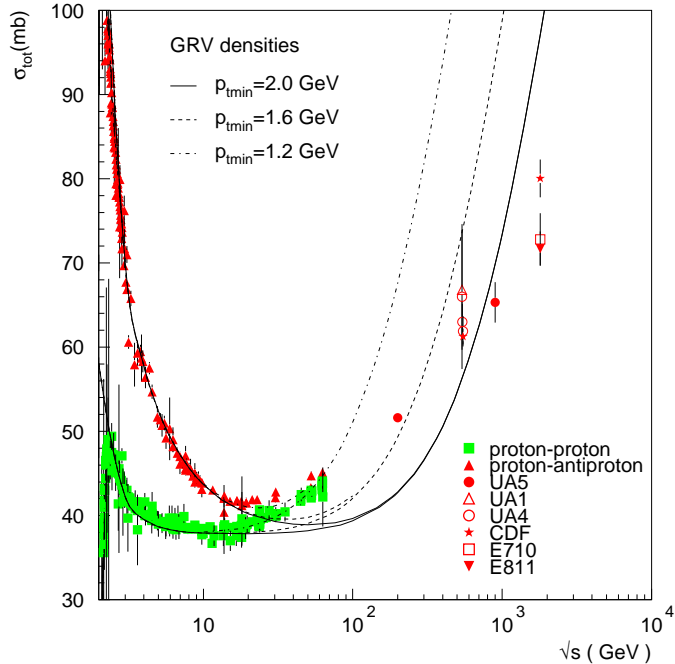


FIG. 1: Comparison between data [8, 32, 33, 34, 35, 36, 37] and the EMM (see text), for different minimum jet transverse momentum.

Different solutions to the above problems have been adopted, including energy dependence of the cut-off parameter p_{tmin} , adding more terms in Eq. (23) or using Eq. (24)[28, 30] with different constants for the low and the high energy part.

We have taken the approach to keep fixed p_{tmin} (for a given beam and target combination) and have an energy dependent overlap function, the energy dependence being modelled by a QCD motivated calculation. The QCD motivated model gives a transverse momentum distribution of the partons which is energy dependent and this, in turn, makes our overlap functions energy dependent. We shall see in the next sections that in this model soft gluon emission has an effect very similar to that of an energy dependent p_{tmin} .

IV. ANALYTICITY REQUIREMENTS ON THE IMPACT PARAMETER DISTRIBUTION

Within the context of QCD, the problem described in the previous section, sometimes referred to as the soft and hard Pomeron problem, can have (at least) two different origins. If complete factorisation between the dependence on the energy \sqrt{s} and the impact parameter

\vec{b} holds, then the abovementioned soft and hard pomeron problem may simply be taken to be indicative that the s -dependence generated through gluon densities, is wrong. On the other hand, given the fact that gluon densities are measured in Deep Inelastic Scattering experiments, one may think of an alternative explanation of the problem. This second, complementary explanation is that not all the s -dependence is due to the jet-cross-sections and there is further energy dependence in the impact parameter distribution. Some general analyticity arguments can be invoked to shed light on the above and thus limit the arbitrariness in the choice of the function $\chi(b, s)$.

Consider the elastic scattering amplitude $T_{el}(s, t)$ (with s and t the usual Mandelstam variables) normalized so that the total cross section is given by

$$\sigma_{tot} = 2 \int d^2\vec{b} [1 - e^{-\Im m \chi(b, s)} \cos \Re \chi(b, s)] = \left(\frac{2}{s}\right) \Im m T_{el}(s, t=0), \quad (25)$$

and the elastic differential cross section by

$$\frac{d^2\sigma_{el}}{d^2\vec{q}} = \frac{1}{4\pi^2 s^2} |T_{el}|^2 \quad (26)$$

Consistently with Eq.(25), one then has

$$T_{el}(s, t) = is \int d^2\vec{b} e^{i\vec{q}\cdot\vec{b}} [1 - e^{i\chi(b, s)}]. \quad (27)$$

For complete absorption, i.e. $\Re \chi = 0$, we get

$$T_{el}(s, t) = is \int d^2\vec{b} e^{i\vec{q}\cdot\vec{b}} [1 - e^{-n(b, s)/2}] \quad (28)$$

Let us briefly examine restrictions imposed on the large b -behaviour of the function $n(b, s)$ by the requirements of analyticity. Consider the Fourier transform of the elastic scattering amplitude,

$$\mathcal{A}(s, b) = \frac{-i}{4\pi s} \int_{-\infty}^0 dt T_{el}(s, t) J_0(b\sqrt{-t}) \quad (29)$$

where $t = -\vec{q}^2$, with \vec{q} the transverse momentum variable. Equivalently, we have

$$T_{el}(s, t) = \pi is \int_0^\infty db^2 J_0(b\sqrt{-t}) \mathcal{A}(s, b) \quad (30)$$

The finite range of hadronic interactions implies that the partial wave expansion converges beyond the physical region, i.e., throughout the Lehmann ellipse. This requires that $T_{el}(s, t)$ be analytic in t up to $t = 4\mu^2$, where μ is the pion mass. For positive t , we continue the

above expression for $\sqrt{-t} = iW$, with W real and positive. In this “unphysical” region, we have

$$T_{el}(s, W^2) = \pi i s \int db^2 I_0(bW) \mathcal{A}(s, b^2) \quad (31)$$

For large b , $I_0(bW) \sim \frac{e^{bW}}{\sqrt{2\pi bW}}$, so that for the integral to converge, one needs

$$|\mathcal{A}(s, b^2)| < e^{-bW_o} \quad \text{with} \quad W_o \simeq 2\mu \quad (32)$$

In the purely absorptive model ($\Re \chi = 0$) thus,

$$1 - e^{-n(b,s)/2} < e^{-bW_o/2}, \quad (33)$$

and we see that $n(b, s)$ must be bounded at least by an exponential. Stronger (but model dependent) constraints arise provided one imposes that the elastic differential cross-section exhibit a “diffraction peak”. That is

$$T_{el}(s, t) \simeq f(s) e^{\hat{b}(s)t} \quad (34)$$

where $\hat{b}(s)$ is the so-called width of the diffraction peak, which has an observed (approximately) logarithmic s dependence. Then, Eq.(29) gives

$$\hat{\mathcal{A}}(s, b) \simeq \frac{if(s)}{4\pi s} \int dq^2 J_0(bq) e^{-\hat{b}(s)q^2} = \frac{if(s)}{2\pi s} \left[\frac{1}{2\hat{b}(s)} \right] e^{-\frac{b^2}{4\hat{b}(s)}}, \quad (35)$$

which requires a Gaussian fall-off of the amplitude in the impact parameter b , with its scale determined by the width of the diffraction peak. In the Regge pole description,

$$\hat{b}(s) \sim \alpha' \ln(s/s_o), \quad f(s) \sim -i\beta(s/s_o)^{1+\epsilon} \quad (36)$$

and

$$\mathcal{A}(s, b) \simeq \frac{\beta(s/s_o)^\epsilon}{4\pi(\alpha' s_o) \ln(s/s_o)} e^{-\frac{b^2}{4\alpha' \ln(s/s_o)}} \quad (37)$$

In the EMM, where the impact parameter distribution is given by the Fourier transform of the proton form factor, a model we refer to as the Form Factor (FF) model, one has

$$n(b, s) = \frac{\nu^2}{96\pi} (\nu b)^3 K_3(\nu b) [\sigma_{soft} + \sigma_{jet}] \quad (38)$$

The modified Bessel functions of the third kind $K_\mu(z)$ are bounded by an exponential at large values of the argument, i.e. $K_\mu(z) \sim \sqrt{\frac{\pi}{2z}} e^{-z} \{1 + \mathcal{O}(\frac{1}{z})\}$. We see then that the EMM in the FF formulation does satisfy the requirements of analyticity in the Lehmann ellipse.

But in the EMM, the observed shrinking of the diffraction peak, corresponding to an energy dependent near gaussian fall off, is not present. Even if one were to introduce an *ad hoc* energy dependence (as is often the practice) instead of the constant scale parameter ν , as in the FF model, still one would be nowhere near the stronger Gaussian decrease at large impact parameter values. This is one reason why the FF model in the eikonal formulation, where jet cross-sections drive the rise, fails to provide an adequate description of the overall energy dependence of the total cross section, without introducing an *ad hoc* modification of the scale parameters.

V. THE SOFT GLUON TRANSVERSE MOMENTUM DISTRIBUTION

The model for the total cross-section presented in this paper is based on the ansatz that QCD provides the main processes at work leading to the observed energy rise of all measured total cross-sections. An important motivation of this model is to make quantitative calculations based on current QCD phenomenology, namely current parton densities with their energy momentum dependence, running behaviour of the coupling constant and soft gluon resummation techniques. Of all these at present soft gluon resummation is the one which presents the toughest technical challenge.

Let us begin by considering the well known function which describes soft gluon emission from a parton-parton pair, namely the soft gluon transverse momentum distribution[38, 39]

$$\frac{d^2 P(\mathbf{K}_\perp)}{d^2 \mathbf{K}_\perp} \equiv \Pi(K_\perp) = \int \frac{d^2 \vec{b}}{(2\pi)^2} e^{i\mathbf{K}_\perp \cdot \mathbf{b} - h(b)} \quad (39)$$

with

$$h(b) = \int d^3 \bar{n}_g(k) [1 - e^{-i\mathbf{k}_\perp \cdot \mathbf{b}}] = \int \frac{d^3 k}{2k_0} \sum_{i,j=colors} |j^{\mu,i}(k) j_{\mu,j}(k)| [1 - e^{-i\mathbf{k}_\perp \cdot \mathbf{b}}] \quad (40)$$

where $d^3 \bar{n}_g(k)$ is the distribution for single gluon emission in a scattering process, and $j^{\mu,i}$ the QCD current responsible for emission. The above expression has been widely used to study the initial state transverse momentum distribution in Drell-Yan processes [40, 41] as well as W-production [42, 43].

In the limit of large $k_\perp b$, one can neglect the exponential term, and the above expression reduces to the well known Sudakov form factor [44], namely

$$h(b) \approx S(b) = \int d^3 \bar{n}_g(k) = \int \frac{d^3 k}{2k_0} \sum_{colors} |j^{\mu,i}(k) j_{\mu,j}(k)| \quad (41)$$

Introducing the running coupling constant and its asymptotic freedom expression, the integration from $1/b$ up to an upper limit Q gives [45]

$$h(b) \approx \frac{4C_F}{(11 - 2n_f/3)} \ln \frac{Q^2}{\Lambda^2} \ln \frac{\ln(Q^2/\Lambda^2)}{\ln(1/b^2\Lambda^2)} \quad (42)$$

For small momenta, however, the above expression is not sufficient to reproduce the observed transverse momentum distribution in various hadronic processes. Thus, an intrinsic transverse momentum (of Drell-Yan pairs or W-boson or partons, depending on the physical process under consideration) has to be introduced. The function now reads [46]

$$h(b) = b^2 p_{\perp int}^2 + S(b) \quad (43)$$

where the intrinsic transverse momentum $p_{\perp int}$ is a constant, of the order of a few 100 MeV, parametrized according to the process under consideration.

In our model, the function in Eq. (40), in addition to describing various hadronic transverse momentum effects, also plays a major role in total cross-section calculations. Our model for the hadronic transverse momentum distributions due to soft gluon resummation differs in two major points from what we have just described. Basically, we focus on the lower and higher limits of integration in Eq. (40). At the lower limit, since this expression refers to soft gluons, we suggest that the correct use of this equation requires to integrate the gluon momentum down to $k_{\perp} = 0$ and avoid the introduction of *ad hoc* quantities such as the intrinsic transverse momentum. At the upper limit, one needs to specify, for each given process, how the maximum soft gluon energy is defined. However, in order to use the above expression for a believable calculation, a number of points need to be clarified, namely

- (i) whether it makes sense to use a parton picture when the gluon momenta become close to zero, with the related question of what is the behaviour of the strong coupling constant α_s when one integrates the gluon momentum down to zero
- (ii) whether the emitting particles are quarks or gluons
- (iii) what are the constraints from kinematics upon the maximum gluon momentum

We shall discuss some of these issues in detail in the sections to come, here we comment briefly on these three points.

Concerning the parton picture, while we use it for the mini-jet contribution to the cross-section, soft gluon Bloch-Nordsieck resummation factors out of the LO basic scattering

process, and is thus independent of parton densities, involving only the momenta and the QCD coupling between soft gluons and the emitting partons. On the other hand when $k_{\perp} \rightarrow 0$, this coupling is not an observable quantity, since it refers to a single soft gluon emission, and one soft gluon is not an observable quantity (only its integrated spectrum is). This has two effects: firstly, one needs to use a non-perturbative expression for the QCD coupling constant, since the momenta are so small, and secondly only the *integral* of moments of α_s will matter. The infrared behaviour of α_s is a matter of speculation. We propose our own model, whose justification rests upon a Regge description and on the Richardson type potential for quarkonium. As will be described in the next sections, a specific form for α_s is chosen, singular in the infrared limit, but integrable [1].

Another issue to address relates to the effect of emission from quarks, valence and sea, and from gluons in the parton processes. In this paper, we only deal with emission from the initial valence quarks, and use the relevant kinematics with their averages. This approximation is justified by the fact that, relative to the emission from the initial valence quarks, emission from the gluon legs is to be considered as emission from internal legs, thus subleading in infrared terms. A complete calculation should of course include also emission from partons other than the valence quarks and hence mostly the low-x gluons. We expect this inclusion may eventually increase the softening effect.

To describe the softening effect quantitatively, one needs to focus on the maximum transverse momentum allowed to single gluon emission. This quantity is energy dependent, as one can easily see using the kinematics of single gluon emission in parton-parton scattering of initial c.m. energy $\sqrt{\hat{s}}$. For the process

$$p_i + p_j \rightarrow gluon + X \quad (44)$$

where $\hat{s} = (p_i + p_j)^2$ and X a final state of given momentum Q, the maximum transverse momentum of the gluon is given by

$$q_{max}(\hat{s}) = \frac{\sqrt{\hat{s}}}{2} \left(1 - \frac{Q^2}{\hat{s}}\right) \quad (45)$$

If we consider the state X to be the final (mini) jet-jet system in the inelastic collision contributing to the cross-section, then the above expression depends on the parton sub-energies and on the final state momentum of the jet-jet system, characterized by a transverse momentum $p_t \geq p_{tmin}$, where p_{tmin} is a scale chosen to separate hard and soft processes. In

principle, for each subprocess of given \hat{s} and Q^2 one should evaluate the function $h(b)$ with the above q_{max} . In practice, we use a value for the maximum transverse momentum allowed to single gluons, which is averaged over the initial and final parton momenta. This is shown more explicitly in [1] and in section VI, but the result is that for a given p_{tmin} cut-off in the minijet distribution, for the valence quarks, the scale q_{max} increases with the c.m. energy of the hadron-hadron system. This can be qualitatively understood by considering that the valence quarks will on the average carry a larger energy and can then shed more soft gluons. Thus, in the picture we present for hadron-hadron collisions, as the overall energy increases, we have more parton-parton collisions for the same p_{tmin} (since the number of low x-gluons increases) but also more energy available to soft gluons both from initial valence quarks and from all the hard partons in general, and thus more of a reduction. It is the balancing of these two effects which we believe to be responsible for the observed softer rise of total cross-sections.

VI. SOFT GLUON EMISSION AND ENERGY DEPENDENCE IN THE IMPACT PARAMETER DISTRIBUTION

In our previous work, we have advocated that a cure for the difficulty in obtaining the early dramatic rise and the softer asymptotic behaviour *simultaneously*, lies within QCD itself; viz., the ubiquitous soft gluon emission accompanying all QCD scatterings which can slow down any abrupt rise in the cross-section. To make this quantitative, we put forward a model for the impact parameter distribution of partons in the hadrons, based on the Fourier transform of the transverse momentum distribution of the soft gluons emitted in the collisions, as described in the Block-Nordsieck (BN) summation procedure. This distribution is energy dependent simply because the maximum energy allowed to each single emitted soft gluon, in turn depends on the energy of the colliding partons. In detail, we have a picture of parton-parton collisions at all admissible subenergy values and with a given transverse momentum due to initial state radiation. In our model the soft gluon resummed transverse momentum distribution of partons in the hadrons and the parton distribution in impact parameter space are Fourier Transforms of each other. In principle, this formalism could be used to obtain impact parameter dependent parton densities, but our aim in the present paper is to obtain a prediction for the total cross-section based on currently used QCD

functions and parameters such as parton densities and Λ_{QCD} . We thus follow our previous proposal [1, 47] to average out the behaviour of partons in their transverse momentum variable and arrive at the following expression

$$2\Im m \chi(b, s) = n(b, s; q_{max}, p_{tmin}) = n_{soft} + n_{jet} = A_{soft}(b)\sigma_{soft} + A_{BN}(b, q_{max})\sigma_{jet} \quad (46)$$

As mentioned earlier, the eikonal formulation provides a natural framework, in which different contributions to the total cross-section can be resolved into their various structural elements: the rise is incorporated in n_{jet} , and the decrease and normalization in n_{soft} . In our previous work, we had parametrized phenomenologically the soft part, and used perturbative QCD for the jet part. In this paper, we study whether soft gluon summation can describe the (experimentally observed) initial decrease in proton-proton scattering.

For this purpose, we write

$$n(b, s) = A_{BN}^{soft}\sigma_{soft} + A_{BN}^{jet}\sigma_{jet} \quad (47)$$

with

$$A_{BN} = \frac{e^{-h(b,s)}}{\int d^2\vec{b} e^{-h(b,s)}} \quad (48)$$

where from Eq.(40) we have

$$h(b, s) = \frac{8}{3\pi} \int_0^{q_{max}} \frac{dk}{k} \alpha_s(k^2) \ln\left(\frac{q_{max} + \sqrt{q_{max}^2 - k^2}}{q_{max} - \sqrt{q_{max}^2 - k^2}}\right) [1 - J_0(kb)] \quad (49)$$

and q_{max} depends on energy and the kinematics of the process[41]. From Eq.(45), the following average expression for q_{max} was proposed in our previous paper [1],

$$M \equiv \langle q_{max}(s) \rangle = \frac{\sqrt{s}}{2} \frac{\sum_{i,j} \int \frac{dx_1}{x_1} f_{i/a}(x_1) \int \frac{dx_2}{x_2} f_{j/b}(x_2) \sqrt{x_1 x_2} \int_{z_{min}}^1 dz (1-z)}{\sum_{i,j} \int \frac{dx_1}{x_1} f_{i/a}(x_1) \int \frac{dx_2}{x_2} f_{j/b}(x_2) \int_{z_{min}}^1 (dz)} \quad (50)$$

with $z_{min} = 4p_{tmin}^2/(sx_1x_2)$ and $f_{i/a}$ the valence quark densities used in the jet cross-section calculation.

M establishes the scale which, on the average, regulates soft gluon emission in the collisions, whereas p_{tmin} provides the scale which characterizes the onset of hard parton-parton scattering. For any parton parton subprocess characterized by a $p_{tmin} \approx 1 \div 2 \text{ GeV}$, M has a logarithmic increase at reasonably low energy and an almost constant behaviour at high energy[1]. The eikonal formalism which we use to describe the total cross-section, incorporates multiple parton parton collisions, accompanied by soft gluon emission from the initial

valence quarks, to leading order. Notice that in this model, we consider emissions only from the external quark legs. In the impulse approximation on which the parton model itself is based, the valence quarks are free, external particles. In this picture, emission of soft gluons from the gluons involved in the hard scattering, is non leading. As the energy increases, more and more hard gluons are emitted but there is also a larger and larger probability of soft gluon emission : the overall effect is a rise of the cross-section, tempered by the soft emission, i.e. the violent mini-jet rise due to semi-hard gluon gluon collisions is tamed by soft gluons. Crucial in this model, are the scale and the behaviour of the strong coupling constant which is present in the integral over the soft gluon spectrum. While in the jet cross-section α_s never plunges into the infrared region, as the scattering partons are by construction semi-hard, in the soft gluon spectrum the opposite is true and a regularization is mandatory. We notice however that here, as in other problems of soft hadron physics[48], what matters most is not the value of $\alpha_s(0)$, but rather its integral. Thus, all that we need to demand, is that α_s be integrable, even if singular [49]. We employ the same phenomenological expression for α_s as used in our previous works, namely

$$\alpha_s(k_\perp) = \frac{12\pi}{(33 - 2N_f)} \frac{p}{\ln[1 + p(\frac{k_\perp}{\Lambda})^{2p}]} \quad (51)$$

Through the above, we were able to reproduce the effect of the phenomenologically introduced intrinsic transverse momentum of hadrons [49], and more recently obtained a very good description of the entire region where the total cross-section rises[1]. This expression for α_s coincides with the usual one-loop expression for large values of k_\perp , while going to a singular limit for small k_\perp . For $p = 1$ this expression corresponds to the Richardson potential[50] used in bound state problems. We see from Eq.(49) that $p = 1$, leads to a divergent integral, and thus cannot be used. Notice that, presently, in the expression for $h(b, s)$, the masses of the emitting particles are put to zero as is usual in perturbative QCD. Thus, for a convergent integral, one requires $p < 1$ and the successful phenomenology indicated in [1] gave $p = 3/4$. However, more study is needed, especially in the full utilization of the Bloch-Nordsieck description, before one can completely define an expression for α_s in the infrared limit. A different possibility is to use a so-called frozen α_s model, for which $\alpha_s(k_\perp^2) = \frac{12\pi}{27 \ln[(k_\perp^2 + a^2 \Lambda^2)/\Lambda^2]}$. These two expressions lead to very different large- b behaviour of the function $n(b, s)$ and, in light of the above discussion concerning the shrinking of the diffraction peak, give quite a different s -dependence in the rising region of the total proton

cross-section.

In [1], we presented analytic approximations for the function $h(b, s)$, obtaining, in the frozen α_s case

$$\lim_{b \rightarrow \infty} h(b, M, \Lambda) = \frac{2c_F \bar{\alpha}_s}{\pi} \left[\frac{1}{4} \ln(2Mb) + 2 \ln(Mb) \ln(a\Lambda b) - \ln^2(a\Lambda b) \right] \quad (52)$$

while for the singular case

$$\begin{aligned} \lim_{b \rightarrow \infty} h(b, M, \Lambda) = & \frac{2c_F \bar{b}}{\pi} (b^2 \Lambda^2)^p \left[\frac{1}{8(1-p)} \left(2 \ln(2Mb) + \frac{1}{1-p} \right) + \right. \\ & \left. \frac{1}{2p} \left(2 \ln(Mb) - \frac{1}{p} \right) \right] \end{aligned}$$

where $M \equiv q_{max}$, $\bar{\alpha}_s = \frac{12\pi}{27 \ln a^2}$, $\bar{b} = \frac{12\pi}{33-2N_f}$. From the above, one can see that an approximately Gaussian limit results in the singular α_s case, but not in the frozen case. Indeed, for the singular case, one has, to the lowest order, $\lim_{b \rightarrow \infty} n(b, s) \sim e^{-b^{2p}}$ i.e. an exact Gaussian limit for the Richardson potential, which corresponds to $p = 1$. This provides the theoretical reason why the entire region where the total cross-section rises is well described by perturbative QCD (jet cross-section) combined with Bloch-Nordsieck summation with a singular α_s in the infrared region. In contrast, neither the frozen α_s model nor, the Form Factor (FF) model are successful there.

Other models for the behaviour of α_s in the infrared region, and studies of the range of variability of the parameters used in Eq(51) will be presented in a forthcoming publication.

VII. THE DECREASE PRIOR TO THE ONSET OF MINI-JETS

Let us now address the question of the lower energy range, prior to the rise, using the same phenomenological and theoretical tools of [1], but abandoning the FF and the frozen α_s models which appear to be inadequate. To study the low energy region, we apply our procedure to proton-proton scattering, where the absence of resonances in the s-channel and leading Regge poles in the t-channel make the picture remarkably simple. At low energies, say before 10 GeV in the proton-proton c.m. system, one observes a very soft decrease, which converts in a rise at an energy of ≈ 15 GeV in the c.m. In this low energy region, we know that gluon densities are still very small, and that (almost) all hard parton-parton scattering takes place among valence quarks : σ_{jet} , as defined through Eq.(17), is a few thousands of the observed σ_{tot} . In the break up of $\Im m \chi$ into a soft and a hard part, the parameter

p_{tmin} separates hard and soft processes, namely for $p_t^{parton} \geq p_{tmin}$ one counts parton-parton processes as part of the jet cross-section, whereas for $p_t^{parton} \leq p_{tmin}$ the process can be counted as part of σ_{soft} . Thus, in this region, we can study the contribution of valence quark scattering without complications from inelastic gluon-gluon collisions.

This region then exhibits the effect of soft gluon emission accompanying gluon exchanges among the valence quarks. At higher energies, these soft interactions still take place and be a substantial part of the cross-section, but they will be shielded by the more dramatic behaviour of the perturbative QCD processes, since as the energy increases, smaller and smaller x-values of the gluon densities are probed and gluon exchanges among gluons start becoming important. Thus we must build a piece of the total cross-section which will survive at high energies, but which does not contribute to the rise. To begin with, we start with a very simple ansatz : that for proton-proton the cross-section σ_{soft} is a constant and the slight decrease comes from the straggling, acollinearity effect of soft gluon emission. We therefore propose, in first instance, the following expression for the average number of soft collisions

$$n_{soft}(b, s) = A_{BN}^{soft} \sigma_0 \quad (53)$$

with A_{BN}^{soft} calculated through Eqs.(48,49) and investigate whether it is possible to find a constant σ_0 and a set of parameters (q_{max}) which can describe pp scattering at low energy. For the soft part, the scale q_{max} corresponds to the maximum energy allowed to soft gluons accompanying scattering with a final parton transverse momentum smaller than p_{tmin} . We are dealing with soft emission (for hard gluons $p_t > p_{tmin}$) and thus we expect q_{max} not to be larger than 10 – 20 % of p_{tmin} . This provides an upper bound for q_{max} for soft processes.

The observation is then that for processes contributing to n_{soft} , a soft gluon will always carry away less energy than for those contributing to n_{hard} . The question is how much lower is the allowed energy. We have proceeded phenomenologically, and found a set of values which, as will be shown in the last section, can give an acceptable description for σ_{pp} before the rise. These values are shown in Table I. Notice that, in order to reduce the number of free parameters, we assume that, at low energies, there is only one value of q_{max} , for both hard and soft processes. However, as the energy increases, the scale characterizing the soft processes does not grow as the one for the hard case, the latter obtained through the kinematics of jet production in a hard parton parton scattering. Here we should expect the scale to start as a slowly increasing function of s , but to become a constant as soon as

TABLE I: Average q_{max} values used for the impact parameter distribution of the soft part of the eikonal

$\sqrt{s}(GeV)$	$q_{max}^{soft}(GeV)$
5.	0.19
6.	0.21
7.	0.22
8.	0.23
9.	0.235
10.	0.24
50.	0.24
100	0.24

hard processes become substantial for $\sqrt{s} \geq 10 \text{ GeV}$. This is necessary, i.e. q_{max} does not increase indefinitely, because as the energy available to soft gluons increases, at a certain point the soft gluons will become hard and then undergo scattering among themselves.

Clearly, a soft scale not larger than 240 MeV is consistent with our understanding of how a proton is structured, if we attribute σ_{soft} , the soft component, as the cross section when the scattering protons (and antiprotons) manifest themselves as a quark and a spin zero diquark. The point is that the slopes for meson and baryon Regge trajectories are justifiably equal only if a baryon is pictured as a quark/diquark system similar to a meson as a quark/antiquark system (thus making the string tensions equal). This mode for the nucleon is soft and diffusely spread over about a fermi[51]. Consequently, the soft- gluon radiation distribution must be limited (lest it break the system). Thus, we estimate for the soft process a $q_{max} \approx 1/(1 \text{ fermi}) = 0.2 \text{ GeV}$.

With the value of p_{tmin} which gave a smooth description of the total cross-section in [1], we plot in FIG. (2) the behaviour of q_{max} as a function of energy, where the upper curve is the one obtained using Eq.(50), for $p_{tmin} = 1.15 \text{ GeV}$, whereas the soft q_{max} starts with the same value obtained for the jet term, and then is made to become a constant when it reaches 240 MeV.

With these values for q_{max} we can now calculate A_{BN} for both the hard and soft terms in the eikonal as a function of the impact parameter b . Both soft and hard A_{BN} are shown

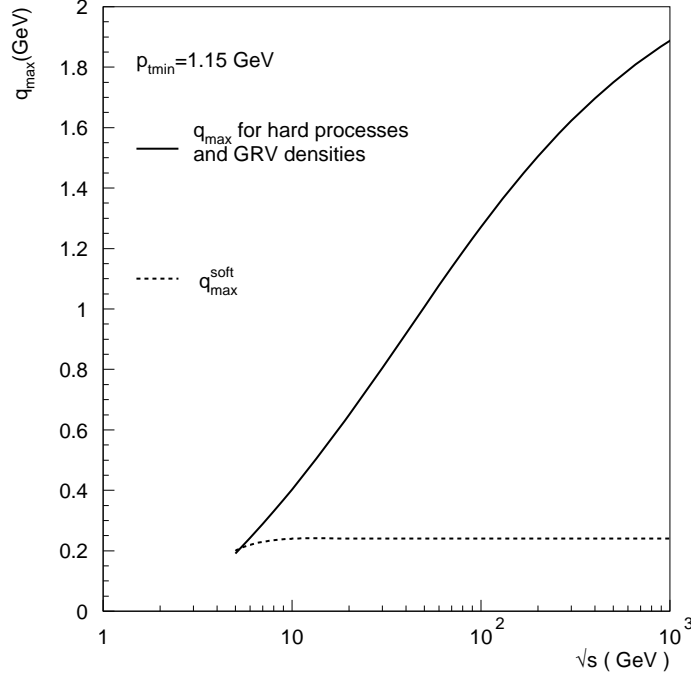


FIG. 2: The maximum transverse momentum allowed (on the average) for single soft gluon emission as a function of the c.m. energy of scattering hadrons.

in Fig. 3 for a set of representative c.m. energies, $\sqrt{s} = 5, 10, 50, 100, 500$ GeV. Notice that A_{BN}^{soft} does not change for $\sqrt{s} \geq 10$ GeV, since q_{\max} remains constant.

VIII. NORMALIZATION AND TOTAL CROSS-SECTIONS

In order to obtain the average number of collisions and thus the total cross-sections, the overall normalization, given by σ_{soft} has to be determined.

As mentioned in the introduction, we are assuming that the entire rise is due to σ_{jet} . For the proton-proton cross-section, one needs only one further parameter for the non perturbative region, namely a constant σ_0 which gives the normalization of the cross-section. As far as the proton-antiproton cross-section is concerned, the rapid decrease after the resonances is interpreted as dual to the Regge trajectory exchange and it should be described by a power $s^{\alpha_R(0)-1} \approx 1/\sqrt{s}$. Neglecting the real part of the eikonal, our model is now complete and reads as follows :

$$\sigma_{\text{tot}} = 2 \int d^2\vec{b} [1 - e^{-\Im m \chi(b,s)}] \quad (54)$$

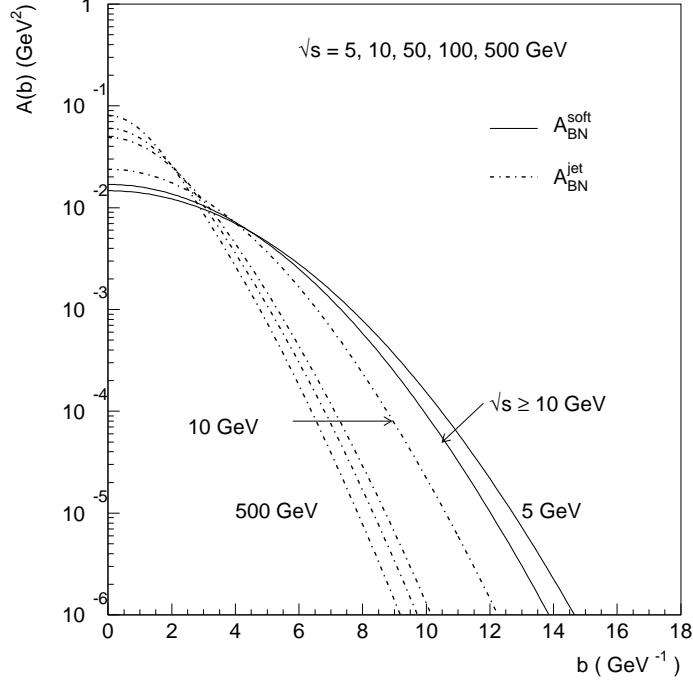


FIG. 3: The impact parameter distribution function calculated for the soft gluon summation model, using the q_{max} values described in the text, for various energy values. Full lines are for the soft term.

with

$$2\Im \chi(b, s) = A_{BN}(b, q_{max}^{soft})\sigma_{soft}^{pp,\bar{p}} + A_{BN}(b, q_{max}^{jet})\sigma_{jet}(s; p_{tmin}) \quad (55)$$

We also have

$$\sigma_{soft}^{pp} = \sigma_0, \quad \sigma_{soft}^{p\bar{p}} = \sigma_0 \left(1 + \frac{2}{\sqrt{s}}\right) \quad (56)$$

We find that, in order to properly reproduce the normalization of the cross-section, we need a value $\sigma_0 = 48 \text{ mb}$, in good agreement with the considerations of Sect.II. We now show in FIG. (4) the average number of collisions as a function of b , distinguishing between hard and soft contributions and using the values of q_{max} shown in FIG. (2). We only show the low energy region, $\sqrt{s} = 10 \div 100 \text{ GeV}$ where the transition between soft and hard processes occurs.

Finally, in FIG. (5) we show the results of our model, putting all the pieces together, for the total cross-section for proton-proton and proton-antiproton collisions. We see that the model gives an overall satisfactory description of the energy behaviour of available data[8].

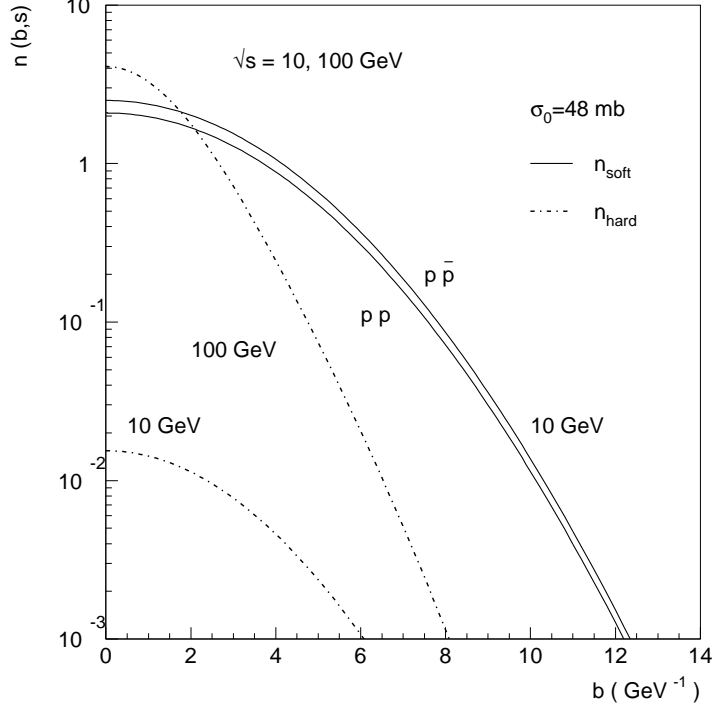


FIG. 4: The average number of collisions at $\sqrt{s} = 10, 100 \text{ GeV}$ is plotted for the soft gluon summation model, using the q_{max} values described in the text. The dot-dashed line corresponds to the jet contribution at $\sqrt{s} = 10, 100 \text{ GeV}$.

IX. ENERGY DEPENDENT $\langle b^2 \rangle$ IN THE BLOCH-NORDSIECK MODEL

The energy dependent transverse overlap function, A_{BN} , discussed in the previous sections, can be used to estimate the energy dependence of the average distance among partons in the transverse space during a scattering process, namely

$$\langle b^2 \rangle = \frac{\int d^2\vec{b} b^2 [A_{BN}(b, q_{max}^{soft}) + A_{BN}(b, q_{max}^{jet})]}{\int d^2\vec{b} [A_{BN}(b, q_{max}^{soft}) + A_{BN}(b, q_{max}^{jet})]} \quad (57)$$

The energy dependence of the average rms distance between partons so defined, is shown in FIG. (6).

One can see from this figure that for the hard part of the eikonal in the Bloch-Nordsieck model (dotted curve), the mean distance between the scattering partons does decrease as the energy increases, thereby increasing the shadowing and taming the rise, as opposed to the Form Factor model where $\langle b^2 \rangle$ is a constant. This is then further reflected in a more modest high energy rise for the BN model as seen in FIG. (5). It is also pleasing to note

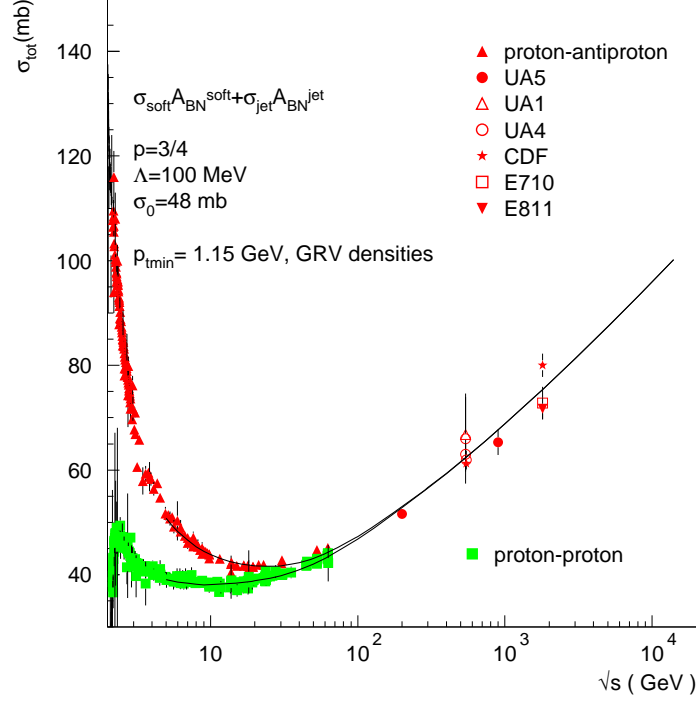


FIG. 5: Comparison of pp and $p\bar{p}$ total cross-section data [8, 32, 33, 34, 35, 36, 37] with results from the Bloch-Nordsieck model described in the text.

the following self consistency. That is, at low values of \sqrt{s} , values of $\langle b^2 \rangle$ are the same for both the soft and hard part of the eikonal in both models, as they must since at low energy the transverse overlap function from the BN model is very similar to that from the FF model.

Observations about the need of a shrinkage in the radius of the proton, have been made in Ref. [52, 53], where multiparticle production in hadron hadron interactions has been studied in detail in an eikonal Monte Carlo model. They find that in the hard multi parton model, a good fit to the CDF data is obtained if the proton radius is decreased [54] by about a factor of 1.7, as compared to the form factor model. Now the observation has been extended to photoproduction as well [55].

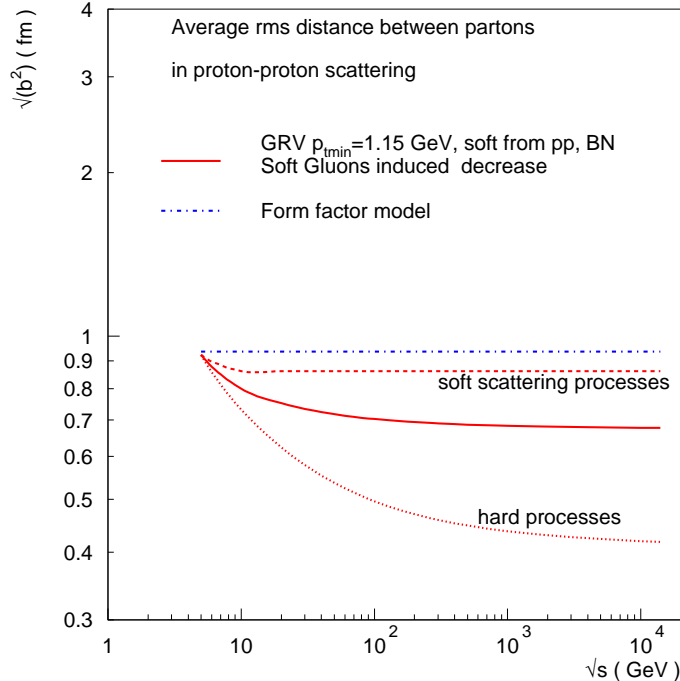


FIG. 6: The average distance between partons in the Bloch Nordsieck model (full line) and the Form Factor model (dot-dashes) is shown as a function of energy. For the former case, the contribution to this distance from the purely soft processes (dashes) with $p_t \leq p_{tmin}$, and purely hard ones (dots) with $p_t \geq p_{tmin}$, is also shown.

X. CONCLUSION

In conclusion, we have shown that standard QCD processes such as hard parton parton scattering and soft gluon emission from valence quarks can account for two salient features of the total proton-proton cross-section, the rise at high energy and the very gentle decrease at low energy. An important characteristic of this treatment is that, as the minijet cross-section rises with energy, soft gluon emission produces an acollinearity of the partons and reduces the probability of collisions. This affects the cross-sections in two ways : at low energy it produces a very soft decrease in σ_{tot}^{pp} and contributes to the faster decrease in $\sigma_{tot}^{p\bar{p}}$, at high energy it tames the rise due to σ_{jet} . It is then possible to have a very small p_{tmin} to see the onset of the rise around $10 \div 20 \text{ GeV}$, without encountering too large a cross-section when the energy climbs into the TeV range and beyond. We stress that the above behaviour

is obtained from leading soft gluon emission from the valence quarks. Subleading emission from internal gluon legs is not considered here. It is to be emphasized that singular α_s appears necessary for this purpose thereby implying that confinement plays a crucial role in the energy dependence of the total cross section.

Further input is needed to understand the scale or the normalization, which plays a dominant role in the early decrease of the proton-antiproton cross-section.

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