# Multi-objective optimization of venturi scrubbers using a three-dimensional model for collection efficiency<sup>†</sup>

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Abstract: Multi-objective optimization of a venturi scrubber was carried out using a three-dimensional model for collection efficiency and non-dominated sorting genetic algorithm (NSGA). Two objective functions, namely (a) maximization of the overall collection efficiency, and (b) minimization of the pressure drop were used in this study. Three decision variables including two operating parameters, viz liquid-gas ratio and gas velocity in the throat, and the nozzle configuration. Optimal design curves (non-dominated Pareto sets) and the values of the decision variables corresponding to optimum conditions on the Pareto set for a pilot-scale scrubber were obtained. The liquid to gas (L/G) ratio, which is a key decision variable that determines the uniformity of liquid distribution, and a staggered nozzle configuration can produce uniform liquid distribution in the scrubber. Multiple penetration using nozzles of two different sizes in a triangular staggered arrangement can reduce liquid loading by as much as 50%, consequently reducing the pressure drop in the scrubber.

Keywords: venturi scrubber; collection efficiency; pressure drop; multi-objective optimization; pareto sets; genetic algorithm

# NOTATION

$C_{\rm d}, C_{\rm p}$	Concentration of the droplets and particles
	$(no m^{-3})$
$E_{\rm d}$	Eddy diffusivity of the droplets $(m^2 s^{-1})$
Ι	Objective functions for the GA
$L_{ m th}$	Length of venture scrubber, m
$N_{\rm c}$	Nozzle configuration
$Q_{\rm d}$	Liquid drop source strength (no $m^{-3}$ -s <sup>-1</sup> )
$Q_{\rm f}^{\rm u}$	Amount of liquid flowing as film on the wall
-	$(no m^{-3} - s^{-1})$
$R_0$	Half-width of the venturi throat parallel to
	water injection, mm
t	Time (s)
u	Decision variable (dimensionless)
$V_{\rm dx}$	Velocity of the droplets (m $s^{-1}$ )
$V_{\rm G,th}^{\rm ux}$	Gas velocity at the throat $(ms)^{-1}$
$W_0$	Width of the venturi throat perpendicular to
0	water injection, mm
$\eta_{0}$	Overall collection efficiency
10	(dimensionless)

## Subscript

*i* Index of decision variable

Superscript l, u Lower and upper limit

## INTRODUCTION

Venturi scrubbers have been used extensively since the 1950s as a major gas-cleaning device for the removal of fine particulates from industrial exhausts. Although there has been significant progress in developing models for the collection efficiency and pressure drop, not much work has been reported on the optimization of venturi scrubbers. In the pioneering work on optimization, Goel and Hollands<sup>1</sup> used the model of Calvert<sup>2</sup> for the determination of the pressure drop and the collection efficiency and developed optimized collection efficiency and pressure drop charts. Leith and Cooper<sup>3</sup> proposed an optimization algorithm based on Calvert's model, which uses a straight duct approach, whereas Cooper and Leith<sup>4</sup> presented an improved approach wherein the scrubber geometry was considered. All these early approaches attempted to optimize the performance of a given scrubber using two important operating variables, viz the ratio, L/G, of

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the liquid-to-gas flow rates, and the gas velocity,  $V_{G,th}$ , in the throat. Several important design variables, such as the throat dimensions, nozzle diameter, and the arrangement of the liquid injection nozzles, that can affect the liquid distribution significantly, were not considered. In addition, these optimization approaches used simplified models that assumed uniform liquid distribution without accounting for film flow. Hence, the above approaches did not provide exact optimized solutions and are scrubber-specific. An improved method for the multi-objective optimization of Pease-Anthony type of scrubbers was presented recently<sup>5</sup> using the recent and more detailed models developed by Viswanathan and co-workers<sup>6,7</sup> for the collection efficiency and the pressure drop, respectively. However, this study was also somewhat limited in scope since it optimized only existing (specified) scrubbers and did not incorporate any design variables.

A detailed optimization study must obtain the best conditions for both the design and the operating variables. In the present study, an important design variable, viz the optimum nozzle arrangement in a Pease-Anthony scrubber, is included (along with some important operating variables) among the 'control' (also called decision) variables used for optimization. A three-dimensional model is used to calculate the collection efficiency of the scrubber. The optimization was conducted using genetic algorithm (GA), a nontraditional search and optimization method, introduced by Holland.<sup>8</sup> GA is known to have several advantages over conventional optimization techniques such as objective functions can be multimodal or discontinuous; information is required only on the objective function, (and no gradients need be computed); a starting (guess) solution is not needed; the search is carried out using a population of several (rather than a single) points simultaneously. In addition, GA is well suited to handle problems involving several design or operating decision variables. Nondominated sorting genetic algorithm<sup>9</sup> (NSGA) is used to solve a variety of complex problems of industrial interest involving a vector of several objective functions. In this study, NSGA is used with the two objective functions, namely, maximizing the overall collection efficiency and minimizing the pressure drop in a pilot-scale venturi scrubber.

# MODELS FOR COLLECTION EFFICIENCY AND PRESSURE DROP

The performance of a venturi scrubber (Fig 1) depends largely on the manner of liquid injection, including the initial liquid momentum, since this essentially determines the size-distribution of the droplets of liquid formed and the liquid-flux distribution. Ananthanarayanan and Viswanathan<sup>10</sup> used a simplified version of the model proposed earlier by Viswanathan,<sup>6</sup> and this model is used in this work to calculate the droplet flux distribution in the scrubber.

The model takes into account the jet penetration length, the initial momentum of the liquid and the non-uniformity in the drop-size distribution. The model assumes that the movement of the drops in the axial (x in Fig 1) and lateral (y) directions is due to convection and diffusion, respectively. It also assumes drops of uniform size, constant film flow, no dropdrop interactions, uniform inlet distribution of the particles, no interaction between the particles, and that particulate collection by the droplets is mainly due to the inertial impaction of the dust particles onto the droplets. An empirical correlation has been used to estimate the fraction of liquid flowing on the walls.

The pressure drop,  $\Delta p$ , in the Pease–Anthony scrubber is determined using the annular flow model developed by Viswanathan *et al.*<sup>7</sup> Pressure losses that occur due to the acceleration of the gas and of the liquid drops and due to frictional losses are estimated. The total pressure drop is calculated by also taking into account the pressure recovery that occurs in the diffuser. The overall collection efficiency and the pressure drop equations are available in Refs 5–7.

The majority of the collection of particulates occurs in the throat because of the presence of a high degree of turbulence in this region caused by large relative velocities between the drops and the particles. A schematic representation of the throat of the scrubber, sub-divided into a three-dimensional grid, is presented in Fig 2. Use of such a three-dimensional approach in

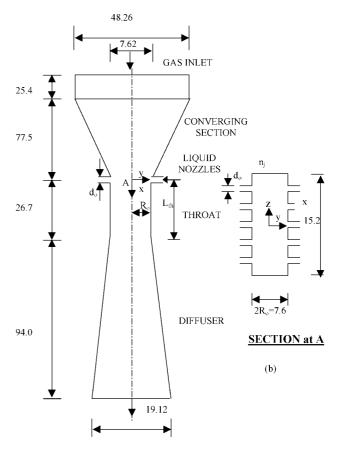


Figure 1. Schematic diagram of the pilot-scale venturi scrubber (all dimensions are in cm).

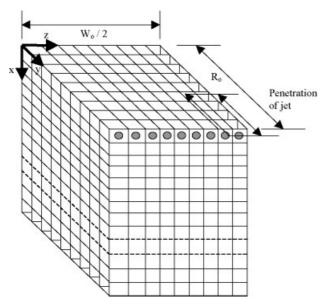


Figure 2. Grid lay-out for optimization.

the present study enables one to obtain the optimum nozzle arrangement for venturi scrubbers, a very important design variable that dictates the flux distribution in this equipment. Due to the symmetry in the problem, only a part of the scrubber-throat,  $0 \le x \le L_{\text{th}}$ ;  $0 \le y \le R_{o}$ ;  $0 \le z \le W_{o}/2$  (see Figs 1 and 2), is required for simulation. This reduces the complexity as well as the computational time required for simulation. The droplets are introduced in the scrubber using a total of 34 nozzles placed at the entrance of the throat (17 on either side of the throat). The physical space inside the throat is divided into cells as shown in Fig 2 (a fixed finite-difference Eulerian grid; 9 grid points used in  $0 \le z \le W_0/2$ , 13 grid points used in  $0 \le x \le L_{\text{th}}$ , 9 grid points used in  $0 \leq y \leq R_0$ ) and the Lagrangian mass particles carry the fluid from cell to cell by the sum of bulk and turbulent velocities. In order to evaluate the movement of each mass particle, the bulk velocity, eddy diffusivity, gas stream drag and the initial liquid momentum are calculated. The jet penetration length is determined and subsequently, the flux distribution at any axial position is obtained by applying a central difference formula to solve the three dimensional steady-state continuity equation for the liquid drops as shown in eqn (1):

$$\frac{\partial}{\partial x}(V_{\rm dx}C_{\rm d}) = \frac{\partial}{\partial y}\left(E_{\rm d}\frac{\partial C_{\rm d}}{\partial y}\right) + \frac{\partial}{\partial z}\left(E_{\rm d}\frac{\partial C_{\rm d}}{\partial z}\right) + Q_{\rm d} - Q_{\rm f}$$
(1)

This results in a sparse matrix, which is then solved by the use of appropriate subroutines obtained from the IMSL library. Particulate matter is introduced in the scrubber as uniformly distributed dust particles moving with the same velocity as the gas stream. The particle distribution at any axial position is then determined in a way similar to the liquid-flux distribution. The overall collection efficiency,  $\eta_o$ , at the end of the scrubber is then computed as:

$$\eta_{\rm o} = 1 - \frac{\int C_{\rm p(x,y)} dy}{\int C_{\rm p(0,y)} dy}$$
(2)

## FORMULATION

The two objective functions chosen in this study, are the:

- (a) maximization of  $I_1$ , the overall collection efficiency,  $\eta_0$ , and
- (b) minimization of  $I_2$ , the pressure drop,  $\Delta p$ .

Three decision variables are identified and their bounds were fixed depending on practical considerations. These are: the liquid-to-gas flow ratio, L/G; the gas velocity at the throat,  $V_{G,th}$ ; and a (dimensionless) parameter,  $N_{\rm c}$ , that describes the geometrical placement of the nozzles at the beginning of the throat. Figure 3 shows five types of nozzle arrangements, of nine nozzles in each case, in the region,  $0 \leq y \leq R_0$ ;  $0 \le z \le W_0/2$  (there are 17 nozzles on each side of the throat, at the grid points). The central nozzle in Fig 3 is shared by the symmetric other quarter on that side of the throat. In the first nozzle arrangement, also called the single file, nine nozzles are arranged side-by-side in the first row, one at each grid-point. The second configuration is that of a triangular pitch where the nozzles in the first row are arranged on alternate gridpoints and those in the second row are staggered with those in the first. The third type of nozzle configuration is that of a *double* file (square). The nozzles in the fourth type of nozzle arrangement are arranged in a triple file. All the nozzles in these four configurations have the same diameter, viz 2.1 mm, and hence take up the same projected area available for liquid injection.

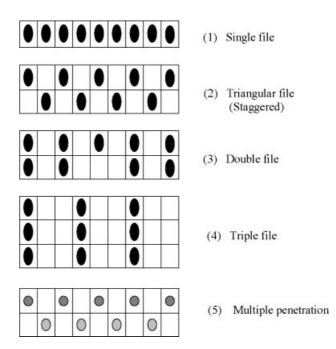


Figure 3. Schematic representation of nozzle configurations studied.

When the nozzles are of the same diameter, the liquid from each of the nozzles penetrates into the same extent into the scrubber. This phenomenon is termed as *uniform* penetration. The fifth configuration in Fig 3 is a staggered arrangement, similar to the triangular file (the second configuration), but with the nozzle diameters in the first and second rows being different. The diameters of the nozzles in the first and second rows are 1 and 1.5 mm, respectively. Thus, in configuration 5, the penetration lengths are different in the first and second rows, and the phenomenon is called *multiple* penetration.

The multi-objective function optimization problem studied here is, thus, described mathematically by

Max 
$$I_1(\mathbf{u}) \equiv I_1\left(\frac{L}{G}, V_{G,\text{th}}, N_c\right) = \eta_o$$
 (3a)

Min 
$$I_2(\mathbf{u}) \equiv I_2\left(\frac{L}{G}, V_{G,\text{th}}, N_c\right) = \Delta p$$
 (3b)

subject to:

 $u_i^{\rm l} \leqslant u_i \leqslant u_i^{\rm u}; i = 1, 2, 3 \tag{3c}$ 

and model equations in Refs 5-7 (3d)

In this problem, dust of a uniform size  $(D_p = 5 \mu m)$  is used in the feed to the venturi scrubber. The optimal value for L/G obtained by Ananthanarayanan and Viswanathan<sup>10</sup> (using only a single decision variable at a time) was  $1.2 \times 10^{-3}$ . Accordingly, a range of  $0.3 \times 10^{-3}$ – $1.4 \times 10^{-3}$  m<sup>3</sup> of liquid/m<sup>3</sup> of air has been selected as the bounds for L/G. Bounds (40–120 m s<sup>-1</sup>) for the throat gas velocity,  $V_{G,th}$ , were decided based on industrial practice. All other dimensions of the venturi scrubber are the same as used by Ananthanarayanan and Viswanathan,<sup>10</sup> and are given in Fig 1.

The decision variables that lead to an increase in the overall collection efficiency in a venturi scrubber may produce a simultaneous undesirable increase in the pressure drop. Similarly, decision variables that lead to a minimization of the pressure drop may lead to lower collection efficiencies in the scrubber. Thus, the two objective functions in eqn (3) are conflicting in nature and would lead, *most likely*, to a Pareto set of optimal solutions.

### **RESULTS AND DISCUSSION**

The multi-objective optimization problem was solved on a CRAY J916 computer. The CPU time required to solve this problem was 1.35 s. In solving the problem, the domains (bounds) of the several decision variables were varied and the Pareto sets obtained. Thereafter, a few of the optimal points on the Pareto set were selected (such that they spanned the entire range of the Pareto set) and simulation results for these cases were analysed (results not shown here due to the scope of the paper). This permitted analysis of the conditions required for optimality and provided considerable insight into the physical phenomena important under optimal conditions.

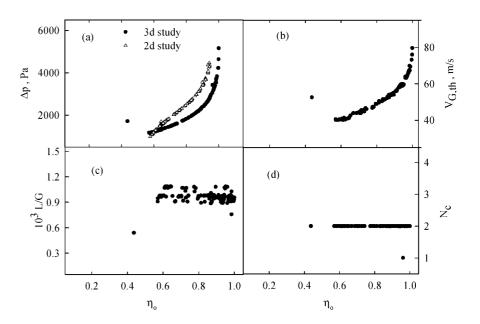
The values of several of the computational parameters used in this study, as well as those of the other variables and parameters describing the venturi scrubber, are listed in Table 1. The results of optimization are shown in Fig 4 (solid points for the 3-D case). A plot of  $\eta_0$  versus  $\Delta p$  is shown in Fig 4(a) (a typical Pareto set, wherein an improvement (increase) in  $\eta_0$  is accompanied with a worsening (increase) of  $\Delta p$ ). Plots of the three decision variables corresponding to the different points on the Pareto set are shown in Fig 4(b-d). It is observed that the gas velocity at the throat,  $V_{G,th}$ , varies along the points on the Pareto. The values of L/G that provide optimal operating conditions are found to be almost constant, varying in a narrow range of about  $0.8 \times 10^{-3}$ -1.1  $\times 10^{-3}$  (Fig 4(c)), similar to what was found in our previous study.<sup>5</sup> The optimum nozzle configuration,  $N_c$  (Fig 4(d)), is configuration 2 wherein the nozzles are arranged in a staggered triangular file.

A comparison has been made with our previous study,<sup>5</sup> which used a two-dimensional model for the collection efficiency. Both the studies retain the basic characteristics of the Pareto curve but the values are slightly different because of the difference in the rigor involved in the models used. Higher collection efficiency at a lower pressure drop in the threedimensional case than that in the two-dimensional case may be due to better liquid distribution in the scrubber due to the optimal nozzle arrangement  $(N_c=2)$  being used here, something that was not possible in our earlier study. The influence and importance of adding on the new design variable is clearly observed. Figure 4 suggests that when all the three decision variables are used simultaneously, the Pareto is far more sensitive to  $V_{G,th}$  than to L/G, and that the scatter in the optimal values of L/G in Fig 4(c)is essentially a reflection of its relative insensitivity. An increase in  $V_{\rm G,th}$  at a nearly constant range of L/G leads

Table 1. Parameters used in the optimization

Parameter	Value
Computational	
Maximum number of generations (maxgen)	100
Population size $(N_{\rm p})$	100
Probability of crossover ( $p_{c}$ )	0.55
Probability of mutation $(p_m)$	0.001
Random seed	0.87619
Spreading parameter ( $\sigma$ )	0.005
Sharing function (α)	2
Grid size (h) (m)	0.004
Model <sup>a</sup>	
Density of liquid (kgm <sup>-3</sup> )	993.0
Density of particles (kgm $^{-3}$ )	2500
Viscosity of gas (Pas)	$1.8 \times 10^{-5}$
Viscosity of liquid (Pas)	$1.0 \times 10^{-3}$

<sup>a</sup> References 5–7.



**Figure 4.** Pareto optimal solution showing pressure drop *versus* efficiency. (a)  $\Delta P$  *versus*  $\eta$ ; (b) gas velocity *versus*  $\eta$ ; (c) Liquid/Gas ratio *versus*  $\eta$ ; (d) nozzle configuration *versus*  $\eta$ .  $\bullet$ , optimal solution for 3-D case  $\triangle$ , optimal solution for 2-D case.

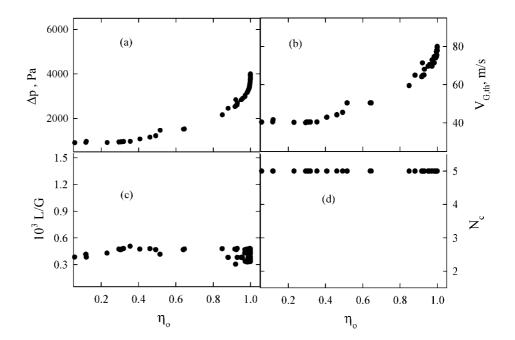
to higher values of  $\eta_0$  because of the formation of a higher number of smaller droplets.

A slightly different case of the multiobjective optimization problem discussed above, namely, the comparison between uniform and multiple penetrations, is attempted next. In this example, the best nozzle configuration from the previous uniform penetration studies ( $N_c = 2$ , corresponding to the triangular file arrangement) is compared with the case with  $N_c = 5$ , wherein nozzles of two sizes are used. The optimal Paretos are shown in Fig 5. A comparison of Figs 4 and 5 shows that multiple penetration (configuration No 5 in Fig 3) gives superior solutions (higher  $\eta_0$  for the same  $\Delta p$  on the Pareto set), indicating a more uniform liquid distribution in the scrubber. In the multiple penetration configuration, the liquid penetrates to different extents in the first two rows along the axis of the scrubber. As a result, there is good spreading of the liquid. As the liquid flows down the scrubber, driven by the momentum of the gas flow, an even greater spread of liquid droplets occurs due to lateral diffusion.

The optimal L/G ratio in multiple penetration is located in a narrow range of  $0.4 \times 10^{-3}$ - $0.5 \times 10^{-3}$ , compared with  $1.0 \times 10^{-3}$  for uniform penetration. The lower values of the optimal L/G suggest considerable savings in the requirement of the scrubbing liquid. Such savings are consistent with the recent findings<sup>11</sup> of scrubber optimization using computational fluid dynamics (CFD) to model the flow in the scrubber.

## CONCLUSIONS

Multi-objective optimization of a venturi scrubber was carried out using a three-dimensional model for the collection efficiency with the NSGA algorithm. Simul-



**Figure 5.** Optimal solutions for multiple penetration nozzle configuration ( $N_c$ =5). (a)  $\Delta P$  versus  $\eta$ ; (b) gas velocity versus  $\eta$ ; (c) liquid/gas ratio versus  $\eta$ ; (d) nozzle configuration versus  $\eta$ . taneous optimization of collection efficiency and pressure drop in the scrubber was conducted using three decision variables. Pareto optimal sets were obtained. The variables include two operating variables (viz L/G and  $V_{G,th}$ ) and one design variable, namely, the nozzle configuration. The ratio of liquid to gas flow (L/G) determines the uniformity of liquid distribution inside the scrubber. An optimal range is obtained for this ratio.  $V_{\rm G,th}$  is found to vary along the Pareto and contributes to the shape of the Pareto design curve. The optimization results (for uniform penetration configurations) indicate that a staggered nozzle configuration, corresponding to  $N_c=2$ , produces relatively uniform liquid distribution in the scrubber and leads to improved performance. Multiple penetration using nozzles of two different sizes in a triangular staggered arrangement,  $N_c = 5$ , can cause an almost 50% reduction in the liquid loading, consequently reducing the pressure drop in the scrubber. The present optimization study, using a three-dimensional simulation model, produces the interesting information that the nozzle arrangement is an important decision variable in venturi scrubbers. The Pareto solutions obtained in this work are essentially design curves that assist in narrowing down the choices of a decision-maker.

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