THE ASTROPHYSICAL JOURNAL, 462:309–315, 1996 May 1

COLLISIONAL BUILDUP OF MOLECULAR CLOUDS: MASS AND VELOCITY SPECTRA

MOUSUMI DAS AND CHANDA J. JOG
Joint Astronomy Programme, Department of Physics, Indian Institute of Science, Bangalore 560012, India
Received 1995 March 27; accepted 1995 October 13

ABSTRACT

We have studied the evolution of a local region of molecular clouds, moving in the mean gravitational potential of the Galactic disk. The clouds interact gravitationally with one another and undergo inelastic collisions. The clouds have been modeled as spheres, and a collision is assumed to take place when two clouds overlap. The outcome of a collision depends on the relative mass and velocity of the clouds and can result in the fragmentation or coalescence of the clouds. We find that the initial random mass distribution of the clouds evolves into a power-law distribution of the form $N(m) \propto m^{\alpha}$, where $\alpha = -1.7$ to -1.9, and is flatter toward the low-mass end. The mean one-dimensional random velocity of the molecular clouds is $\sim$8–10 km s^{-1} and is found to be independent of cloud mass. These results agree well with the observed mass and velocity distribution of molecular clouds. We also conclude that cloud collisions in a sheared galactic disk, rather than local gravitional interactions, are important in determining the mass and random velocity distribution of molecular clouds. Cloud collisions thus play a dominant role in the formation of molecular clouds.

Subject headings: ISM: clouds — ISM: kinematics and dynamics — ISM: molecules

1. INTRODUCTION

During the past two decades, surveys of molecular gas in galaxies have revealed a great deal about the distribution, structure, and kinematics of molecular clouds (Scoville & Sanders 1988). Molecular clouds have been found to form a collisional, dissipational system of self-gravitating gas in which matter is continuously recycled through the process of star formation. The clouds are a major component of the neutral gas in a galaxy; in our Galaxy they constitute 50% of the total neutral gas. However, many issues regarding these clouds are still not fully understood, one of them being their formation. Cloud collisions, instabilities in the interstellar medium, and the accumulation of dense gas behind shocks are some of the mechanisms for molecular cloud formation suggested by various authors in the literature.

Cloud collisions lead to fragmentation, when the colliding clouds have supersonic velocities and coalescence when the collisions are nonsupersonic (Hausman 1981). Coalescence also occurs when one cloud is much more massive than the other (Gilden 1984). Gravity is important in the collisional buildup model of molecular clouds because the collisional cross section of the clouds is enhanced by gravitational interactions. The collisional buildup of clouds has been studied by a number of authors, both analytically and numerically (Oort 1954; Field & Saslaw 1965; Scoville & Hersh 1979; Casoli & Combes 1982; Kwan & Valdes 1987; Nozakura 1990; Sotnikova & Volkov 1994).

Gravitational instabilities can also form molecular clouds. The self-gravity of the gas helps to bind the clouds when they start forming, and the subsequent cooling of the shocked gas leads to cloud formation on small scales as well as on the scale of cloud complexes (Elmegreen 1989). The stellar and gas systems have been treated as a gravitationally coupled two-fluid system by Jog & Solomon (1984). They find that even though the stellar and gas systems may be separately stable, owing to the gravitational interaction of the two systems, the two-fluid system may become unstable. Interstellar gas compressed by stellar winds and supernova explosions can also lead to molecular cloud formation. The hot, shocked gas cools rapidly and becomes gravitationally unstable, leading to the formation of small clouds. The Bok globules and the Cometary globules seem to have been formed by such mechanisms.

In this paper, we have studied the evolution of a system of clouds under their mutual gravitational interaction and cloud collisions in a sheared disk. We show that this is the main mechanism of molecular cloud formation which acts on the clouds all over the sheared disk. To examine this idea, we did some simulations in which molecular clouds were evolved in a local region of the disk under their mutual interaction and cloud collisions. Since the gravitational interaction between the clouds was included, at each time step in the simulation approximately N^2 summation operations had to be performed. Hence, it was not possible to include the entire distribution of clouds in the molecular ring, and so only a local distribution of clouds was evolved in the simulations.

Our study of cloud collisions and formation differs from previous studies of cloud formation in that the gravitational interaction of the clouds in a sheared disk has been included and the initial cloud distribution was random in both mass and velocity. Cloud collisions have been treated in a simplified manner; the rules governing the outcome of a collision are based on the detailed hydrodynamic simulations of cloud collisions by previous authors in the literature. The system of clouds was evolved, and the resulting mass and velocity spectra were obtained simultaneously. These results were compared with observed spectra. We have found that the results of our simulations are very similar to the observed mass spectra of molecular clouds. We have obtained a power-law mass spectrum which agrees with the observed mass spectra for molecular clouds in the Galactic disk (e.g., Solomon & Rivolo 1989). The mass distribution has an index of $\alpha \approx -1.8$, where $N(m) \propto m^{\alpha}$ and the one-dimensional cloud velocity is nearly independent of cloud mass. Although this is contrary to that expected for a system in energy equipartition, observations of molecular cloud velocities indicate that cloud velocities do not depend on cloud mass (Casoli, Combes, & Gerin 1984; Stark 1984).

Thus, the clouds do not appear to be in energy equipartition.

In § 2, we discuss the parameters used in the model, for both the molecular clouds and the disk potential. Section 3 deals with the treatment of cloud collisions in the simulations. We have discussed in detail the various rules governing the outcome of a collision. In § 4, the details of the numerical procedure have been discussed, and the results are presented in § 5. We end by summarizing our conclusions in § 6.

2. MODEL PARAMETERS

In this section we discuss the various cloud and Galaxy parameters used in the simulations. The parameters are based on observations as closely as possible. A summary of the initial conditions is listed in Table 1.

2.1. *Molecular Cloud Parameters*

The clouds have been treated as hard spheres with a finite dimension. The radius of a cloud depends on its mass through the empirical relation

$$\left(\frac{m_c}{M_\odot}\right) = 100\left(\frac{r}{\text{pc}}\right)^2 , \tag{1}$$

where m_c is the cloud mass and r is its radius in parsec (Falgarone & Puget 1986; Larson 1981). This means that the clouds have a density profile peaked toward the center, i.e., of the form $\rho \propto 1/r$. Since cloud coalescence and fragmentation have been included in the simulations, assigning a cloud radius was important. The internal velocity dispersion within a cloud σ_v also depends on the cloud radius. We have used the virial relation $\sigma_v = 0.36(R/\text{pc})^{0.5}$. This is used to determine whether a collision in the simulation is supersonic or not. The initial cloud distribution was taken to be a uniform, random distribution with a mass range between 100 $M_\odot$ and 10^5 $M_\odot$. After evolving the system, some of the cloud masses build up to 10^6 $M_\odot$.

2.2. *Galactic Parameters*

The Galactic potential used in the simulation is taken from Carlberg & Innanen (1987). It is a cylindrical potential which gives a rotation velocity of 235 km s^{-1} at the Solar neighborhood.

The surface density $\Sigma(r)$ of molecular gas has been found to vary over the Galaxy. The molecular gas distribution peaks in the central region and also in the molecular ring in the disk (3–9 kpc). In our simulations we have evolved molecular clouds in three regions of the disk, from 3 to 4.5 kpc, 4.5 to 7 kpc, and 7 to 9.5 kpc. The initial, random distribution of cloud masses was varied, so that the gas surface density of molecular gas in the simulations matched the observed average $\Sigma(r)$ values over the Galaxy as observed by Sanders, Scoville, & Solomon (1985). For each range of disk radius, the approximate average value of $\Sigma(r)$ was used. Since the mass of the clouds was generated randomly, an approximate mean value of the surface density, close to the observed value was used in the simulations (Table 1).

TABLE 1

INITIAL CONDITIONS FOR THE THREE REGIONS OF MOLECULAR CLOUDS IN THE GALACTIC DISK

Radial Extent of Local Region (kpc)	Vertical Extent of Local Region (pc)	Mass Surface Density $\Sigma(M_\odot\ \text{pc}^{-2})$	Number of Clouds	Mass Range $(M_\odot)$
3.0–4.5	200	4.9	222	10^2–10^5
4.5–7.0	200	9.0	1110	10^2–10^5
7.0–9.5	200	9.0	1110	10^2–10^5

3. MODELING CLOUD COLLISIONS

Cloud collisions have been studied extensively using both N-body and hydrodynamic simulations. Hydrodynamic simulations study the detailed evolution of a collision and the resulting coalescence or fragmentation of the clouds (Hausman 1981; Lattanzio et al. 1985; Gilden 1984). They indicate that the outcome of a collision depends on the relative masses of the clouds, their relative sizes, and their relative velocities. It also depends on the degree of overlap between the two clouds. Collisions which are supersonic, relative to the internal velocities in the clouds, lead to the fragmentation of the clouds, and nonsupersonic collisions lead to the coalescence of the clouds. When one cloud is more massive compared to the other cloud, a bow shock forms and the larger cloud engulfs the smaller one, resulting in the coalescence of the two clouds. However, these papers do not consider the long-term evolution of the clouds and the development of their mass and velocity spectra.

The N-body treatment in the literature, however, considers the effect of cloud collisions on the overall distribution and kinematics of molecular clouds (Kwan & Valdes 1987; Fukunaga & Tosa 1989; Sotnikova & Volkov 1994). The clouds are treated as particles, and the collisions are treated inelastically. In the literature, the coagulation and fragmentation of clouds has been included in the simulation of cloud motion in a galaxy, and in some cases the long-range gravitational interaction of the clouds has also been included (e.g., Combes & Elmegreen 1993; Thomasson, Donner, & Elmegreen 1991). However, these papers do not include the two-body gravitational interaction between the clouds. They also do not explicitly study the collisional buildup of the mass and velocity distribution of a dissipational system of molecular clouds.

In this paper we study the evolution of molecular clouds through gravitational interactions and cloud collisions. In our N-body approach, the clouds have been modeled as spheres of finite radii having an internal velocity dispersion which varies with cloud mass. The cloud model is based on observed cloud parameters (§ 2.1). The collisions in our simulation are based on the results of detailed hydrodynamic modeling of cloud collisions in the literature (e.g., Hausman 1981; Lattanzio et al. 1985; Gilden 1984). We have simplified the collisions so that the outcome of a collision depends on the relative sizes and velocities of the colliding clouds. Since the gravitational interaction between the clouds was included in the simulation, the entire disk of clouds could not be treated, as it would involve too many clouds. Instead, a small region of molecular clouds in the disk was evolved. The number of clouds was chosen sufficiently large enough so that the final result was fairly general but small enough to be numerically manageable. The system was evolved for 4 × 10^7 yr. Star formation was not included in the simulations, even though it probably plays a vital role in the long-term distribution and evolution of molecular gas, because the motivation for this work was to study the effect of the local gravitational interactions and collisions on the evolution of molecular clouds. The mass and velocity spectra were computed at intervals of 0.5×10^7 yr during the evolution.

Two clouds are said to collide when their separation is less than the sum of their radii. The Mach number of a collision M is the ratio of the relative velocity v_r of the two clouds to the internal velocity dispersions within the clouds, σ_{v1} and σ_{v2}. If $M > 1$ for either cloud, the collision is supersonic and will result in the fragmentation of the clouds. If the collision is nonsupersonic, the clouds coalesce. The ratio of the cloud masses is also computed and if it is larger than 3, the collision is assumed to result in the coalescence of the two clouds. The orbits of the colliding clouds are evolved until the separation between the cloud centres reaches a minimum. Only then are the rules for coalescence or fragmentation implemented. Below we discuss the outcome of a collision in detail. The cloud masses are denoted by m_1 and m_2, their internal dispersions by σ_{v1} and σ_{v2} and their relative velocity by $\boldsymbol{v}_r = \boldsymbol{v}_1 - \boldsymbol{v}_2$.

3.1. *Coalescence*

When the ratio of the cloud masses is greater than 3 or the Mach number of the collision is less than one, i.e., $v_r/\sigma_{v1} < 1$ or $v_r/\sigma_{v2} < 1$, the collision leads to the coalescence of the two clouds. When the overlap of the clouds reaches a maximum, the two clouds are replaced by a single cloud of mass $m_c = (m_1 + m_2)$ moving with the center-of-mass velocity of m_1 and m_2, $\boldsymbol{v}_c = (m_1 \boldsymbol{v}_1 + m_2 \boldsymbol{v}_2)/m_1 + m_2$. The energy dissipation due to the coalescence of the two clouds is

$$\Delta E = \frac{m_1 m_2 (\boldsymbol{v}_1 - \boldsymbol{v}_2)^2}{2(m_1 + m_2)} . \tag{2}$$

3.2. *Fragmentation*

Supersonic collisions of similar mass clouds tend to produce fragmentation or disruption of the colliding clouds. Modeling the outcome of such a collision is difficult because the result of the collision can vary due to many factors such as the angle of the collision, the densities of the clouds, and the relative velocities of the clouds. Hydrodynamic simulations have shown that head-on collisions lead to cloud disruption (Gilden 1984), whereas highly off-center collisions lead to cloud mass being sheared off and forming streamer-like extensions which later dissipate away in the interstellar medium (Hausman 1981). In our simulations, supersonic cloud collisions resulting in an overlapping mass less than 100 $M_{\odot}$ are neglected. The mass is assumed to dissipate into the interstellar medium. Though the molecular gas system loses some mass during the evolution, the loss is negligible compared to the overall system mass.

When the mass of the overlapping region is greater than 100 $M_{\odot}$, the collision results in a third cloud being formed. The collision is evolved until the separation of the two clouds reaches a minimum. The mass of the resulting cloud fragment m_f is calculated as follows. Let the overlapping region in the two clouds of mass m_1 and m_2 have angular radii of θ_1 and θ_2, respectively. Then the volume of the overlapping region is

$$V = \frac{\pi r_1^3}{3} [2(1 - \cos \theta_1) - \sin^2 \theta_1 \cos \theta_1] + \frac{\pi r_2^3}{3} [2(1 - \cos \theta_2) - \sin^2 \theta_2 \cos \theta_2] . \tag{3}$$

Hence, the mass of the fragment is

$$m_f = \frac{3V}{4\pi} \left(\frac{m_1}{r_1^3} + \frac{m_2}{r_2^3} \right) . \tag{4}$$

But the cloud radius is related to the cloud mass by $r_1 = 0.1(m_1)^{1/2}$ (Falgarone & Puget 1986). Hence,

$$m_f = 238.7 V \left(\frac{1}{\sqrt{m_1}} + \frac{1}{\sqrt{m_2}} \right) . \tag{5}$$

The remaining cloud masses are

$$m_1' = m_1 - 238.7 \frac{V}{\sqrt{m_1}} , \tag{6}$$

$$m_2' = m_2 - 238.7 \frac{V}{\sqrt{m_2}} , \tag{7}$$

The momentum of the collision is conserved during the collision. So if the colliding clouds have the same velocities $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ after the collision, the velocity of m_f is given by

$$m_1 \boldsymbol{v}_1 + m_2 \boldsymbol{v}_2 = m_1' \boldsymbol{v}_1 + m_2' \boldsymbol{v}_2 + m_f \boldsymbol{v}_3 , \tag{8}$$

$$\boldsymbol{v}_3 = \frac{(m_1 - m_1')\boldsymbol{v}_1 + (m_2 - m_2')\boldsymbol{v}_2}{m_f} . \tag{9}$$

The energy loss due to fragmentation is

$$\Delta E = \frac{(m_1 - m_1')(m_2 - m_2')(\boldsymbol{v}_1 - \boldsymbol{v}_2)^2}{2(m_1 + m_2 - m_1' - m_2')} . \tag{10}$$

After the collision, the new cloud of mass m_f starts moving with a momentum $m_f \boldsymbol{v}_f$ from the center of mass of the system. The other two clouds of reduced masses m_1' and m_2' are advanced by a hundred time steps along their initial directions with momenta $m_2' \boldsymbol{v}_2$. Though this may seem artifical, it does not affect the overall dynamics of the system but moreover prevents the clouds from colliding again and forming unnatural "ribbon-like" structures in the simulations.

4. NUMERICAL PROCEDURE

We have studied the collisional buildup of molecular clouds, under the effect of local gravitational interactions, in three regions in the disk; between Galactic radii 3–4.5 kpc, 4.5–7 kpc, and 7–9.5 kpc. The initial cloud masses have been chosen randomly between 10^2 and 10^5 $M_{\odot}$ in all three regions so that the initial mass distribution is linear and uniformly random. This means that if there are N clouds having a mass between 10^2 and 10^3 $M_{\odot}$, there should be $10N$ clouds having a mass between 10^3 and 10^4 $M_{\odot}$, and so on. The clouds were given an initial random velocity between -3 and 3 km s^{-1} along each of the axes. This leads to an initial mean, one-dimensional random velocity of approximately 2 km s^{-1}. The details of the initial distribution for the three regions in the disk have been summed up in Table 1.

In this problem, only local gravitational interactions have been considered. This means that clouds on the edge of the distribution experience less gravitational interaction with other clouds than clouds deep within the distribution. To reduce this effect, the ratio of the perimenter of the region to the area of the region in the x-y plane should be minimum. Hence, we have tried to keep the distribution of clouds over a square region in the disk. However, due to differential rotation, such a distribution will soon be highly sheared into a rhombus. To avoid this, we have taken the initial surface distribution of clouds in the disk at $t = 0$ yr to be over a rhombus, of such dimension that at $t = 1.5 \times 10^7$ yr the areal distribution of clouds is sheared into a square and at $t = 3 \times 10^7$ yr, the distribution is again sheared by differential rotation into a rhombus of opposite sense as that at $t = 0$ yr. To check whether a steady mass and velocity distribution has been attained, the clouds were evolved for another 10^7 yr until $t = 4 \times 10^7$ yr.

The system of clouds is evolved in the disk potential of Carlberg & Innanen (1987), and the equation of motion of the *i*th cloud is

$$\ddot{\boldsymbol{R}}_i = -(\nabla\Phi_c)_{\boldsymbol{R}_i} - \sum_j \frac{G m_j(\boldsymbol{R}_i - \boldsymbol{R}_j)}{(|\boldsymbol{R}_i - \boldsymbol{R}_j|^2 + \epsilon^2)^{3/2}}\,, \qquad (11)$$

where ϵ is the softening and is taken to be a constant. It is taken to be 5% of each dimension of the distribution. $\boldsymbol{R}_i$ is the position vector of the center of the *i*th cloud, and Φ_c is the potential. All the simulations are in a three-dimensional system, and the potential has a cylindrical form. The second term on the right is a result of the gravitational interactions between the clouds and is calculated by summing over all the cloud pairs. A very competent differential equation solver ODEN was used to evolve the equation of motion for each cloud in the system.

The system of clouds were evolved in three different regions of the disk for 4×10^7 yr. The mass and velocity spectra were plotted at intervals of 5×10^6 yr to see how the distribution of clouds changes with time. The mass spectrum was determined by first dividing the mass range into bins at logarithmic intervals. The number of clouds in a bin was divided by the size of the bin to get (dN/dm). Then log (dN/dm) was plotted against log (m). The slope of the curve is equal to the index α, where $N(m) \propto m^{\alpha}$. Similarly, to obtain a velocity spectrum, the random velocity of a cloud was first determined by subtracting the rotational velocity from the total cloud velocity. The rotational velocity was obtained from the potential Φ_c used in the problem. The velocity dispersion was assumed to be isotropic, and the mean, one-dimensional random velocity of the cloud distribution was determined. Then the mean random velocity of the clouds, in each mass bin, was plotted against log (m) to obtain the velocity spectrum of the clouds.

5. RESULTS

The mass and velocity spectra obtained in our simulations are very similar to the observed spectra, indicating that cloud collisions and interactions play a major role in cloud formation. The main results are described below.

5.1. *Mass Spectrum*

1. For disk radii greater than 4.5 kpc, where the gas surface density $\Sigma \sim 9\ M_\odot$ pc^{-2}, the cloud system soon settles down to a steady mass spectrum within 2×10^7 yr (Fig. 1). The spectrum does not change much with time after this period. However, for the region between disk radii 3–4.5 kpc, where the surface density of molecular gas is much lower, the time taken for the clouds to reach a steady mass spectrum is much longer. Only after a time $t = 3 \times 10^7$ yr does the high-mass end of the mass spectrum develop (Fig. 2). This indicates that cloud collisions are important in the collisional buildup of molecular clouds only in the molecular ring region, at radii greater than 4 kpc, where the surface density of molecular gas is high.

2. We have fitted the mass spectra for masses greater than $\sim 10^4\ M_\odot$. The region below $10^4\ M_\odot$ is flatter, and so we have fitted power-law slopes to the high-mass end. This is similar to the power-law fit of observational mass spectra in the literature. The region 3–4.5 kpc has an index $\alpha = -1.86$ at $t = 4 \times 10^7$ yr; the molecular ring region between radii 4.5–7 kpc has an index $\alpha = -1.78$, and the region 7–9.5 kpc has an index $\alpha = -1.72$ at $t = 4 \times 10^7$ yr. These results are similar to the observed mass distributions of molecular clouds in the molecular ring of our Galaxy (Solomon et al. 1987; Solomon & Rivolo 1989).

3. There is a distinct flattening in the mass spectra at the low-mass end of our simulations (Fig. 1). This flattening in the low-mass end is seen in the observationally obtained mass spectra of molecular clouds in the Galactic disk (Solomon et al. 1987; Solomon & Rivolo 1989) and is also present in the observed mass spectra of clumps or cores inside molecular clouds (Nozawa et al. 1991; Stutzki & Gusten 1990; Loren 1989). The shape of the curve may not be explained as due to undersampling of low-mass clouds or clumps, as was stated in the above observational papers, but instead it may indicate a genuine lower fraction of low-mass clouds than that expected for a power law fit. Our simulations support this conclusion.

4. In Fig. 3, we have plotted the fraction of cloud mass within a certain mass range against the mass for the region between Galactic radii 4.5–7 kpc. This figure shows how much gas is in the form of high-mass clouds. Most of the molecular cloud mass appears to be in the large mass clouds. We also found that 50% of the molecular clouds are more massive than $3.5 \times 10^5\ M_\odot$. These results are similar to those obtained by Sanders et al. (1985).

5.2. *Velocity Spectrum*

1. The final, steady, random velocity for molecular clouds in the region 4.5–9.5 kpc is 8–10.5 km s^{-1} (Table 2). The results for the region 4.5–7 kpc are plotted in Figure 4. For the region 3–4.5 kpc, the random velocity is higher and is about 11 km s^{-1} (Fig. 5). These values of the random velocities are larger than the observed velocities which lie between 5–7 km s^{-1} (Casoli et al. 1984; Stark 1984). This is perhaps because we have not included energy losses such as those due to gas shocking and radiation, which will occur during cloud collisions. These processes may retard the clouds as some of their relative kinetic energy may be dissi-

TABLE 2

MEAN RANDOM CLOUD VELOCITY AT DIFFERENT TIME INTERVALS IN THE EVOLUTION

TIME OF EVOLUTION (10^7 yr)	MEAN RANDOM VELOCITY (km s^{-1})		
	3–4.5 kpc	4.5–7 kpc	7–9.5 kpc
0.0	1.70	1.67	1.64
0.5	4.84	5.79	4.04
1.0	7.90	8.62	5.83
1.5	10.28	10.78	7.23
2.0	9.93	11.11	8.35
2.5	9.71	10.47	8.44
3.0	10.54	10.22	8.24
3.5	12.10	10.36	8.18
4.0	11.50	10.55	8.17

1996ApJ...462..309D

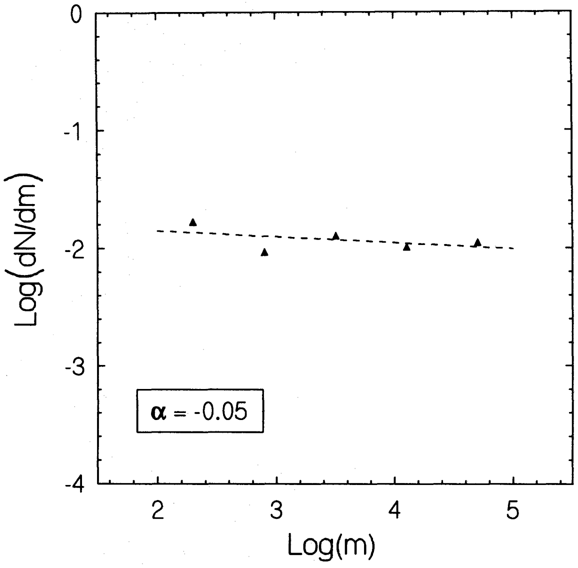

FIG. 1*a*

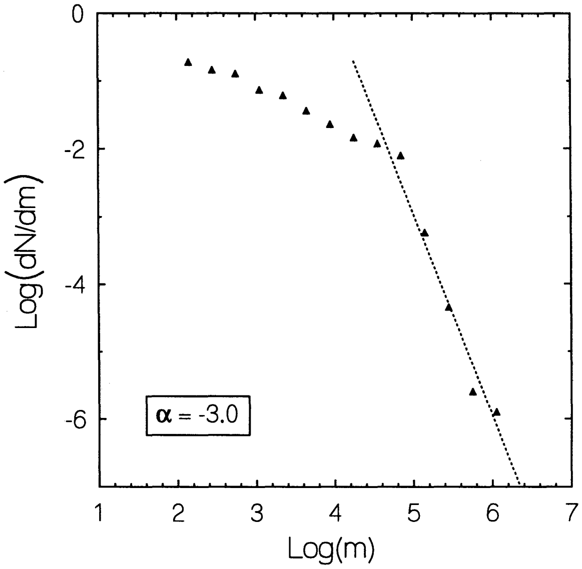

FIG. 1*b*

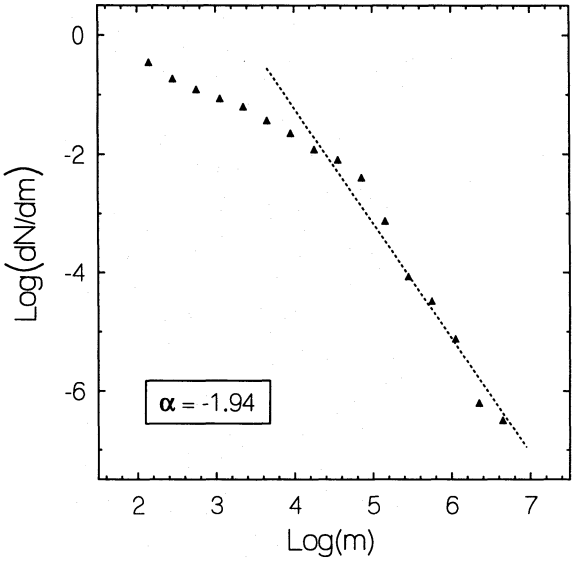

FIG. 1*c*

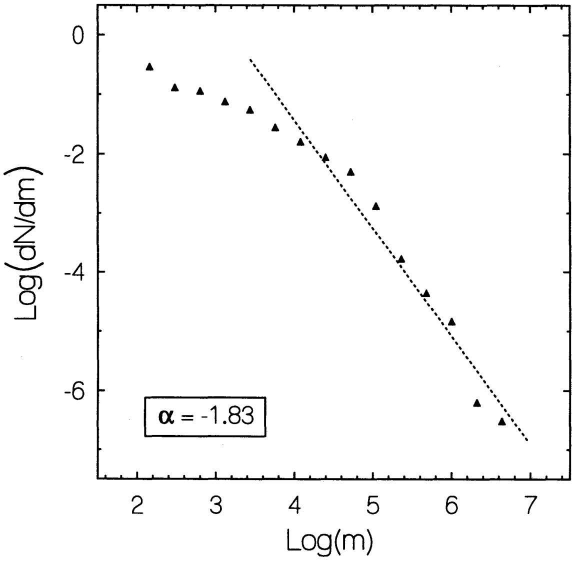

FIG. 1*d*

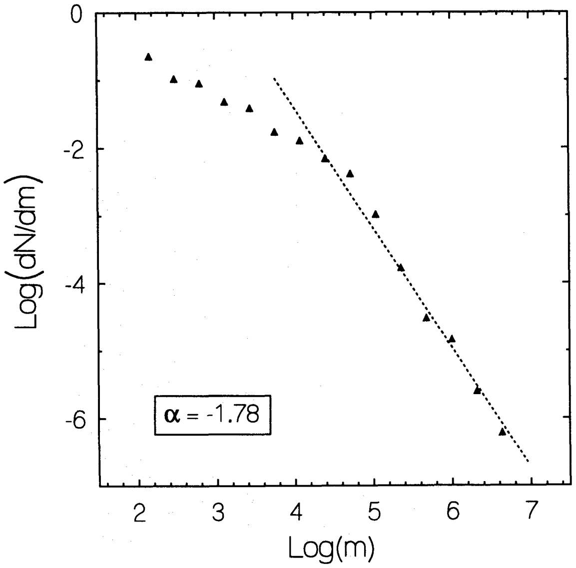

FIG. 1*e*

FIG. 1.—Evolution of the molecular cloud mass spectra with time. The clouds lie in the disk between radii 4.5–7 kpc. (*a*) $t = 0$ yr; (*b*) $t = 1 \times 10^7$ yr; (*c*) $t = 2 \times 10^7$ yr; (*d*) $t = 3 \times 10^7$ yr; (*e*) $t = 4 \times 10^7$ yr.

pated away as heat. Since we have not included energy losses due to dissipation in our simulations, the resulting cloud velocities may be an overestimate of the real velocities. This can partly explain why our velocities are higher.

2. The random velocity does not depend on cloud mass, as can be seen from Figures 4 and 5. In the determination of the velocity spectrum, the mean velocity of clouds in each mass bin was determined. But this averaging is not efficient for the large-mass bins, as there are not enough large-mass clouds. This leads to some fluctuation in the high-mass end of the spectra.

3. The lack of dependence of the random velocity on mass is also observed in the molecular clouds in the Galactic disk (Casoli et al. 1984; Stark 1984). This indicates that the clouds are not in equipartition, as otherwise the square of the random velocity would depend on the inverse of cloud mass, i.e., $v \propto m^{-1/2}$. Also, if gravitational interaction was the only mechanism accelerating the clouds, then one would expect the smaller mass clouds to have a much higher velocity than the large-mass clouds. Since this is not the case, there must be some other mechanism acting to equalize cloud velocities. Cloud collisions leading to

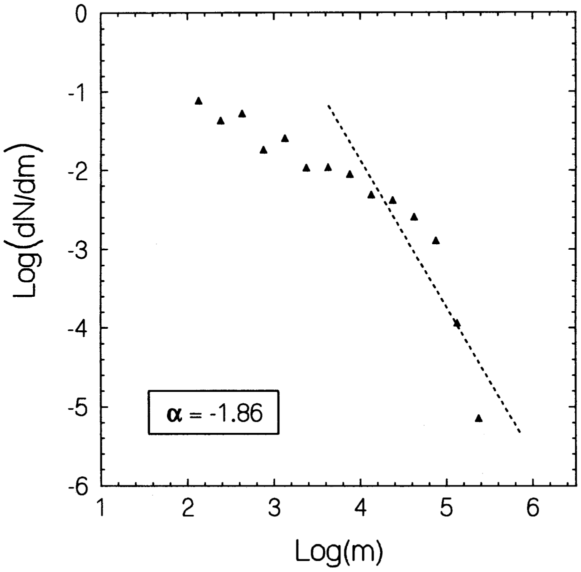

FIG. 2.—The molecular cloud mass spectrum for clouds lying between 3–4.5 kpc in the disk, at a time $t = 4 \times 10^7$ yr in the simulation.

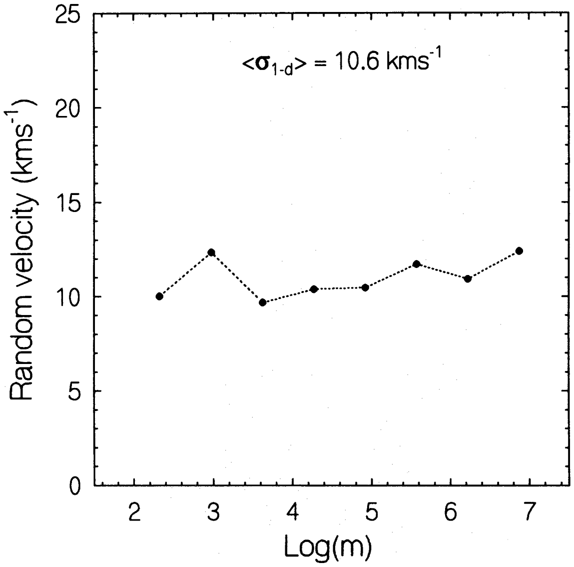

FIG. 4.—The molecular cloud velocity spectrum for clouds lying between 4.5–7 kpc in the disk, at a time $t = 4 \times 10^7$ yr in the simulation.

coalescence and fragmentation may prevent the smaller clouds from being accelerated between collisions if the collisions are frequent enough. For a power-law type of mass spectrum, there are a greater number of small-mass clouds, and their repeated collisions may be the reason behind the nonequipartition of energy between clouds of different masses.

6. DISCUSSION

1. The random cloud velocities obtained in our simulation are ~8–10 km s^{-1} for the region 4.5–9.5 kpc and ~11 km s^{-1} for the region 3–4.5 kpc. These values are somewhat higher than the observed cloud velocity dispersion, which is $\sigma_{1d} \sim 3$–7 km s^{-1}. The reason is probably because we have evolved a local region of molecular clouds in our simulations. To reduce this effect, the ratio of the perimeter of the region to its area in the disk P/A should be a minimum (see § 4). To study this effect, we evolved a distribution of clouds within a smaller area in the disk between radii 5–6 kpc which has a ratio $P/A \sim 4$. The mass spectrum has a form similar to that for a larger distribution, and the spectral index $\alpha \sim -1.8$. The steady random velocity of the clouds is ~11.2 km s^{-1}. However, for the distribution between 4.5–7 kpc, $P/A \sim 1.6$, and the steady random velocity is only ~10.5 km s^{-1} (Fig. 1). For the same reason, the cloud random velocity in the region 3–4.5 kpc is higher than that for clouds in the outer disk, which have been distributed over a larger area. Another reason for the higher velocities in the inner region could be that there is a higher velocity shear at lesser radii, due to differential rotation in the disk. The higher shear results in a higher P/A ratio in the simulation, thus affecting the velocity dispersion.

2. As mentioned in the previous section, the main factor influencing the collisional buildup of molecular clouds is the gas surface density in the disk. For a low gas surface

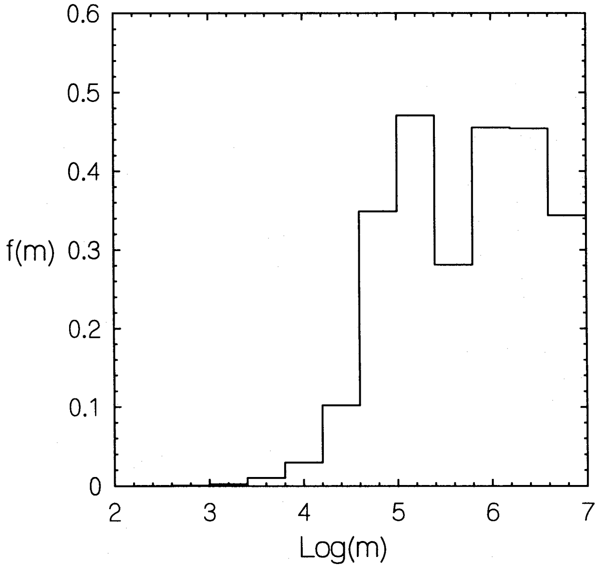

FIG. 3.—The fraction of cloud mass, $f(m) = 1/M \ d(m)/d \log(m)$, is plotted against the logarithm of the mass range; the clouds lie between 4.5–7.0 kpc in the disk at time $t = 4 \times 10^7$ yr in the simulation. Note that most of the cloud mass is in clouds more massive than $10^5 \ M_\odot$.

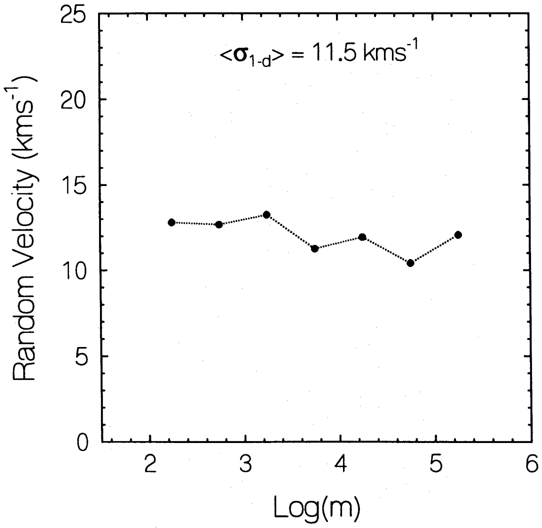

FIG. 5.—The molecular cloud velocity spectrum for clouds lying between 3–4.5 kpc in the disk, at a time $t = 4 \times 10^7$ yr in the simulation.

density, as in the disk for radii less than 4 kpc, the collisional buildup is very slow. But it is important for clouds in the molecular ring region beyond 4 kpc.

3. Star formation was not included in our simulation of cloud interactions and collisions in the disk of the Galaxy because the aim of the work was to study the effect of local interactions on the collisional buildup of molecular clouds. The clouds in the simulations attain a steady distribution after 2×10^7 yr, and they are evolved for a further 2×10^7. We feel that neglecting star formation for a period of 2×10^7 yr will not produce a major effect on the cloud distribution. This is confirmed by the fact that our simulation results agree well with the observed mass and velocity distributions.

4. In the simulations, the clouds initially had a mean random velocity of $\sigma_{1d} \sim 2$ km s^{-1}. Within a few times 10^7 yr, the clouds interact, and the random velocity becomes $\sigma_{1d} \sim 8$ km s^{-1}. Hence, the clouds acquire random motion from galactic rotation through their gravitational interactions and physical collisions. This energy input mechanism has been proposed and examined in detail in the literature (Fukunaga & Tosa 1989; Jog & Ostriker 1988), but only for a pair of molecular clouds of a particular mass. Our present simulation is a more general model of gravitationally interacting clouds in which the clouds are of different masses. Further, we have simultaneously evolved the mass and velocity spectra of molecular clouds. Hence, we could show that the resulting velocity spectrum did not depend on the cloud mass.

5. To study the relative importance of local gravitational interactions and cloud collisions, we evolved the cloud distribution between 4.5–7 kpc without including gravitational interaction forces. The initial conditions were the same as before. We find that the velocity distributions are similar, but the mass distribution is slightly steeper when local gravitational interaction between the clouds is not included. The mass spectrum has a slope $\alpha = -1.94$ for the latter case and $\alpha = -1.78$ when the gravitational interaction between the clouds is also taken into account. This means that gravitational interaction enhances the collisional buildup of molecular clouds. However, the difference between the two cases is not large enough to be significant. Thus, collisions rather than gravitational interaction in a sheared disk are the dominant effect for determining the dynamics of molecular clouds.

6. The initial mass distribution used in the simulations has a linear, random form. There is thus a larger fraction of high-mass clouds, and hence the distribution may appear to be more favorable for the collisional buildup model of molecular clouds. Note, however, that the mass doubling time by collisions of equal mass clouds is 10^8 yr (Elmegreen 1988). When the gravitational focusing term $\{1 + [2G(m_1 + m_2)]/[(r_1 + r_2)|\boldsymbol{v}_1 - \boldsymbol{v}_2|^2]\}$ is included, this is further reduced to a few times 10^7 yr. Hence, the initial mass spectrum used in the simulations will not alter the collisional buildup time of the clouds by a significant amount.

7. CONCLUSIONS

We have studied the evolution of a local region of gravitationally interacting and colliding molecular clouds in a sheared galactic disk. The outcome of a collision is either coalescence or fragmentation, depending on the relative masses or velocities of the clouds. We have plotted the mass and velocity spectra of the clouds, and they are similar to the observed mass and velocity distribution of molecular clouds in the Galactic disk. For molecular clouds at radii greater than 4 kpc in the disk, the mass spectrum has a slope of ~ -1.8, and 50% of the total molecular gas mass is in clouds more massive than 3.5×10^5 $M_\odot$. The mean one-dimensional random velocity of the clouds obtained is $\sim$8–10 km s^{-1}. This is slightly larger than the observed value, and the discrepancy is due to the fact that a local distribution of clouds was evolved in the simulations, and second, the energy losses due to gas shocking during cloud collisions have not been included. This was done to make the problem numerically tractable. The random velocity is independent of cloud mass, indicating a non-equipartition of energy between the clouds. This could be due to the repeated inelastic collisions between the clouds, which prevents them from being accelerated by gravitational interactions with other clouds. We also find that collisions rather than local gravitational interactions are important for determining the mass and velocity distribution of molecular clouds. Thus, collisions and gravitational instabilities together are perhaps the most important factors in the formation of molecular clouds.

M. D. would like to thank Rajaram Nityananda for discussions and valuable suggestions throughout the course of this work. We would like to thank the referee for his suggestions regarding the relative importance of gravitational interactions versus collisions. The numerical computations of this work were done at SERC, Indian Institute of Science, Bangalore. This work is part of the thesis work of M. D. and was supported by a research grant from CSIR, India [award number 9/79(269)/89-EMRI].

REFERENCES

Carlberg, R. G., & Innanen, K. A. 1987, AJ, 94(3), 666
Casoli, F., & Combes, F. 1982, A&A, 110, 287
Casoli, F., Combes, F., & Gerin, M. 1984, A&A, 133, 99
Combes, F., & Elmegreen, B. G. 1993, A&A, 271, 391
Elmegreen, B. G. 1988, in Interstellar Processes, ed. D. J. Hollenbach & H. A. Thronson, (Dordrecht: Reidel), 254
———. 1989, ApJ, 344, 306
Falgarone, E., & Puget, J. L. 1986, A&A, 162, 235
Field, G. B., & Saslaw, W. C. 1965, ApJ, 142, 568
Fukunaga, M., & Tosa, M. 1989, PASJ, 41, 241
Gilden, D. L. 1984, ApJ, 279, 335
Hausman, M. A. 1981, ApJ, 245, 72
Jog, C. J., & Ostriker, J. P. 1988, ApJ, 328, 404
Jog, C. J., & Solomon, P. M. 1984, ApJ, 276, 114
Kwan, J., & Valdes, F. 1987, ApJ, 315, 92
Larson, R. B. 1981, MNRAS, 194, 809
Lattanzio, J. C., Monaghan, J. J., Pongracic, H., & Schwarz, M. P. 1985, MNRAS, 215, 125
Loren, R. B. 1989, ApJ, 338, 902
Nozakura, T. 1990, MNRAS, 243, 543
Nozawa, S., Mizuno, A., Teshima, Y., Ogawa, H., & Fukui, Y. 1991, ApJS, 77, 647
Oort, J. H. 1954, B.A.N., 12, 177
Sanders, D. B., Scoville, N. Z., & Solomon, P. M. 1985, ApJ, 289, 373
Scoville, N. Z., & Hersh, K. 1979, ApJ, 229, 578
Scoville, N. Z., & Sanders, D. B. 1988, in Interstellar Processes, ed. D. J. Hollenbach & H. A. Thronson, Jr. (Dordrecht: Reidel), 21
Solomon, P. M., & Rivolo, A. R. 1989, ApJ, 339, 919
Solomon, P. M., Rivolo, A. R., Barrett, J., & Yahil, A. 1987, ApJ, 319, 730
Sotnikova, N., & Volkov, E. 1994, A&A, 288, 942
Stark, A. A. 1984, ApJ, 281, 624
Stutzki, J., & Gusten, R. 1990, ApJ, 356, 513
Thomasson, M., Donner, K. J., & Elmegreen, B. G. 1991, A&A, 250, 316