Quark number susceptibilities, strangeness and dynamical confinement

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We report first results on the strange quark number susceptibility, χ_s, over a large range of temperatures, mainly in the plasma phase of QCD. χ_s jumps across the phase transition temperature, T_c, and grows rapidly with temperature above but close to T_c. For all quark masses and susceptibilities in the entire temperature range studied, we found significant departures from ideal-gas values. We also observed a strong correlation between these quantities and the susceptibility in the scalar/pseudo-scalar channel, supporting ideas of "dynamical confinement" in the high temperature phase of the QCD plasma.

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As seen recently in the Quark Matter 2001 conference [3], the first runs of the RHIC have provided many exciting results. With the anticipated much longer runtime, at a variety of colliding energies and nuclear species, the existence and properties of the high temperature phase of quantum chromodynamics (QCD) will be probed and tested further. This is an immediate reason to improve our theoretical knowledge of such matter.

Fully non-perturbative computations of quark number susceptibilities are important for three reasons. Firstly, there have been attempts to link them directly to experimental measurements of event-to-event fluctuations in particle production [4]. Secondly, experimental observations of a relative enhancement of strange quarks have been attributed to the formation of a QCD plasma [5]; a hypothesis which can be quantitatively tested against the computation of the strange quark susceptibility. Finally, earlier results [6–9] showed a strong jump in the susceptibility across the phase transition, but indicated a statistically significant 20% departure from weak-coupling behaviour; this is the only (lattice) evidence that physics at finite chemical potential is not weak-coupling physics. In this paper we present the first extensive systematic study of such susceptibilities and relate them to other widely studied variables.

The partition function of QCD with three quark species can be written on a discrete space-time lattice as

$$Z(T,\mu_u,\mu_d,\mu_s) = \int_{U,\psi,\bar\psi} \exp[-S(T)] \times \prod_{f=u,d,s} \det M(T,m_f,\mu_f), \qquad (1)$$

where the temperature T enters through the size of the Euclidean time direction, $S(T)$ is the gluonic part of the action, the determinants of the Dirac matrices, M, contain as parameters the quark masses, m, and the chemical potentials μ. The chemical potentials for specific flavours, μ_f ($f = u, d, s$), can be expressed as linear combinations of the chemical potentials, μ_α (α=0,3,8), corresponding to the diagonal generators of flavour $SU(3)$. We work with the choice $\mu_0 = \mu_u + \mu_d + \mu_s$, $\mu_3 = \mu_u - \mu_d$ and $\mu_8 = \mu_u + \mu_d - 2\mu_s$, because μ_0 is then the usual baryon chemical potential.

We follow convention in defining quark numbers

$$n_i(T,\mu_u,\mu_d,\mu_s) = \frac{T}{V}\frac{\partial \log Z}{\partial \mu_i}, \qquad (2)$$

and number susceptibilities

$$\chi_{ij}(T,\mu_u,\mu_d,\mu_s) = \frac{T}{V}\frac{\partial^2 \log Z}{\partial \mu_i \partial \mu_j}. \qquad (3)$$

Here V denotes the spatial volume, and i and j are either of the index sets f or α. Transformations between these bases are straightforward. We lighten our notation by writing the diagonal susceptibilities χ_{ii} as χ_i. We determine the susceptibilities at zero chemical potentials, $\mu_f = 0$. In this limit, of course, $n_i(T) = 0$ for all i. It is also easy to prove that $\chi_{03} = 0$ if $m_u = m_d$. Since all current lattice computations are made in this approximation, the off-diagonal susceptibility cannot be measured at present. Also, in this limit, $\chi_{us} = \chi_{ds}$, where the strange quark mass, $m_s \neq m_{u,d}$.

Previous works have reported the values of the flavour $SU(2)$ singlet, χ_0, and triplet, χ_3, susceptibilities. Measurements of $\chi_{0,3}$ in 2-flavour dynamical QCD showed a statistically significant 20% departure from free field theory (FFT) for $m/T = 0.1$ and 0.15 [6,8]. Later computations with $m/T = 0.05$ reported $\chi_{0,3}$ compatible with FFT but with errors of about 20% [9]. Quenched computations deviated from the first set of dynamical QCD results by 5–10% [7]. All this work covered a range of temperatures up to $1.5T_c$. However, the quark mass varied with temperature, since the quantity that was fixed was m/T.

In order to facilitate a comparison of our results with these, we report the same susceptibilities. In the remainder of this paper, χ_0 and χ_3 refer to these 2-flavour quantities. We also report the first determination of the strange quark susceptibility, χ_s. This work improves on previous studies in four other ways— by covering a larger range of temperatures, by using a series of different quark masses at fixed m/T_c, by investigating finite spatial volume effects systematically, and by analysing the effect of increasing statistics in the stochastic determination of Fermion operators on the lattice. The study of volume dependence gives control over extrapolation to the thermodynamic limit. Our study of many different quark masses gives the strange quark susceptibility. The systematic study of the stochastic method yields a vastly improved determination of the flavour-singlet susceptibility, χ_0.

These computations have been made on lattices with lattice spacing $a = 1/4T$ in the quenched approximation. Fermion loops are neglected in this approximation, making it substantially easier to handle numerically. Apart from an overall normalisation of the temperature scale, this approximation is known to reproduce all the qualitative features of the full QCD simulations. Furthermore numerical agreement between the quenched and 2-flavour dynamical QCD results for $\chi_{0,3}$ are obtained by 5–10% correction of the former [7]. It has been shown recently that one can extract continuum results from the lattice spacing we employ [10]. Nevertheless, in future studies we will analyse the effects of relaxing these two approximations.

Successive configurations used in our computations are separated by 1000 sweeps of a Cabbibo-Marinari heat bath algorithm, so that the gauge fields are completely decorrelated by any measure one may choose to use. At $\beta = 5.8941$, corresponding to $T = 1.5T_c$ we have generated configurations on 4×8^3, 4×12^3 and 4×16^3 lattices. At other couplings, corresponding to $T = 0.75T_c$, $1.1T_c$, $1.25T_c$, $2T_c$ and $3T_c$, we have used only the 4×12^3 lattice. We have used quark masses $m/T_c = 3$, 1.5, 1, 0.75, 0.30 and 0.03. Using estimates of T_c=275–290 for the quenched theory [10], we see that the strange quark mass lies between $0.3T_c$ and $0.5T_c$.

In this letter we use staggered Fermions exclusively. Since these are defined for four flavours, we have to normalise the susceptibilities appropriately [11]. We have

$$\chi_0 = \frac{1}{2}(\mathcal{O}_1 + \frac{1}{2}\mathcal{O}_2),$$
$$\chi_3 = \frac{1}{2}\mathcal{O}_1, \qquad (4)$$
$$\chi_s = \frac{1}{4}(\mathcal{O}_1 + \frac{1}{4}\mathcal{O}_2),$$

where the two operators involved are

$$\mathcal{O}_1 = \frac{T}{V}\left\langle \mathrm{Tr}\ \left(M''M^{-1} - M'M^{-1}M'M^{-1}\right)\right\rangle,$$
$$\mathcal{O}_2 = \frac{T}{V}\left\langle \left(\mathrm{Tr}\, M'M^{-1}\right)^2\right\rangle. \qquad (5)$$

The traces here are sums over lattice points and colour indices, and angular brackets are averages over gauge field configurations. Primes denote derivatives of the Dirac matrix with respect to appropriate chemical potentials. The quark mass to be used in the Dirac operator for evaluating χ_s is, of course, different from that for $\chi_{0,3}$.

The traces are evaluated by the usual stochastic technique,

$$\mathrm{Tr}\, A = \frac{1}{N}\sum_{i=i}^{N} R_i^\dagger A R_i, \qquad (6)$$

where R_i are a set of N uncorrelated vectors with components drawn independently from a Gaussian ensemble with unit variance. Each vector has three colour components at each site of the lattice. We improve on the definitions in eq. (5) by using half lattice versions of the Dirac operator for staggered Fermions. A detailed discussion of the stochastic evaluation of the squared trace in eq. (5) can be found in [6]. A systematic study of the optimum value of N is shown in Figure 1. We find that $N \approx 80$ is needed in order to see that $\chi_0 = \chi_3$ within statistical errors [12]. In all our subsequent work we have used $N = 80$ [9]. Such a large value of N also seems to decrease the variance in the average over gauge configurations.

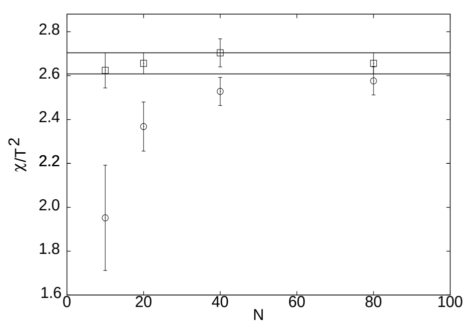

FIG. 1. Variation of χ_0 (circles) and χ_3 (squares) with the number of vectors, N, used per configuration. The averages are taken over 20 thermal configurations at $T = 1.5T_c$ on a 4×8^3 lattice for $m/T_c = 1$.

In Figure 2 we exhibit the spatial volume dependence of our results at $T = 1.5T_c$. Note that the volume dependence is smaller than the statistical errors on χ at this temperature, and also much smaller than what could be expected for an ideal gas of Fermions. In view of this, we have chosen to perform the remaining computations with lattices of size 4×12^3. This volume allows us to measure thermodynamic quantities up to $3T_c$. At larger T, spatial deconfinement sets in and distorts the results unless larger lattices are used [13]. Measurements of finite volume effects on the QCD equation of state indicates that this lattice size can also be used down to $1.1T_c$, below which the first order phase transition of the quenched theory causes strong finite volume effects. As a result, the quenched theory approximation to full QCD is expected to fail close to T_c. Another finite size effect appears in Fermion computations in the quenched theory. As the quark mass decreases at fixed temperature, the scalar/pseudo-scalar screening length increases. If this length becomes comparable to the spatial dimensions of the lattice, then finite volume effects cannot be neglected. Among all our computations, this happened only for $m/T_c = 0.03$ at $T = 1.1T_c$.

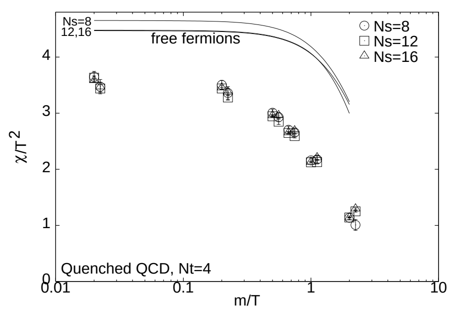

FIG. 2. Quark number susceptibilities at $T = 1.5T_c$. Values of χ_0 are displaced slightly to the right for visibility.

In Figure 3 we collect together all our data. At all these points, $\chi_0 = \chi_3 = 2\chi_s$ within statistical errors [14]. For $T < T_c$ all the susceptibilities are consistent with zero. Notice that the results lie significantly below the expectation from FFT even at temperatures as high as $3T_c$. It is interesting to note that the departures from an ideal gas become stronger with increasing quark mass. We discuss later our checks that this is not a lattice artifact. The jump in χ_s across T_c, its rapid increase with temperature, and its becoming comparable with $\chi_{0,3}$ for $T \geq 2T_c$, all have observable consequences in the pattern of relative strangeness enhancement, including strange baryon enhancement, in going from CERN to RHIC energies [15].

It is interesting to speculate upon the reasons for departure from the weak coupling theory. In view of the long-standing observation that screening masses in the scalar/pseudo-scalar channel (so-called pion screening masses) also depart strongly from the weak coupling theory, we would like to check whether these observations are related. Below T_c, at vanishing chemical potential, pair production of quarks can take place only by pair production of mesons. The lightest meson, the pion, will be most effective at producing quark pairs. Hence the pion susceptibility [16]

$$\chi_\pi = G_\pi(k_0 = 0, \mathbf{k} = 0) = \frac{1}{N_z} \sum_z C_\pi(z) \qquad (7)$$

(here G_π is the pion propagator in momentum space and C_π is the zero-momentum screening correlator) should determine $\chi_{0,3}$. For $T > T_c$, this logic does not necessarily follow, but in view of the strong correlation in the pion sector, it is interesting to test such a hypothesis.

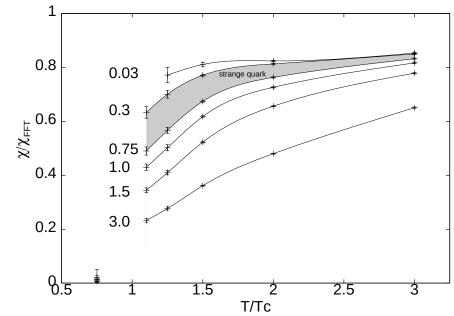

FIG. 3. The ratio of χ_3 to its value in a free fermion theory at the same lattice volume and quark mass, shown as a function of temperature for the values of m_q/T_c indicated. The lines are cubic spline fits to the data. The shaded area is the range of masses relevant for χ_s.

All the susceptibilities are functions of m and T. Since QCD with two light dynamical flavours is expected to have a second order chiral phase transition for $m = 0$ and $T = T_c$, near the critical point we should expect these functions to scale as powers of $T - T_c$ or m. These powers are determined by the ratio of various critical exponents. The scaled quantities would then vary as universal functions of each other. At the critical point, there is a zero mass correlation in the scalar/pseudoscalar channel which is solely responsible for the susceptibilities, since the correlations in the other channels are massive. The universal scaling is then a signal for a restricted form of dynamical confinement [17] near the critical point.

In quenched QCD, where there is no critical point, we can instead make an expansion such as

$$\chi_i = a_i^0(T) + a_i^1(T) m + a_i^2(T) m \log m + \mathcal{O}(m^2), \qquad (8)$$

where $i = 0$, 3, s or π [18]. Eliminating m between χ_π and χ_3, we can always expand one in terms of the other, albeit with temperature dependent coefficients. The surprise, shown in Figure 4, is that the curves for different T can be scaled to lie on top of each other. The scaling function which achieves this goes to 1 as $T \to T_c$. Thus, for $T \leq 2T_c$ and $m/T \geq 1.5$, the co-variance of χ_π and χ_3 at T_c determines that at higher temperatures. Since the scalar/pseudoscalar screening mass is smallest at finite temperature, and very much smaller than other screening masses, therefore the observed scaling may be interpreted as a demonstration of dynamical confinement in much the same way as at a second order phase transition.

Why is this not a trivial observation? After all, as $m \to 0$, χ_π/χ_3 is a T-dependent number, which defines the scaling function. The point is that the scaling is observed not only for $m = 0$, but for a whole range of masses. This defines an universal curve at T_c, not just a single value. As a result, this scaling also gives us a partial ability to test for lattice artifacts. The very fact that scaling is seen for all $m/T_c \geq 1.5$, when results for the lightest mass are consistent with previous computations in two flavour dynamical QCD, implies that there are neither strong quenching artifacts, nor other strong lattice artifacts in our measurements.

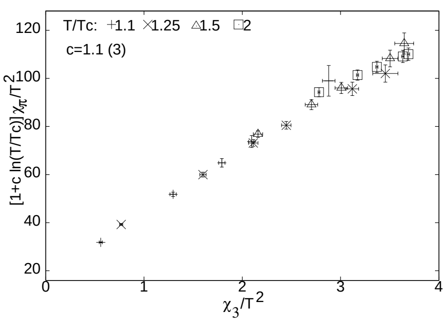

FIG. 4. Scaled values of χ_π vary with χ_3 independently of the values of T and m/T at which they are measured, provided $T \leq 2T_c$ and $m/T_c \leq 1.5$. The meanings of the symbols are the same in the two panels. χ_3 increases with decreasing m/T_c at fixed T.

In summary, we have presented new and precise results on quark number susceptibilities over a wide range of temperatures and quark masses in the high temperature phase of QCD. The susceptibilities differ significantly from the ideal gas expectations (Figure 3). These deviations increase with mass and decrease at higher T. As a result, we expect the relative strangeness enhancement seen in heavy-ion collisions to increase with temperature as quantitatively determined here. The linear relation between χ_π and χ_3 (Figure 4) is the clearest evidence to date for the hypothesis of "dynamical confinement" in the high temperature phase of the plasma [17]. However, it also shows that such a picture becomes less effective with increasing temperature. It is perhaps not a coincidence that the temperature at which the quenched plasma becomes free of this phenomenon is also the temperature at which dimensional reduction becomes quantitatively correct [19]. For $T \geq 2T_c$, the light quark susceptibility is known accurately enough to test dimensional reduction and various other ideas which have emerged in trying to explain lattice results on the QCD equation of state.

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