Noise Assisted Directed Motion at the Molecular Level – 2

Mangal C Mahato and A M Jayannavar

Mangal C Mahato is at Department of Physics, Guru Ghasidas University, Bilaspur.

A M Jayannavar is at Institute of Physics, Bhubaneswar.

The term noise is used to describe fluctuations about the mean deterministic stationary value of a physical quantity. It is now being increasingly realised that noise is an important ingredient to bring order in dynamical processes. Though it appears counterintuitive, noise seems to help in directing transport processes in biological systems at the molecular level. In Part 2 of the article, we discuss some more examples of noise assisted directed motion.

Inhomogeneous Ratchets

Frictional forces offer resistance to motion. The larger the coefficient of friction the larger becomes the resistance to motion. Therefore, if the medium is inhomogeneous the resistance to motion will vary in space accordingly. If the coefficient of friction varies periodically (such systems can be fabricated or found to exist in Nature, mostly in biology) so will the force of resistance. Also, as we see from our prototype potential of Figure 1(Part 1)[1], the force acting on the particle, derived from the potential function, varies periodically in space. Is it possible to combine these two ingredients together to obtain macroscopic current? Yes, it is possible, though it requires the presence of external noise. The noise need not be correlated as is required for the rocking and flashing ratchets. Also, the periodic potential function need not be spatially asymmetric in order to obtain macroscopic current in this minimal model.

[1] Part 1. Noise Assisted Directed Motion at the Molecular Level - 1, *Resonance*, Vol.8, No.7, pp. 33-45, 2003.

Keywords
Noise fluctuation, Brownian motion.

It has been shown that if the periodicity of the coefficient of friction and the potential function are the same but are shifted by a phase difference, ϕ other than 0 and π

macroscopic current is obtained. Under this condition, a particle moving in the medium, in the presence of external noise, will feel as though it is moving in a periodic potential ¯eld in combination with a constant force. For a plausible mechanism behind the macroscopic current, see Box 1. The direction of macroscopic current depends on the phase di®erence Á. As mentioned above, the original periodic potential need not be symmetric. The asymmetry of the potential, however, provides an

Box 1.

In the presence of external (parametric) noise the particle on an average absorbs energy from the noise source (without having to satisfy the condition of °uctuation-dissipation theorem). The particle spends larger time in the region of space where the friction is higher and hence the energy absorption from the noise source is higher in these regions. Therefore, the particle in the high friction regions feels e®ectively higher temperatures. Thus, in the presence of external (parametric) noise the problem of motion of a particle in a space dependent friction becomes equivalent to the problem in a space dependent temperature. Let us consider Figure 1 as an illustration of a special case of the equivalent problem. Let the darkened regions represent the regions of higher temperature. A particle in the darkened regions on the average gains more energy as compared to other regions and thus ¯nds it easier to cross the peak of the potential and go over to the left side well, whereas for a particle on the left side of the peak it is not as easy to cross over to the right side well. Hence a current in the left direction is assured. This follows as a corollary to the Landauer's blow-torch theorem that the notion of stability changes dramatically in the presence of temperature inhomogeneities. In such cases the notion of local stability, valid in equilibrium systems, does not hold.

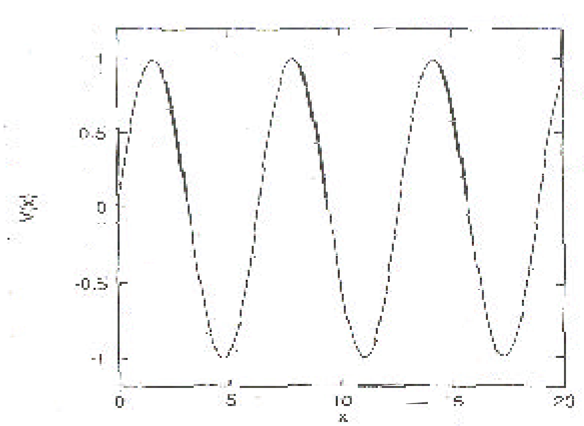

Figure 1. Shows a model potential periodic in space. The temperature, however, is nonuniform in space. The darkened parts show regions of higher temperature.

The macroscopic current can also be obtained in a symmetric potential system in a homogeneous medium but the system needs to be driven by a zero average but temporally asymmetric periodic field.

extra control parameter. A proper choice of asymmetry helps in reversing the direction of the macroscopic current as a function of the strength of the fluctuating forces. Here too one can think of many variants of the model. The macroscopic current can also be obtained in a symmetric potential system in a homogeneous medium but the system needs to be driven by a zero average but temporally asymmetric periodic field.

Driven Ratchets

Some eucaryotic cells (for example, sperm cells) have long (macroscopically) uniform (but microscopically structurally periodic polymeric) tails (just like microtubules), in some cases, called flagella. They swim in viscous fluids and are helped by flagellar flappings. Each flapping consists of two half-cyclic strokes: power and reverse. To complete the power stroke it takes less time than the reverse stroke, that is, one is swift and the other gentler. Both taken together form a period (of flapping). The transverse flappings in the viscous medium (that is, the nonuniform relative motion between the flagellum and the viscous medium) help propel the cell (as a whole) longitudinally ahead. Here effectively the (macroscopically stationary) medium in contact with the flagellum exerts the necessary force on the flagellum and hence on the cell. (If the head of the cell (swimmer) were somehow pinned in space would the fluid acquire a macroscopic motion?) Consider a similar situation but keep the flagellum (or a microtubule) stationary and let a particle loosely in contact with it experience a nonuniform time varying force in conjunction with the viscous medium. The stationary flagellum (microtubule) offers a periodic potential. Apply the oscillating force (on the particle) along the length of the microtubule. The force is such that it changes (per period) from its maximum ($+|F|$, say) value to the minimum ($-|F|$) in a shorter time than the time it takes to change from the minimum value to the maximum (such that the time integral of

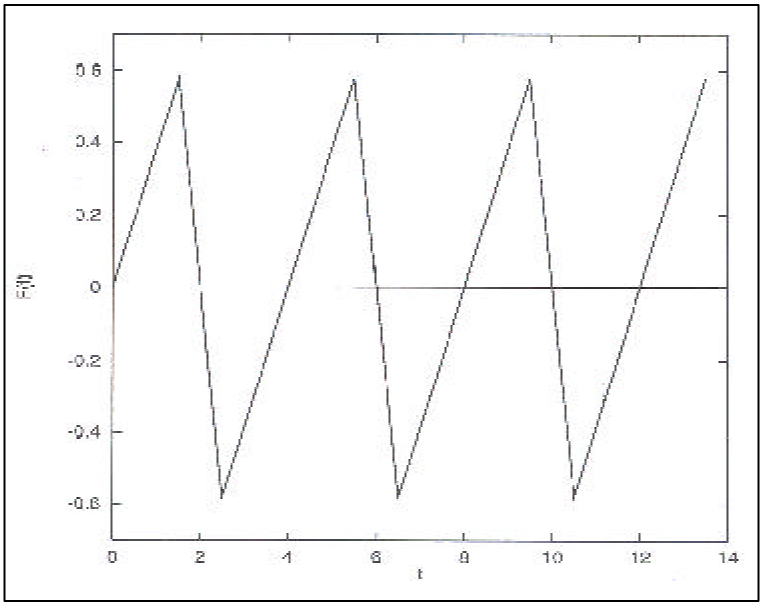

Figure 2. Temporally periodic force acting on a particle. The force is asymmetric in time as seen from the slopes in each period average force per period is zero.

the force over a period is zero). Will the particle have a macroscopic motion along the length of the stationary microtubule?

Let us concretize the problem. Consider a particle in a symmetric periodic potential. The particle is in thermal contact with the medium (Gaussian white noise). It is subjected to a temporally periodic but asymmetric (zero average) external forcing (Figure 2). Will the particle have a net unidirectional motion? Yes, indeed, the particle shows macroscopic motion. Also, the macroscopic current shows a peak as a function of noise strength. That is, the current shows stochastic resonance (Box 1, Part 1) behaviour as well. When the system is driven by an asymmetric field the motion of particles becomes more synchronized in one direction than the other. As one can see when $F < F_{cl}$ the potential barrier does not vanish but becomes the smallest (largest, for crossing in the opposite direction) when the field value is the largest. It is in that situation that the barrier crossings become most probable (least probable in the opposite direction). The passage also depends on the length of

The efficiency of a machine is defined as the ratio of the amount of useful work extracted from it to the amount of energy (or Gibbs free energy) supplied to it in order to get that much useful work.

duration the particle sees a low potential barrier and the energy it has gained during the external field cycles (dragging the particle along) as the field value approaches its maximum. Since the field sweeps in the two half cycles are not the same, passages are not symmetric on both the directions giving rise to macroscopic current. This macroscopic current depends in a complex manner on various parameters including the noise strength. This important model, however, has not received much attention.

Efficiency of Ratchets

In all the model ratchets discussed so far we require to spend energy in order to obtain macroscopic (particle) current. Ratchets are, thus, tiny machines to generate current (like electric current, if the particles are charged, for example). Machines are useful only if some work can be efficiently extracted out of it. (Molecular motors in the living cells function with very high efficiency.) The efficiency of a machine is defined as the ratio of the amount of useful work extracted from it to the amount of energy (or Gibbs free energy) supplied to it in order to get that much useful work. In all the examples of ratchets that we have considered so far no useful work seems to have been accomplished. It is because the particle moving in the periodic potential system ends up with the potential energy even after crossing over to the adjacent potential minimum. That is to say, no extra energy is stored in the particle which can be usefully expended when desired. Therefore, in order to calculate efficiency of the ratchet we need to apply a load, L. In such a case the particle moves against the load performing thereby some work (W). Here the input energy (E_{in}) coming from the source of nonequilibrium (i.e., the external agent that provides the energy to alternately change the potential profile) is transformed into mechanical energy related to the load. The thermodynamic efficiency (η) is, therefore, given by $\eta = \frac{W}{E_{in}}$. The calculation of W and E_{in} are based on Langevin equation using a formalism of stochastic energetics. Using this method one can readily establish the compatibility between the Langevin equation approach to Brownian motion and the laws of thermodynamics. It is important to note that an analysis of °uctuations is essential for the calculation of e±ciency of a ratchet system at the molecular level. These °uctuations are completely ignored for the working of the conventional heat engines at larger scale. The e±ciency of Brownian motors (ratchets) is extremely sensitive to system parameters and exhibits several counter-intuitive behaviour. Noise for example, may facilitate energy conversion, i.e., increasing the strength of noise can make a ratchet engine more e±cient. By going away from quasistatic limit (adiabatic limit, by for example, increasing the frequency of the external pumping agent) e±ciency can be increased, contrary to what is known for the macroscopic reversible heat engines.

It is the flashing ratchet which shows the promise for large efficiency owing to the fact that macroscopic current results due to the sliding of particles down the potential slope.

It is the °ashing ratchet, however, which shows the promise for large e±ciency owing to the fact that macroscopic current results due to the sliding of particles down the potential slope. For independent particle motion the e±ciency remains low (usually $< 5\%$, but with suitable choice of ratchet parameters it can be increased.) but when the particles are coupled the e±ciency shows a marked increase (' 50%). This could be because of the possibility of a particle sliding away from its parent potential valley to pull along another coupled particle which otherwise would have slid down to the minimum of the parent potential valley. With a suitable choice of the ratchet parameters this mechanism may work to enhance the value of macroscopic current and hence the e±ciency. The result is quite intriguing because given all the other parameters same the coupled particles (with larger e®ective mass) should give lower current than the independent particles.

Applicability of Ratchets

It was mentioned earlier that by choosing the parameters of the ratchet operation it is possible to reverse the direction of the macroscopic current as a function of noise strength or any other parameter. This is a very interesting and important result from the practical point of view. Noise strength, however, is related to the friction coe± cient ° (see Box 2, Part 1) which, in turn, also depends on the shape, size, etc. of the macroscopic particle. Therefore, di®erent types of particles will have di®erent values of °. Thus, it is possible to tune the parameters of the ratchet operation such that in a mixture of the two types of particles the current for one type of particle will have opposite direction than the current for the other types of particles (with di®erent ° value) for the same (other) operating parameters. The ratchet mechanism, therefore, can be used to separate them by exploiting their opposite motional properties in the appropriate domain of parameter space. The possibility of such micromachines is under intense investigation these days.

It has been suggested that the understanding gained in obtaining noise-induced transport can be exploited and applied in diverse ¯elds including game theory. Indeed, a new area has emerged under the subject of Parrondo's paradoxes in game theory. Here, for example, two separately losing (with probability one) gambling games when played in combination in random sequence may lead to a winning game with probability one. These games are inspired by °ashing Brownian ratchets and are discrete time version of ratchet models. The °ashing ratchet can be viewed as the combination of two separate dynamics: Brownian motion in an asymmetric potential and Brownian motion on a °at potential as discussed in the section on °ashing ratchets in Part 1. In each of these cases, the particle does not exhibit any asymmetric motion. However, when they are alter-

It has been suggested that the understanding gained in obtaining noise-induced transport can be exploited and applied in diverse fields including game theory.

nated the particle moves to the left. The effect persists (i.e., the direction of net current being to the left) even if we add a small uniform external force pointing to the right. In that case, the two dynamics discussed above yield motion separately to the right, but when they are combined the particle moves to the left. This apparent paradox points out that two separate dynamics, in which a given variable decreases (or increases), when combined together the same variable, in certain circumstances, can increase (or decrease) in the resulting combined dynamics. This basic fact is utilized in Parrondo's games.

A parallel development in mesoscopic physics has led to the discovery of quantum pumps, where one can obtain currents (in the absence of bias).

Recent Developments

We now mention a few recent developments related to the subject. In adiabatically rocked classical ratchets (for $|F| < F_d$) at temperature $T = 0$ the macroscopic current identically vanishes. However, quantum mechanically the macroscopic current can arise due to the possibility of tunneling through the barriers. It turns out that the direction of this current is opposite to the classical macroscopic current obtained at high temperatures for the same ratchet system of Figure 1a, Part 1. In a string of triangular quantum dots (simulating effectively a ratchet potential) in GaAs/AlGaAs heterostructures the change in the direction of macroscopic current as a function of temperature has been observed experimentally. This clearly indicates a case of cross-over from quantum to classical regime. Such cross-over effects are of interest in the area of foundation of quantum mechanics. A parallel development in mesoscopic physics has led to the discovery of quantum pumps, where one can obtain currents (in the absence of bias). For this purpose one needs to vary at least two system parameters periodically in time but with a phase difference. The phase difference determines the direction of current. Its analogous aspects are being explored in classical systems.

In an another related recent development in ratchet systems (for single particle case) the phenomenon of absolute negative mobility (as opposed to negative differential mobility) has been predicted. In these nonequilibrium systems, currents are zero in the absence of bias. However, with the application of a small bias the current flows in the direction opposite to the direction of bias. The existence of such phenomena has also been predicted in a system of coupled particles even in periodic potentials which exhibit symmetry breaking transition in nonequilibrium situations. Also, studies of ratchet systems in higher dimensions have indicated the possibility of rerouting the particles in any desired direction by appropriately choosing the ratchet potentials and other parameters.

It is remarkable that fluctuating random forces help in obtaining deterministic (ordered) current, as seen in all the ratchet examples, and by controlling the strength of the randomly fluctuating forces one can maximise it too.

Currently, the notion of reversible ratchets has been of considerable interest. In these systems energy dissipation or entropy production are essentially zero. A deep connection between efficiency, entropy and information are being pursued intensively. These investigations may help in furthering fundamental developments in the area of driven nonequilibrium systems.

In summary, we have discussed qualitatively the phenomenon of noise-induced transport, in the absence of bias, in periodic (mostly) asymmetric (ratchet) potentials. For such macroscopic currents not only is the presence of noise essential but its presence with optimal strength helps in making the current peak with appreciable value. The mechanism of various categories of ratchets as discussed in Part 1 differ in details. The essential idea behind this nonequilibrium phenomenon, however, remains the same. It is remarkable that fluctuating random forces help in obtaining deterministic (ordered) current, as seen in all the ratchet examples, and by controlling the strength of the randomly fluctuating forces one can maximise it too. Stochastic resonance helps in tuning the effect optimally. Both the ratchet effect and the stochastic resonance (separately or together) are seen to play important roles in diverse systems including biological systems. A few examples of the phenomena where noise plays constructive role have been given in the introduction. The important role of noise in these phenomena has led to a new paradigm in natural sciences wherein attempts are being made to harness noise for useful purposes.

Suggested Reading

[1] J Maddox, Making models of muscle contraction, *Nature*, Vol. 365, p. 203, 1993.

[2] J Maddox, More models of muscle contraction, *Nature*, Vol. 368, p. 287, 1994.

[3] J Maddox, Directed motion from random noise, *Nature*, Vol. 369, p. 181, 1994.

[4] R D Astumian, Thermodynamics and kinetics of a Brownian motor, *Science*, Vol. 276, p. 917, 1997.

[5] M Bier, Brownian ratchets in physics and biology, *Contemp. Phys.*, Vol. 38, p. 371, 1997.

[6] F Jülicher, A Ajdari and J Prost, Modeling molecular motors, *Rev. Mod. Phys.*, Vol. 69, p. 1269, 1997.

[7] P Reimann, Brownian motors: noisy transport far from equilibrium, *Physics Reports*, Vol. 361, p. 57, 2002.

[8] For Parrondo's games, see G P Harmer and D Abbott, *Nature*, Vol. 402, pp. 22, (1999); or search under "Parrondo's games" or "Parrondo's paradox" at http://www.google.com.

Address for Correspondence
Mangal C Mahato
Department of Physics
Guru Ghasidas University
Bilaspur 495 009, India.

A M Jayannavar
Institute of Physics
Sachivalaya Marg
Bhubaneswar 751 005, India.

Please Note

***Resonance*, Vol.8, No.7, August 2003**

Page 26: In the author introduction it was mentioned that Dr. Poornima Sinha was the first PhD student of Prof. S N Bose. This is an error; Dr. Poornima Sinha was only the first woman PhD student of Prof. S N Bose.