

# Fermions obstruct dimensional reduction in hot QCD

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We have studied, for the first time, screening masses obtained from glueball-like correlators in Quantum Chromodynamics with four light dynamical flavours of quarks in the temperature range  $1.5T_c \leq T \leq 3T_c$ , where  $T_c$  is the temperature at which the chiral transition occurs. We have also studied pion-like and sigma-like screening masses, and found that they are degenerate in the entire range of  $T$ . These obstruct perturbative dimensional reduction since the lowest glueball screening mass is heavier than them. Extrapolation of our results suggests that this obstruction may affect the entire range of temperature expected to be reached even at the Large Hadron Collider.

11.15.Ha, 12.38.Mh

TIFR/TH/00-17, hep-lat/0004011

The quark-gluon plasma phase of QCD is currently of great experimental interest. The recent announcement that the CERN heavy ion program may have seen this phase, and the forthcoming start of the BNL Relativistic Heavy Ion Collider are just two reasons for this interest. The phase structure of baryon-free QCD matter is pretty well determined through lattice simulations [3]. While this was sufficient in the past, experiments will soon begin to demand much more detailed information on the high temperature phase. In this letter we report work that bears on the detailed structure of this phase.

Lattice simulations of finite temperature ( $T$ ) equilibrium field theories use a discretisation of the Euclidean formulation for partition function—

$$Z(T) = \int \mathcal{D}\phi \exp \left[ - \int_0^{1/T} dt \int d^3x \mathcal{L}(\phi) \right], \quad (1)$$

where  $\phi$  is a generic field,  $\mathcal{L}$  the Lagrangian density, and the Euclidean “time” runs from 0 to  $1/T$ . The path integral is over Bosonic (Fermionic) field configurations which are periodic (anti-periodic) in Euclidean time. For the lattice problem,  $Z(T)$  is the trace of an appropriate power of the transfer matrix,  $\mathcal{T}$ , in one of the spatial directions. All the information in the theory is encoded in the set of eigenvalues of  $\mathcal{T}$ ,  $\Lambda_i(T)$ .

Thermodynamics is determined by the largest eigenvalue,  $\Lambda_0$ , and its derivatives with respect to various parameters of the theory. The spectrum of screening masses (inverse correlation lengths),

$$\mu_i(T) = \log \left[ \frac{\Lambda_0(T)}{\Lambda_i(T)} \right], \quad (2)$$

is in one-to-one correspondence with the remaining eigenvalues, and hence contain all the remaining information about the  $T > 0$  physics of the theory. Due to the lack of symmetry between the space and Euclidean time directions in eq. (1),  $\mathcal{T}$  has only a subgroup of the rotational symmetry of the  $T = 0$  Euclidean theory. Hence the spectrum of  $\mu_i$  is classified by the irreducible representations

(irreps) of this new symmetry group. These symmetries and their irreps have been studied in [4,5].

Dimensional reduction has provided a good qualitative picture of the high temperature limit of field theories. This reduction yields an effective theory describing equilibrium physics of the (3+1)-dimensional theory at distance scales much greater than  $1/T$ . At these distances only fields with low momentum ( $< T$ ) excitations survive; field excitations with momentum scales of order  $T$  or more are integrated out. In practice this procedure is carried out perturbatively [6]. Since the Fourier modes of Boson fields have momenta  $2n\pi T$  in the Euclidean time direction and Fermion modes have momenta  $(2n+1)\pi T$  (where  $n$  is a non-negative integer), therefore all Fermion modes and all non-zero gauge field modes are integrated out. The result of this integration is a 3-dimensional theory of gauge fields coupled to adjoint scalars.

All the screening masses remaining at long distances should then belong to the glue sector of the (3+1)-dimensional theory, with Fermions appearing only in loop corrections. This approach has had great success in dealing with the electroweak sector of the standard model [7], and has been proposed as a way of dealing with high temperature QCD [8,9]. A key question is about the temperature range where this can be done reliably.

Lattice measurements test dimensional reduction in three ways—

1. The degeneracies of  $\mu_i$  test whether the effective symmetries at high temperatures are those of a theory in lower dimension. This form of dimensional reduction has been seen in (3+1)-dimensional pure gauge  $SU(2)$  and  $SU(3)$  theory at temperatures as low as  $2T_c$  [4,10].
2. Perturbative matching of the couplings of the dimensionally reduced theory can be tested by comparing its correlators and screening masses with those of the full theory. Comparison of the results of [4,10] with those in 3-d  $SU(2)$  [11] and  $SU(3)$  [12] gauge theories with an adjoint scalar demon-

strate such dimensional reduction, as do the results of [13]. More recently, this reduction has also been tested in the gauge sector of  $(2 + 1)$ -dimensional QCD [14].

3. All low-lying screening masses must be shown to arise in the gauge sector of the theory if the perturbative matching is correct. Here we report that  $(3+1)$ -dimensional QCD with 4 degenerate flavours of light dynamical quarks fails this test.

We have performed Hybrid Monte Carlo simulations of  $SU(3)$  gauge theory with 4 flavours of dynamical staggered quarks at temperatures above the first-order phase transition temperature  $T_c$  [15]. The simulations were done at three different couplings, and the lattice size in the Euclidean time direction,  $N_\tau$ , was chosen to be 4. The temperature assignments at the couplings  $\beta = 5.1$  ( $T = 1.5T_c$ ) and  $\beta = 5.15$  ( $T = 2T_c$ ) are made using the results of previous simulations at  $N_\tau = 6$  and 8 [16,17]. We also made a simulation at  $\beta = 5.35$ , corresponding to  $T = (2.9 \pm 0.1)T_c$  [18]. We have chosen the bare quark mass to be  $m = 0.02T_c$ . We performed a finite size scaling study at  $2T_c$  using  $4 \times 16 \times 10^2$ ,  $4 \times 24 \times 10^2$ ,  $4 \times 24 \times 12^2$  and  $4 \times 16^3$  lattices. At  $1.5T_c$  we worked with a lattice size of  $4 \times 16 \times 10^2$  and at  $2.9T_c$  with  $4 \times 18^3$ . The lattice sizes have been chosen so that the correlators can be followed in at least one direction to spatial distance of  $2/T$  or more, and the transverse size of the lattice is large enough to avoid spatial deconfinement. In thermal equilibrium, the HMC trajectory lengths were taken to be one unit of molecular dynamics time except for the cubic lattice at  $2T_c$  where it was taken to be half an unit. Thermalisation was monitored using four variables—the spatial and temporal plaquettes, the Wilson line, and the quark condensate.

We have analysed correlation functions constructed from Wilson loops of various sizes (glueball-like correlators) for the first time in full QCD at  $T > 0$ . We have also analysed meson-like correlators, *i. e.*, those constructed with a  $\bar{q}q$  pair. The particular operators that we have examined belong to the scalar and vector irreps of the  $T = 0$  theory. Under the symmetry of the spatial transfer matrix for  $T > 0$ , *i. e.*, the group called  $D_4^h$ , the former belong to an irrep called the  $A_1^+$ . The vector irrep of  $T = 0$  reduces under the  $T > 0$  symmetry group: two linear combinations give the irrep  $A_1^+$  and one gives an irrep called  $B_1^+$ .

The glueball-like operators used in this study were the plaquette and the planar 6-link loop. Noise reduction was done through a variant of the fuzzing technique. The operators were constructed at each of five levels of fuzzing, and a variational method was used to extract the lowest mass in the channel and project out the corresponding correlator. More details of the methodology can be found in [4]. We could follow the correlation function in the  $A_1^+$

sector up to a distance of 4, enabling us to obtain its mass with a reasonable degree of confidence.

The meson-like operators used here are local operators corresponding to the  $\pi$ ,  $\sigma$ ,  $\rho$  and  $A$  at  $T = 0$ . These screening correlators have been studied for  $T > 0$  in the  $A_1^+$  channel earlier [20]. We report below our measurements in this channel. The first measurement in the  $B_1^+$  channel was reported recently [21]. At all three temperatures studied here, the  $B_1^+$  correlators vanished within errors, consistent with free field theory and unlike the  $B_1^+$  correlators in the glue sector obtained from the same simulations.

We made direct measurements of the autocorrelation function for the  $\pi$ -like correlator,  $C_\pi(z)$ , at  $T = 2T_c$  on the  $4 \times 24 \times 10^2$  lattice. For the correlators at  $z = 0$  and  $z = 6$  we found comparable autocorrelation times, somewhat less than 1 MD trajectory. For  $z > 6$  autocorrelations are hard to measure directly. We limit them by the following argument. For measurements of a correlator  $C(z)$  where the relative errors  $e(z)/C(z)$  are expected to be similar in magnitude, strong decreases signal large autocorrelations. Plotting  $e(z)/C(z)$  for the  $\pi$ -like correlator as a function of  $z$ , we found no decrease for  $z$  close to the center of the lattice. This makes the existence of strong autocorrelations rather unlikely, and eliminates possible simulation algorithm related systematic errors in mass measurements.

In the meson-like channels we have estimated the screening masses by two independent techniques—fitting and constructing local masses. On all lattices these two methods gave consistent results.

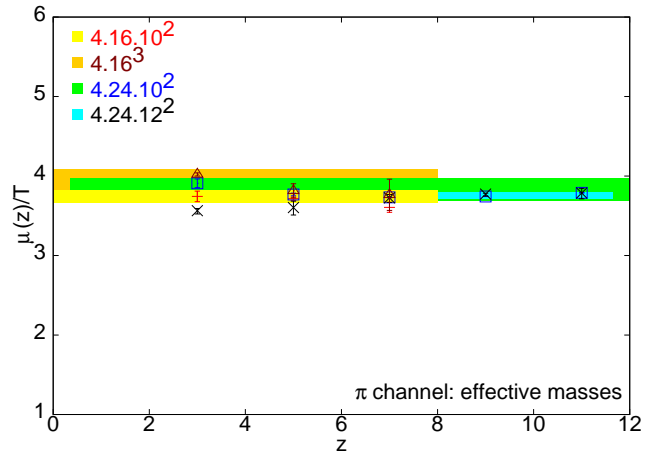


FIG. 1. The  $A_1^+$  local mass obtained using local pionic operators at  $T = 2T_c$  on four different lattices. Also shown are the  $1\text{-}\sigma$  band on the corresponding fitted masses.

Our finite size scaling study showed that finite volume effects are under control. Figure 1 shows the local masses obtained from the pion-like correlator on the four different lattices at  $2T_c$ . Also shown as bands are our estimates of the screening masses obtained by a fit to the

same correlator—

$$\mu_\pi(A_1^+)/T = \begin{cases} 3.74 \pm 0.08 & (4 \times 16 \times 10^2 \text{ lattice}), \\ 3.92 \pm 0.16 & (4 \times 16^3 \text{ lattice}), \\ 3.83 \pm 0.14 & (4 \times 24 \times 10^2 \text{ lattice}), \\ 3.76 \pm 0.04 & (4 \times 24 \times 12^2 \text{ lattice}). \end{cases} \quad (3)$$

Comparing the long ( $N_z = 24$ ) and short ( $N_z = 16$ ) lattices, we find that the effective masses stabilise very well to a common value. At the same time, this estimate of the screening mass is independent of the transverse spatial size of the lattice, indicating that the low lying eigenvalues of the transfer matrix have become independent of the lattice size. Therefore we have determined the infinite volume screening mass accurately. Similar lattice size independence of screening masses was observed in the other channels as well.

The screening masses from the  $\pi$ -like and  $\sigma$ -like correlators were equal within errors, as were those from the  $\rho$ -like and the  $A$ -like correlators. We investigated possible mass splittings more carefully by making simultaneous fits to these pairs of correlators, taking into account the covariance of the data due to the fact that the correlators are constructed from the same configurations. This analysis showed no hint of any mass splitting. The results are collected in Table I.

In principle, the  $\sigma$ -like correlator in the 4-dimensional theory could mix with the vacuum and hence with the  $A_1^+$  glueball-like correlator. However the  $\pi$ -like channel does not mix with the 4-dimensional vacuum, and the above degeneracy of  $\mu_\pi(A_1^+)$  and  $\mu_\sigma(A_1^+)$  cannot be explained if the latter is small due to mixing with  $\mu_g(A_1^+)$ . Hence screening masses seen in these two channels are characteristic of fermion-bilinear currents in the theory. Furthermore, our results taken in conjunction with earlier studies at different cutoffs [20] indicate that  $\pi$ - $\sigma$  degeneracy and the small value of their common screening mass, at these temperatures, are cutoff independent.

Our best estimates of the  $A_1^+$  projection of the  $\rho$ -like screening mass is—

$$\mu_\rho(A_1^+)/T = 5.6 \pm 0.1 \quad (4)$$

at  $T = 2T_c$ . For  $N_\tau = 4$ , free field theory gives  $\mu_\rho(A_1^+)/T = 5.27$ . At other temperatures we also found values of  $\mu_\rho(A_1^+)/T$  close to, but slightly higher than, this perturbative result. The ratio  $\mu_g(B_1^+)/T$  was found to be close to this, but much larger statistics are needed to have any reliable estimate of this number.

In Figure 2 we show  $\mu_\pi(A_1^+)/T$  and  $\mu_g(A_1^+)/T$  as a function of  $T/T_c$ . In contrast to the slow variation of  $\mu_g(A_1^+)/T$  with changing  $T$ , we see that  $\mu_\pi(A_1^+)/T$  is increasing. The latter ratio is smaller up to  $2T_c$ , but there is a cross-over near  $2.9T_c$ . It is possible that at sufficiently large  $T$  the perturbative value  $\mu_\pi(A_1^+)/T = \mu_\rho(A_1^+)/T = 5.27$  is reached [22]. In accordance with this, we tried various smooth and monotonic two parameter fits to the data which have this limit as  $T \rightarrow \infty$ , and

have a finite and non-zero value at  $T = T_c$ . Two such fits are shown in the figure. Extrapolating these to higher temperatures, we find that  $\mu_\pi(A_1^+)$  becomes compatible with free fermion field theory only for  $T_p \approx 10T_c$  or more. It would be interesting to test this extrapolation by direct simulation [23].

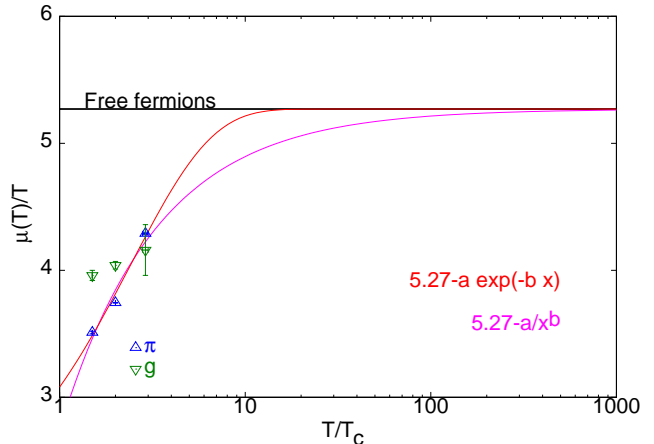


FIG. 2. The  $A_1^+$  screening masses obtained using local pion and glueball operators shown as a function of temperature along with phenomenological fits to the former. The fit formulae are in the same vertical order as the fit lines at large  $T/T_c$ .

Our results have implications for the dimensionally reduced theory. If the perturbative approach to dimensional reduction were accurate at these temperatures, then one should be able to integrate out the Fermions altogether. Their contribution would be visible only in the effective couplings between the zero momentum gauge modes. At the same time,  $\mu(A_1^+)$  should be independent of whether it was measured in the  $\pi$ -like or  $\rho$ -like channel and, being close to the free-field value, should decouple from the long distance physics. However, something non-perturbative happens in the Fermion sector at experimentally accessible temperatures of up to about 1 GeV.

Since  $\mu_\pi(A_1^+) < \mu_g(A_1^+)$  up to  $3T_c$ , this excitation cannot be integrated out of the effective theory that describes physics at the length scale  $1/\mu_g(A_1^+)$ . This state of affairs cannot be accommodated into a perturbative description of dimensional reduction, since all Fermions and multi-Fermion states need to be integrated out in such an approach. For  $3T_c < T < 10T_c$ ,  $\mu_\pi(A_1^+)$  still deviates sufficiently from its perturbative value to obstruct perturbative approaches to dimensional reduction, although the obstruction becomes progressively smaller.

The values of  $\mu_g(A_1^+)/T$  we measured in 4-flavour QCD are 53% higher than in quenched QCD [4]. Although comparisons of glueball masses at  $T = 0$  in quenched and full QCD show much smaller shifts [24], the increase

in the Debye screening mass, in going to the four-flavour theory is about 30% in leading order of the perturbation expansion at any  $T$ . At the same order, the increase in  $\mu_g(A_1^+)/T$  is expected to be about 60%. Although perturbation theory may not be reliable here, this estimate shows that the large shift is not unnatural. In contrast,  $\mu_\pi(A_1^+)/T$  decreases by 17% when going from quenched to 4-flavour QCD. The obstruction to dimensional reduction appears very clearly in 4-flavour QCD due to these two movements. However it is present even in quenched QCD, albeit in a subtle form, since  $\mu_g(A_1^+) < \mu_\pi(A_1^+) < \mu_g(B_1^+)$  for  $T \approx 1.5-2T_c$ .

This first measurement of  $\mu_g(A_1^+)$  in full QCD thus sheds new light on the effective long distance theory at  $T > T_c$ . As far as gauge invariant correlators are concerned, the situation up to  $2T_c$  is reminiscent less of the usual dimensional reduction than of the  $T = 0$  situation where the long distance effective theory is built from scalar and pseudo-scalar fermion bilinear composites. Thermodynamics, governed by  $\Lambda_0(T)$  alone, cannot be explained by this effective model. In any case, there are differences between the  $T = 0$  and  $T > 0$  effective theories. At  $T = 0$  chiral symmetry is broken and the scalar and pseudo-scalar masses are different. Here, on the other hand, this symmetry is restored and the masses are equal.

Finally we mention that a weaker form of dimensional reduction may still be valid in this range of temperatures. This can be investigated by looking for degeneracies of all measurable screening masses and checking whether they reflect the 4-dimensional  $T > 0$  symmetries or an effective 3-dimensional symmetry. This requires a measurement of correlations of non-local Fermion bilinear operators, and is left for the future. Also left for the future is a detailed investigation of the continuum limit of our results. A similar investigation in the more realistic case of 2 light and one medium heavy flavour of quarks is a desirable future computation, specially with Wilson Fermions.

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[18] From [19] we know that the ratio of the lattice spacings, as determined by the  $T = 0$   $\rho$  mass, at  $\beta = 5.15$  and  $5.35$  is  $1.45 \pm 0.05$ . The same result is also obtained by setting the scale through the proton mass.  
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[22] In free field theory we have  $\mu_\pi(A_1^+) = \mu_\rho(A_1^+)$ . However, since  $\mu_\rho(A_1^+)$  is shifted a little, indicating small interaction effects, the same effects could induce a splitting between these two masses. On the other hand, at high temperatures, if the large effects in  $\mu_\pi$  do go away, then this small shift in  $\mu_\rho$  could also become smaller. For this reason we have taken the asymptotic value of  $\mu_\pi(A_1^+)$  to be the free field value. However, no significant change in  $T_p$  occurs with the choice  $\mu_\pi(A_1^+)/T = 5.6$ .  
[23] Extending our computations to higher temperatures is not simple. Finding  $\beta$  corresponding to  $T \approx 10T_c$  needs a sequence either of direct computations of the critical couplings at larger  $N_\tau$ , or of  $T = 0$  computations to determine lattice spacings.  
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$T$	statistics	$\Delta\mu/T$	$\mu_\pi(A_1^+)/T$	$\mu_g(A_1^+)/T$
1.5	900	$0.04 \pm 0.14$	$3.51 \pm 0.01$	$3.96 \pm 0.04$
2.0	1875	$0.06 \pm 0.24$	$3.744 \pm 0.001$	$4.04 \pm 0.03$
2.9	825	$0.03 \pm 0.06$	$4.29 \pm 0.01$	$4.16 \pm 0.20$

TABLE I. Statistics and measurements of various screening masses and the splitting  $\Delta\mu \equiv \mu_\pi(A_1^+) - \mu_\sigma(A_1^+)$  at different  $T$ . For  $T = 2T_c$  the results come from the  $4 \times 24 \times 10^2$  lattice.