

## Resolved photon processes

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**Abstract.** After giving a very brief introduction to the resolved photon processes, I will summarize the latest experimental information from HERA, on resolved photon contribution to large  $p_T$  jet production as well as to direct photon production. I will point out the interesting role that resolved photon processes can play in increasing our understanding of the dynamics of the quarkonium production. I will then discuss the newer information on the parton content of virtual photons as well as the  $k_T$  distribution of the partons in the photon. I will end by giving predictions of an eikonized minijet model for  $\sigma_{\gamma\gamma}^{\text{inel}}$  which crucially uses the experimental measurement of the abovementioned  $k_T$  distribution and comparing them with data.

**Keywords.** Photon structure; hard processes; total cross-sections.

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### 1. Introduction

The study of resolved photon processes [1] i.e., photon induced processes where the partons in the photon take part in the 'hard' scattering process, is useful to improve our understanding of the interactions of high energy photons with other hadrons as well as with each other. In addition, these processes serve as an extra laboratory to study aspects of perturbative and nonperturbative QCD. The status of the study of the 'resolved' processes at HERA and TRISTAN/LEP, in relation to the measurements of  $F_2^\gamma$  at the  $e^+e^-$  colliders like PETRA/PEP/TRISTAN/LEP, is similar to the study of large  $p_T$  jet production at ISR which confirmed the parton structure of proton which had been revealed in the DIS experiments at SLAC. The first experimental verification of the 'resolved' processes came at TRISTAN [2]. The higher energy and statistics at HERA [3] have since then provided a lot of interesting information about photons and their interactions, in the study of these processes [4]. In this talk I summarize the current experimental information on  $F_2^\gamma$  and highlight interesting features of the new analysis of jet production in resolved processes at HERA. I discuss the role that the study of resolved contributions can play in clarifying our understanding of the dynamics of the charmonium production. I will then proceed to discuss new issues such as resolved processes with photons of nonvanishing virtuality as well as the experimental determination of the transverse momentum ( $k_T$ ) distribution of partons in the photon. Lastly I will talk about predictions for  $\sigma_{\gamma\gamma}^{\text{inel}}$  of an eikonized minijet model, which uses the above measurement.

2. Photonic parton densities

As is well known the photon structure function  $F_2^\gamma$  describes the ‘strong’ interactions that the  $\gamma$  develops through  $\gamma \rightarrow q\bar{q}$  vertex. A large amount of data on  $F_2^\gamma$  has been accumulated in the ‘hard’  $e\gamma$  DIS scattering in the single tag  $e^+e^-$  experiments since the first measurements of  $F_2^\gamma$  by PLUTO [5]. The predictions of perturbative QCD have two basic features:

- $F_2^\gamma$  rises linearly with  $\log Q^2$  and the scaling violations are of an entirely different nature as compared to the case of the proton structure function.
- The parton densities in a photon  $\vec{q}_\gamma(x_\gamma)$  peak at large  $x_\gamma$  unlike the parton densities in a proton.

Both these features have been experimentally verified. Figure 1, taken from ref. [6], shows the expected linear rise of integrated  $F_2^\gamma$  (a factor of  $\alpha_{em}$  has been taken out) with  $\log Q^2$ . A few more points can be made.

1. The measurements of  $F_2^\gamma$ , along with certain theoretical assumptions about the charm quark contribution, yield directly the  $q_\gamma(x_\gamma, Q^2)$  for light quarks over a wide range of  $x_\gamma$ :  $0.1 < x_\gamma < 1.0$ , and up to  $Q^2 = 200 \text{ GeV}^2$ .
2. The gluon distribution  $g_\gamma(x_\gamma, Q^2)$  is obtained from the DIS measurements only indirectly, using QCD evolution equations.
3. The charm quark distribution in photon, is correlated with the gluon distribution  $g_\gamma(x_\gamma, Q^2)$  and is not at all well determined. LEP2 offers a good chance for its determination.

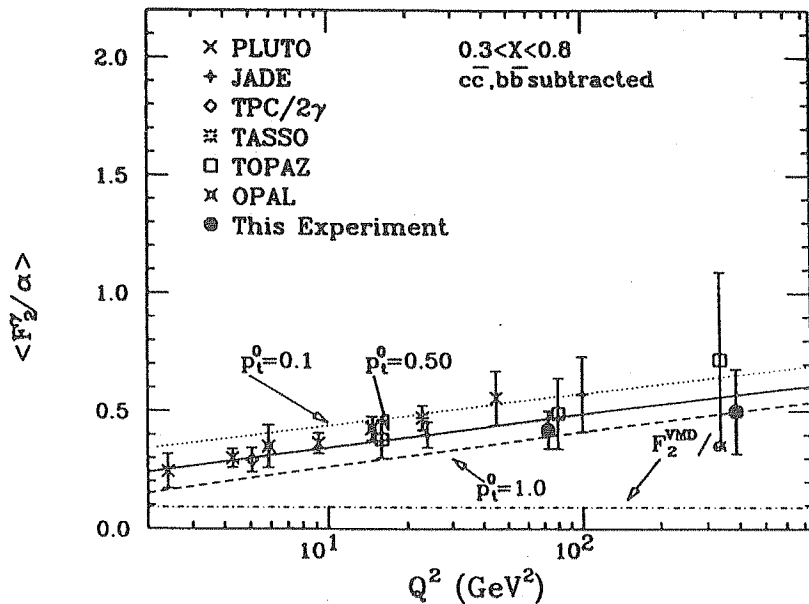


Figure 1.  $x$  integrated  $F_2^\gamma$  data as a function of  $Q^2$ .

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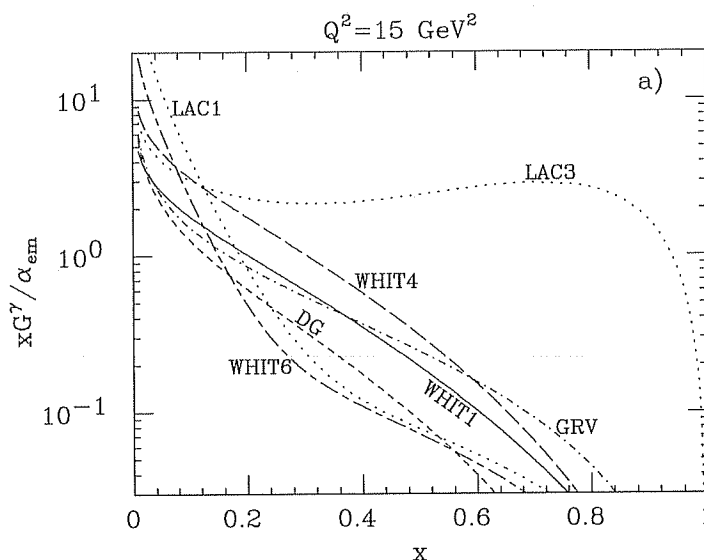


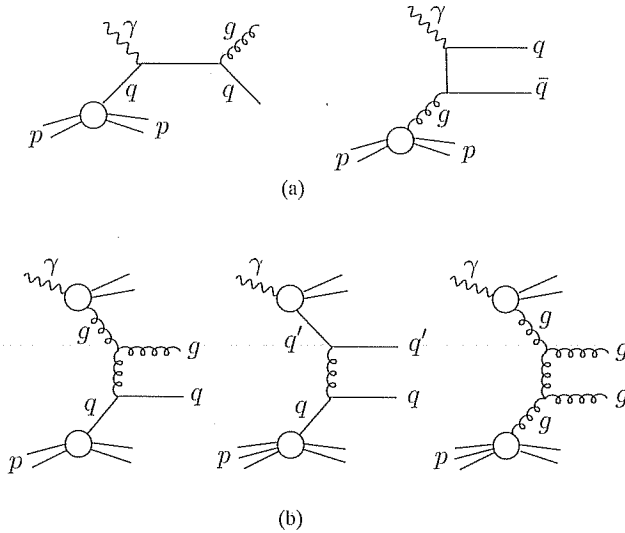
Figure 2. Gluon distributions in various parametrizations.

4. As opposed to 1984, when there was only one parametrization [7] for  $\vec{q}_\gamma(x_\gamma)$ , obtained using fits to the early DIS data, now at least twenty different parametrizations [8] (including LO and NLO fits) are available. These are not just different fits but involve different assumptions about the nonperturbative part of the  $F_2^\gamma$  and hence an accurate determination of  $\vec{q}_\gamma(x_\gamma)$  will test these assumptions as well.

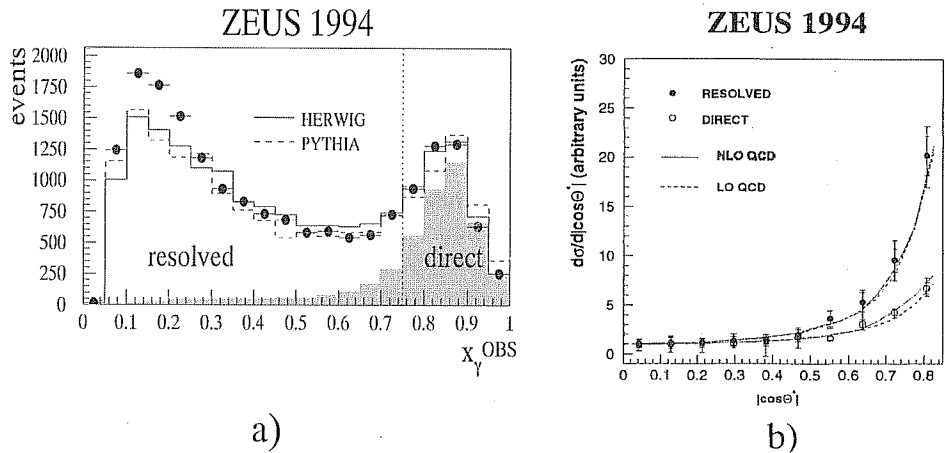
As a result of (1) above the  $q_\gamma(x_\gamma, Q^2)$  in all the parametrizations are very similar. It is not so for  $g_\gamma(x_\gamma, Q^2)$ . This can be seen from figure 2. Hence, it is important to be able to obtain information about  $g_\gamma(x_\gamma, Q^2)$  from alternative sources. Production of high  $p_T$  jets in effective  $\gamma P$  and  $\gamma\gamma$  collisions, which can be studied in photoproduction experiments at  $eP$  collider (HERA) and in no-tag two-photon experiments in  $e^+e^-$  colliders (TRISTAN, LEP), was found to be dominated by the ‘resolved’ contributions [9, 10] and hence a good place to study the gluon density in the photon.

### 3. Jet production in resolved processes

Although the original experimental observation [2] of the resolved contribution was in jet production in  $e^+e^-$  collisions, the more recent experimental and theoretical development has happened in the study of jet production in resolved processes in  $eP$  collisions. The two diagrams in figure 3, give the ‘direct’ and ‘resolved’ contributions respectively. In the ‘direct’ (‘resolved’) processes all (only a fraction) of the photon energy is available for the hard scattering process. Hence the fraction of the photon energy carried by the jets,  $x_\gamma$ , is  $1 (< 1)$  for these two processes. This can separate the two processes if  $x_\gamma$  can be reconstructed experimentally. Note here that a neat separation between the two in terms of the diagrams is possible *only* at the LO level. At NLO one has to be more careful in the separation between the two in the theoretical calculations. However, operationally the



**Figure 3.** ‘Direct’ and ‘resolved’ contributions to jet production in  $\gamma P$  ( $eP$ ) collisions.



**Figure 4.** Experimental separation between ‘direct’ and ‘resolved’ contributions to inclusive jet production at HERA.

difference between the two is clear: only for the resolved processes does one have a remnant jet activity in the direction of the photon (i.e. the electron). This, along with the other two original predictions [9], viz., that the ‘hard’ resolved jets will be closer to the proton direction than for the ‘direct’ process and that the former will dominate upto large values of  $p_T$ , were qualitatively confirmed by the earliest measurements. The dijet angular distributions expected in the LO direct(resolved) contribution are  $|1 - \cos \theta^*|^{-1(-2)}$  corresponding to the spin 1/2(1) exchange in the subprocess responsible for the jet production. The  $x_\gamma^{\text{OBS}}$  in figure 4(a), taken from ref. [11], is essentially the fraction of the

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photon's momentum that appears in the high- $p_T$  jets. As is seen from figure 4(b), the angular distributions obtained by separating between the resolved and direct processes based on  $x_\gamma^{\text{OBS}} < 0.75$  and  $x_\gamma^{\text{OBS}} > 0.75$  respectively, show this difference quite clearly.

As a matter of fact early study of the differential distributions of the inclusive jet cross-sections, showed that photonic parton densities as given by the LAC3 parametrisation, which was already disfavoured by the  $e^+e^-$  jet data, was quite strongly ruled out, despite the uncertainties in the analysis which we shall mention next.

Since the early analysis, newer developments have been theoretical NLO calculations [12] and the incorporation of these in the analysis of the newer 1994/1995 HERA data [4]. Even using LO analysis some general observations can be made. It is seen [1] that the dominance of the inclusive jet spectrum by the resolved contributions is somewhat reduced from the naive expectations, by stricter jet cuts; however, sensitivity to the  $g_\gamma(x_\gamma, Q^2)$  can be increased by rapidity cuts. It is therefore indeed possible [1] to try and use the data on inclusive jet photoproduction and dijet photoproduction to determine the  $\vec{q}_\gamma(x_\gamma)$ . However, inspite of the clear qualitative evidence for the resolved processes provided by the data, a detailed analysis had showed certain discrepancies between the data and theoretical predictions at low  $E_T$  and high  $\eta$  of the jet. It was realized that part of this is caused by the 'underlying' event which can also contribute to the transverse energy activity around a 'hard' jet. In the 'resolved' events the remnant photon jet causes changes in the hadronic activity of the event as compared to the direct events. It was demonstrated [13] that choice of a cone angle of 0.7 rather than the usual 1.0, is more appropriate for

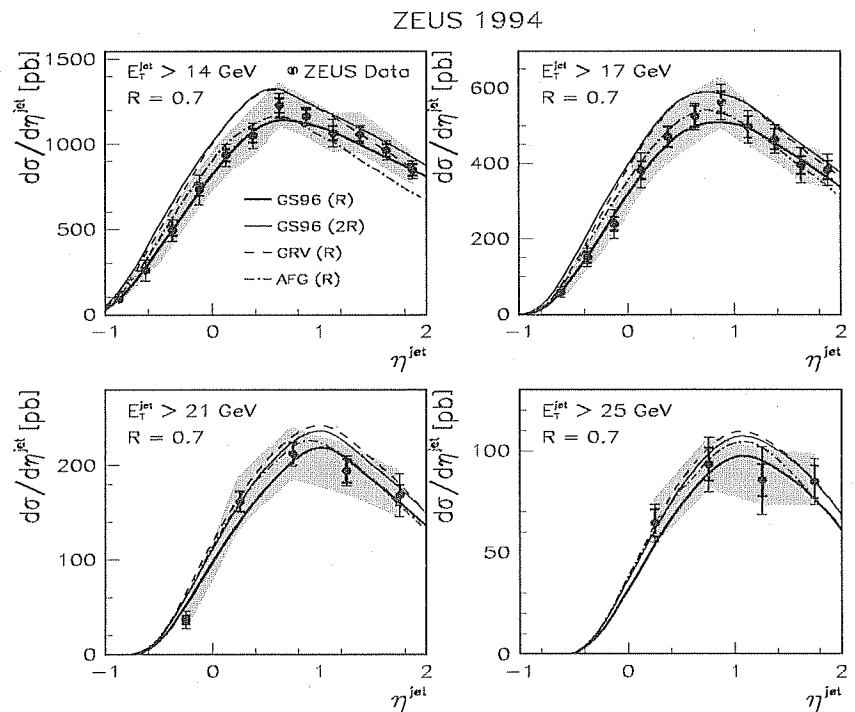


Figure 5.  $E_T$  integrated rapidity distributions for inclusive jet production.

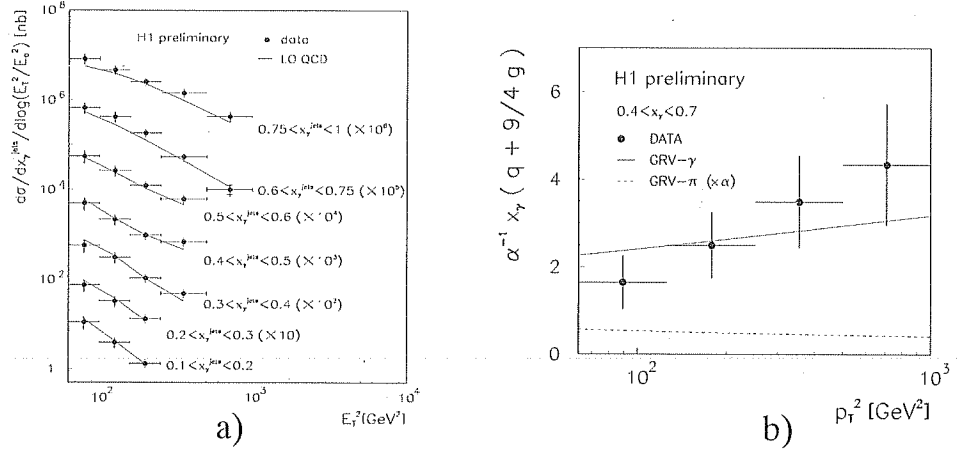


Figure 6. Dijet cross-sections in  $eP$  collisions and extraction of an effective parton distribution in the photon.

photoproduction of jets in the resolved processes. Figure 5, taken from [13], shows that indeed the data are well described by the NLO calculations for all  $E_T$  and  $\eta$ . In general, the data are better described by a MC which includes multiple interactions (MI) [14]. The H1 data [15] also show a direct correlation between the energy flow outside the jet and  $x_\gamma^{\text{OBS}}$ , again demonstrating the importance of an underlying event and need to understand it completely to analyse the dijet and inclusive jet data.

The dijet data have been used to extract an effective photon structure function given by  $(g_\gamma(x_\gamma, Q^2) + \bar{q}_\gamma(x_\gamma, Q^2) + 4/9g_\gamma(x_\gamma, Q^2))$ , by H1 in analogy to the  $pp/p\bar{p}$  case. The dijet data are shown in figure 6, taken from [16]. The effective parton densities in the photon shown in figure 6(b) show a striking linear rise with  $\log Q^2$ . However, the present data and analyses do not distinguish between the recent NLO parametrizations like GRV, SAS and GS [8].

The above discussion shows that the analysis of the inclusive jet cross-sections and dijet data has now come of age. A combined analysis with the jet data from  $e^+e^-$  experiments, both TRISTAN and LEP-2, along with the HERA data, are likely to increase our understanding of the photonic parton densities. Our increased knowledge of the protonic parton densities from HERA will play a very important role here.

#### 4. Direct photon and charmonium production

The gluon distribution of a target is probed effectively if one can isolate hard processes where the gluon contributes dominantly. Both at HERA and at the  $e^+e^-$  colliders, the direct photon and heavy quark production are subjects of experimental investigations to get more information about  $g_\gamma(x_\gamma, Q^2)$ . The event rates for these are much smaller than the for jet production. However, the extra information they can provide makes the study well worth it. For direct photon studies, complete NLO calculation is available [17]. In the case of resolved processes there are additional technical issues involved as compared to the corresponding  $pp/p\bar{p}$  case, due to the additional contributions coming from the

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fragmentation of a parton into a photon. All of these issues have been satisfactorily addressed in the theoretical calculations. The experimental investigations have just started [18] and the experiment has demonstrated the ability of picking out the direct photon events. Unlike the case of jet production, here the resolved contribution, though significant, is not dominant. The current sample shows clearly only the existence of the direct contribution. But analysis of more data certainly holds promise.

Heavy quark and quarkonium production is mostly dominated by the direct contributions and the use of these to determine  $\vec{q}_\gamma(x_\gamma)$  requires (1) a good knowledge of the  $g_p(x_p, Q^2)$  and (2) good set of cuts to separate the two from each other. Actually, since heavy quark production at HERA is used as a probe for determination of  $g_p(x_p, Q^2)$  the latter are essential for that purpose as well. The same kinematic feature which makes the jets in resolved processes come out closer to the proton direction, can be used to isolate the two by using simple rapidity cuts [1].

In case of quarkonium production the second use of study of resolved processes is manifest. The dynamics of quarkonium production and the colour octet contribution to the production, has been a subject of a lot of attention in the past few years. The size of the colour octet matrix elements as extracted from the HERA data and from the Tevatron data are in disagreement. One possible explanation is the apparent breakdown of some of the basic assumptions of the NRQCD calculations in the region of large inelasticity  $z$ . As has been discussed elsewhere in the proceedings [19], it has been shown that the colour octet resolved contribution to  $J/\psi$  production, dominates at small inelasticity of  $J/\psi$ , where the uncertainty in the NRQCD calculations is small. Hence studying the  $J/\psi$  production at small  $z$  at HERA will help pin down the colour octet matrix elements and shed light on the discrepancy and might even rule out [20] the NRQCD model of  $J/\psi$  production. This is where one can see the use of resolved processes as a laboratory of QCD.

Study of inclusive charm production at HERA can also be used to obtain an effective LO measurement of the charm density  $c_\gamma(x_\gamma, Q^2)$  [21]. There also one can use effectively the same rapidity cuts as mentioned above to separate the much more abundant direct contribution.

Thus the field of better determination of gluon and charm quark density in the photon using the above mentioned hard processes is just opening up. Theoretically the gluon and the charm densities in the photon are correlated. The perturbative part of  $F_2^{\gamma,c}$  is easily calculable. It is the nonperturbative part, which can not be calculated, that is closely correlated to the  $g_\gamma(x_\gamma, Q^2)$ . The upcoming analysis of the two photon data will offer a way to measure the charm contribution to  $F_2^\gamma$  and hence shed more light on the issue.

### 5. Internal $k_T$ distribution of photonic partons and resolved processes involving virtual photons.

In a hadron like proton the internal transverse momentum distribution of the partons is taken to be normally Gaussian. However, in the case of photon since the parton content can in principle be traced back to a hard  $\gamma q\bar{q}$  vertex, the transverse momentum distribution is expected to be different. It is expected to be  $f(k_T) \propto 1/(k_T^2 + k_0^2)$  [22, 23]. ZEUS performed a determination of this distribution by essentially measuring the transverse momentum of the remnant jet (i.e the jet in the direction of the electron) in a jet event.

This measures the  $k_T$  distribution of the parton which participated in the hard scattering process. The measurement by ZEUS [24] gave a distribution in agreement with the above expectation and yielded a measurement of  $k_0 = 0.66 \pm 0.22 \text{ GeV}$ . This is indeed a preliminary measurement. Inclusion of such intrinsic  $k_T$  has also improved the agreement with data on dijet rates and transverse momentum distribution in  $e\gamma$  scattering in the analysis by OPAL. In the next section I will discuss an eikonalised model prediction for  $\sigma_{\gamma\gamma}^{\text{inel}}$  which makes use of this measurement.

The parton densities in virtual photon can be completely computed in perturbative QCD when the virtuality of the photon ( $P^2$ ) is much higher than  $\Lambda_{\text{QCD}}^2$ . For quasi real photons ( $Q^2 \simeq 0$ ) the parton densities can not be calculated from first principles, but those are the ones that are parametrised from data. Since, in most photon-induced processes, the contribution to the resolved cross-sections from virtual photons will be significant only for low virtuality,  $P^2 \sim \Lambda_{\text{QCD}}^2$ , it is essential to have a model for these [25–27]. A measurement of jet production with virtual photons with small but *nonvanishing* virtuality, is now available from H1 [28] and ZEUS [29] both. The current results are well described by some of the available parametrisations [25, 26]. However, feasibility of the measurement is demonstrated and we can expect more results from the analyses.

### 6. $k_T$ distribution of photonic partons and model for total inelastic cross-sections for $\gamma\gamma$ and $\gamma P$ processes

The resolved contribution to hard processes, like jet production (say), increases strongly with the energy of the photons. As a matter of fact it was observed [30] that the minijet cross-section,

$$\sigma_{\text{minijet}} \equiv \sigma(\gamma\gamma \rightarrow \text{jets})|_{p_{T\text{min}}}^{\sqrt{s}} \equiv \int_{p_{T\text{min}}} \frac{d\sigma}{dp_T}(\gamma\gamma \rightarrow \text{jets}) \quad (1)$$

is completely dominated by the resolved contribution and rises like a power of  $s$ , where  $s$  is the square of the centre of mass energy. This combined with the phenomenon of beamstrahlung can cause very large backgrounds at the future linear colliders [31]. To assess this correctly one has to see how much of this rise gets reflected in the rise of total inelastic cross-sections. This is also another example where investigations in issues involving resolved processes, function as a theoretical laboratory. To that end here I will describe a model for the calculation of total inelastic cross-section in minijet model, trying to make a special effort to determine the various parameters/inputs from the data involving resolved photons rather than as model assumptions.

The ‘minijet’ cross-section has to be eikonalized so that unitarity is not violated. In general for photon induced processes, the inelastic cross-section obtained by eikonalization (and hence unitarization) of the minijet cross-section is given by

$$\sigma_{ab}^{\text{inel}} = P_{ab}^{\text{had}} \int d^2\vec{b} [1 - e^{-n(b,s)}] \quad (2)$$

with the average number of collisions at a given impact parameter  $\vec{b}$  given by

$$n(b, s) = A_{ab}(b) \left( \sigma_{ab}^{\text{soft}} + \frac{1}{P_{ab}^{\text{had}}} \sigma_{ab}^{\text{jet}} \right), \quad (3)$$



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where  $P_{ab}^{\text{had}}$  is the probability that the colliding particles  $a, b$  are both in a hadronic state,  $A_{ab}(b)$  describes the transverse overlap of the partons in the two projectiles normalised to 1,  $\sigma_{ab}^{\text{soft}}$  is the non-perturbative part of the cross-section while  $\sigma_{ab}^{\text{jet}}$  is the hard part of the cross-section (of order  $\alpha_{\text{em}}$  or  $\alpha_{\text{em}}^2$  for  $\gamma p$  and  $\gamma\gamma$  respectively). Notice that, in the above definitions,  $\sigma_{\text{soft}}$  is a cross-section of hadronic size since the factor  $P_{ab}^{\text{had}}$  has already been factored out. We have,

$$P_{\gamma p}^{\text{had}} = P_{\gamma}^{\text{had}} \equiv P_{\text{had}} \quad \text{and} \quad P_{\gamma\gamma}^{\text{had}} \approx (P_{\gamma}^{\text{had}})^2. \quad (4)$$

The overlap function  $A_{ab}(b)$  is then,

$$A_{ab}(b) = \frac{1}{(2\pi)^2} \int d^2\vec{q} \mathcal{F}_a(q) \mathcal{F}_b(q) e^{i\vec{q}\cdot\vec{b}}, \quad (5)$$

where  $\mathcal{F}$  is the Fourier transform of the  $b$ -distribution of partons in the colliding particles. Normally,  $A_{ab}$  is obtained using for  $\mathcal{F}$  the electromagnetic form factors of the colliding hadrons. In general, for photons people have normally used the form factor for a pion. We [23] take a slightly different approach and calculate the ‘ $b$ -distribution’ of the partons by taking the Fourier transform of the transverse momentum distribution of the partons, which in the case of the photons is expected to be, at least for the perturbative part,

$$f(k_T) = \frac{C}{(k_T^2 + k_0^2)}. \quad (6)$$

As said in the earlier section  $k_0$  has actually been measured by ZEUS to be  $0.66 \pm 0.22$  GeV. It turns out that the form of  $A_{\gamma\gamma}$  with this transverse momentum ansatz and that for the pion form factor ansatz, are the same, differing only in the value of the parameter  $k_0$  which is 0.735 GeV for the  $\pi$  form factor case. Thus one can assess the effect of changing the ansatz for the  $A_{ab}$  for photons by simply changing the value of  $k_0$ .

For the soft part of the cross-section we use a parametrization,

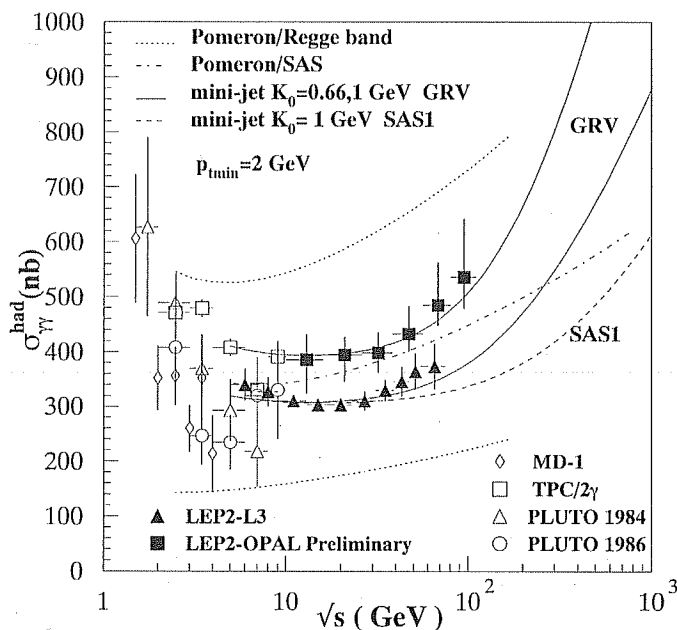
$$\sigma_{\gamma p}^{\text{soft}} = \sigma^0 + \frac{A}{\sqrt{s}} + \frac{B}{s}. \quad (7)$$

we then calculate values for  $\sigma^0, A$  and  $B$  from a best fit [32] to the low energy photoproduction data, starting with the quark parton model (QPM) ansatz  $\sigma_{\gamma p}^0 \approx (2/3)\sigma_{pp}^0$  and the form factor ansatz for the  $A_{\gamma p}$ . The best fit value for  $p_{T \text{ min}}$  that we get is 2 GeV. It might be possible to improve quality of the fit by using a energy dependent  $P_{\text{had}}$ , but this needs to be investigated further. The value of 2 GeV is also comparable to the value 1.6 GeV obtained [33] from a fit to the description of minimum bias events in  $pp/\bar{p}p$  collisions.

For  $\gamma\gamma$  collisions, we repeat the QPM suggestion and propose

$$\sigma_{\gamma\gamma}^{\text{soft}} = \frac{2}{3} \sigma_{\gamma p}^{\text{soft}}. \quad (8)$$

We now apply the criteria and parameter set used in  $\gamma p$  collisions to the case of photon-photon collisions, i.e.  $P_{\gamma}^{\text{had}} = 1/204$ ,  $p_{T \text{ min}} = 2$  GeV,  $A(b)$  from the transverse momentum ansatz with the value  $k_0 = 0.66$  GeV. The results of our calculation are shown in figure 7. The highest of the two full lines corresponds exactly to the same parameter set used in the photoproduction case and appears to be in good agreement with the preliminary results



**Figure 7.** Total inelastic photon-photon cross-section in the eikonized mini-jet model with  $p_{T\text{min}} = 2$  GeV, compared with data and Regge/Pomeron parametrization (see [23] for details). The two lower mini-jet curves correspond to  $k_0 = 1$  GeV with GRV and SAS1 densities. The highest one is for GRV densities and  $k_0 = 0.66$  GeV.

from the OPAL [34] collaboration, whereas the L3 results [35], everything else being the same, would favour a higher  $k_0$  value.

The following can be noticed here from the newer data and model calculation:

- The data on  $\sigma_{\gamma\gamma}^{\text{inel}}$  rise faster with the energy than the predictions of the Regge-Pomeron ideas and the rise is consistent with predictions of the eikonized minijet models where the parameters are fixed by fits to the photoproduction case.
- These calculations use also more up-to-date photonic parton densities and the parameters of the eikonized minijet model are related here to measured properties of photon induced reactions.

### 7. Conclusion

Thus we see that the HERA data have really lived up to the expectations [9] that the photoproduction data, dominated by the resolved processes, would yield information on the photon structure. The study of two photon processes at the  $e^+e^-$  collider LEP2, and further analysis of heavy flavour (closed and open) production, direct photon production at HERA is sure to give more information on the parton content of the photon as well as on the way high energy photons interact with each other and matter. In addition the resolved processes are proving their potential as a laboratory for QCD.

### Acknowledgements

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