

A STUDY OF SYMMETRY RESTORATION AT FINITE TEMPERATURE IN THE $O(4)$ MODEL USING ANISOTROPIC LATTICES *

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Results of investigations of the $O(4)$ spin model at finite temperature using anisotropic lattices are presented. In both the large N approximation and the numerical simulations using the Wolff cluster algorithm we find that the ratio of the symmetry restoration temperature T_{SR} to the Higgs mass m_{H} is independent of the anisotropy. We obtain a lower bound of 0.59 ± 0.04 for the ratio, $T_{\text{SR}}/m_{\text{H}}$, at $m_{\text{H}}a \simeq 0.5$, which is lowered further by about 10% at $m_{\text{H}}a \simeq 1$.

1. INTRODUCTION

Finite temperature investigations of spontaneously broken gauge theories are of importance to the physics of the very early universe. Two prime examples are the inflationary universe and the generation of the baryon asymmetry. Although symmetry restoring phase transitions in spontaneously broken gauge theories are crucial for these areas, our knowledge about them comes chiefly from perturbation theory. Motivated by the desire to learn more about their non-perturbative aspects, lattice investigations[1,2] of these theories at finite temperature have been made. Our investigation of the $O(4)$ spin model on anisotropic lattices is one more step in this direction. Recall that this model is obtained from the fundamental $SU(2)$ Higgs-gauge model in its weak gauge coupling limit. Both the models are expected to be trivial, giving rise to an upper bound on the Higgs mass in their respective scaling regions. The finite temperature investigation

is aimed at studying the model in this scaling region to find out about the symmetry restoring phase transition and to obtain a lower bound on the symmetry restoration temperature T_{SR} . Employing anisotropic lattices has the advantage of being able to distinguish the finite temperature effects, which could be called as a special type of finite size effects, from arbitrary finite size effects since the former have to be independent of the anisotropy in the scaling region. In addition, anisotropic lattices allow one to study the finite temperature effects at a correlation length of order unity.

2. THE ANISOTROPIC $O(N)$ MODEL

On anisotropic lattices $L^3 \times \xi L_t$, the $O(N)$ spin model is defined by the action

$$S = -N\beta(\gamma \sum_x S_x \cdot S_{x+\hat{0}} + \frac{1}{\gamma} \sum_{x,j} S_x \cdot S_{x+\hat{j}}) \quad (1)$$

where $S_x \in O(N)$, $\forall x$, and β (or $\kappa = N\beta/2$ for $O(4)$) is the coupling on the isotropic lat-

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tices for which the anisotropy coupling γ is unity. The physical volume and temperature are respectively given by $V = L^3 a^3$ and $T = 1/\xi L_t a_t$. Since $\xi = a/a_t$, varying ξ on the lattices above amounts to holding the temperature constant in units of a^{-1} , apart from possible quantum renormalization effects.

In both the large N approximation and the numerical simulations our procedure to investigate finite temperature effects was the following. For a given value of the anisotropy coupling γ , we obtained the critical coupling on an $L^3 \times \xi L_t$ lattice by setting γ to its classical value ξ . Our results justify this choice *a posteriori*. β_c in the large N limit is obtained by solving numerically the saddle point equation

$$\beta_c = \frac{1}{L_t L^3} \sum_p' \frac{1}{D(p)} \quad (2)$$

for $L \rightarrow \infty$, where $D(p)$ is given by

$$D(p) = 4\xi^2 \sin^2(\frac{1}{2}p_0) + 4 \sum_j \sin^2(\frac{1}{2}p_j) \quad , \quad (3)$$

with the momenta p_μ given by $p_\mu = 2\pi n_\mu/N_\mu$, $n_\mu = 0, \dots, N_\mu - 1$, where $N_0 = \xi L_t$ and $N_j = L$. The prime on the sum in Eq. (2) indicates that the zero mode, $p = 0$, is being left out. In Monte Carlo(MC) simulations the unique crossing point of the cumulant $g_R = \langle M^4 \rangle / \langle M^2 \rangle^2$ for various volumes L^3 yields $\kappa_c(\infty, \xi L_t)$. Here M is the order parameter, defined by $M = \langle |\sum_x S_x| \rangle / \xi L^3 L_t$. Alternatively, one may use the peak position of the susceptibility, $\chi = \langle M^2 \rangle - \langle M \rangle^2$, to define $\kappa_c(L, \xi L_t)$. Using the critical exponents of the $O(4)$ model in three dimensions, $\kappa_c(\infty, \xi L_t)$ can then be obtained using the finite size scaling theory.

The Higgs mass at zero temperature was then determined at the $\kappa_c(\infty, \xi L_t)$ by studying the zero momentum connected correlation functions of the spin variables on $L^3 \times \xi L$ lattices in both

Fig. 1. g_R and χ as a function of κ for $\xi = 1.5$ on $L^3 \times 6$ lattices for $L = 18$ and 24 .

the spatial and the temporal directions. From the exponential fall-off of the correlation functions, the Higgs mass, $m_H a$, can be obtained using standard methods, such as fits or local distance dependent masses. Demanding Euclidean invariance and by appropriately scaling the temporal direction to match these correlation functions, we determined corrections to the relation $\xi = \gamma$. They were found to be 5-10% for all γ -values we studied, indicating that the quantum corrections to the anisotropy are small. The same conclusion was also obtained in the large N limit in the symmetric phase.

3. RESULTS

Fig. 1 exhibits our results for both g_R and χ on $18^3 \times 6$ and $24^3 \times 6$ lattices for $\xi = 1.5$. We used the spectral density method to obtain the smooth curves shown from our data, shown by crosses. Similar results have also been obtained for other values of ξ and L_t . We have used $\xi = 1$,

ξ	1	1.5	2
L_t			
2	-	-	0.54 ± 0.04
4	0.59 ± 0.04	0.58 ± 0.03	0.59 ± 0.03
6	0.80 ± 0.01	-	-

Table 1

$T_{\text{SR}}/m_{\text{H}}$ as a function of ξ and L_t .

1.5 and 2, $L_t = 2, 4, 6$ and $L = 18$ and 24. In each case we obtained $\kappa_c(\infty, \xi L_t)$ by using both the crossing point of g_{R} and the finite size scaling of the peak position of the susceptibility. Both estimates were always found to be consistent, although we preferred to use the former for determining m_{H} .

At each coupling, the Higgs mass m_{H} was obtained from the plateau in the local distance-dependent masses, defined as $\ln(C(t)/C(t+1))$, where $C(t)$ is the zero momentum correlation function. Again we checked that a fit to the data of an exponential form yielded consistent results with these estimates. Using these results, the ratio $T_{\text{SR}}/m_{\text{H}} = (L_t m_{\text{H}} a)^{-1}$ shown in Table 1 is obtained for various ξ and L_t . The ξ -independence of the ratio is obvious. Recall that $m_{\text{H}} \rightarrow 0$, as one approaches κ_c , i.e., as L_t grows. Thus, depending on the choice of value of the correlation length up to which an effective theory can be defined, one obtains a lower bound on the ratio $T_{\text{SR}}/m_{\text{H}}$. From Table 1, one sees this bound to be 0.59 ± 0.04 for a correlation length of ~ 2 , which decreases by 10% for $m_{\text{H}} a \simeq 1$.

Considering the fluctuations around the saddle point of the large N limit, one can obtain the Higgs mass m_{H} at $\beta_c(\xi L_t)$, while the corresponding renormalized vacuum expectation value of the field is given by $v_{\text{R}}^2 = \beta_c(\xi L_t) - \beta_c(\infty)$. Fig. 2 shows these large N results for $T_{\text{SR}}/m_{\text{H}}$ and $T_{\text{SR}}/v_{\text{R}}$. Both are seen to be clearly independent of ξ . Further, the latter seems to be independent of L_t for $L_t \geq 4$. Qualitatively, the large N

Fig. 2. Large N results for $T_{\text{SR}}/m_{\text{H}}$ and $T_{\text{SR}}/v_{\text{R}}$ as a function of ξ for various L_t .

limit seems to reproduce all the features of the Monte Carlo(MC) data well. However, quantitatively, the large N results seem to lie systematically lower than the MC results by $\sim 15\%$. It would be interesting to check whether the early scaling evident in the MC data for $L_t = 2$ is real by simulating the theory at more ξ values and also by studying the $T_{\text{SR}}/v_{\text{R}}$ ratio.

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