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Can the Quark-Gluon Plasma in the Early Universe be supercooled?

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ABSTRACT

The quark-hadron phase transition in the early universe can produce inhomogeneities in the distribution of nucleons, which in turn affect the primordial nucleosynthesis. In all the investigations of this problem it has been assumed that the degree of supercooling of the quark-gluon plasma after the phase transition is large enough to produce a significant rate of nucleation of hadrons. Using the latest results of finite temperature lattice QCD and the finite size scaling theory, we argue that the degree of supercooling is in fact extremely small and hence the nucleation rate is negligible.

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The implications of the transition from the deconfined quark-gluon plasma phase to the confined hadron phase in the early universe for the primordial nucleosynthesis have been studied extensively in recent years [1-11]. The following phase separation scenario has been used in these investigations. In the early universe when the temperature is high the unconfined quarks and gluons form a plasma. As the universe cools due to expansion and reaches the phase transition temperature, T_c, the confined hadron phase does not appear immediately since the phase transition is assumed to be of first order. The universe has to cool somewhat below T_c before the probability for nucleating bubbles becomes significant. The supercooled phase is metastable and decays into hadronic bubbles.

The rate of nucleation of hadron bubbles has been calculated [4,12] using the theory of homogeneous nucleation [13]. In this theory the new phase is assumed to arise through spontaneous fluctuations in the supercooled phase. The nucleation rate per unit volume, $p(T)$, at a temperature, T, is given by

$$p(\eta) \simeq CT_c^4 \exp(-\frac{16\pi}{3}\frac{\sigma^3}{T_c L^2 \eta^2}) \tag{1}$$

where C is a constant of order unity, σ, the surface tension of the hadronic bubble, L, the latent heat per unit volume of the transition and η, the degree of supercooling, $\eta = (T_c - T)/T_c$. As the supercooling is assumed to be small, after a hadron bubble has been nucleated it grows explosively as a deflagration bubble [12,14] and drives a weak shock-wave which expands into the surrounding quark phase with a velocity just above sound speed $v_s \simeq 1/\sqrt{3}$. The plasma gets reheated by the shock-wave and further nucleation is inhibited due to the steep fall-off in Eq. (1). The fraction of the universe affected by the shock front preceding the growing bubble is

$$g(t) = \int_{t_c}^{t} p(t')\frac{4\pi}{3}\, v_s^3 (t-t')^3 \, dt' \quad , \tag{2}$$

where t_c is the time when the universe cools through T_c. Here the temperature T in $p(T)$ has been replaced by time t using the Einstein relation between T and the age of the universe,t,

$$t = \frac{3}{2\pi(41\pi)^{1/2}}\frac{M_{Pl}}{T^2} \tag{3}$$

In calculating t the quark degrees of freedom have been included[†]. The total

[†] We have used here the ideal gas expression for the energy density. While it

number density of nucleation sites is then

$$N_n = \int_{t_c}^{\infty} f(t)p(t)dt \quad , \tag{4}$$

where

$$f(t) = 1 - g(t) \quad . \tag{5}$$

Since the supercooling, that is, η is small, $p(t)$ rises rapidly with t and therefore $f(t)$ behaves almost like a step function $\theta(t_h - t)$ [4, 12], where t_h is the time when the universe has been reheated. N_n can then be approximated as

$$N_n = \int_{t_c}^{t_h} p(t)\, dt \quad . \tag{6}$$

By expanding $p(t)$ around t_h one obtains [4],

$$p(t) = p(t_h) \exp(-\alpha(t_h - t)) \quad , \tag{7}$$

where

$$\alpha = 1.6528 \times 10^{-7} \frac{\gamma^2}{\eta_h^3} \left(\frac{T_c}{MeV} \right)^2 m^{-1} \quad , \tag{8}$$

γ^2 is a dimensionless combination of σ, L, and T_c,

$$\gamma^2 = \frac{\sigma^3}{L^2 T_c} \tag{9}$$

and η_h can be calculated from the condition $f(t_h) = 0$, i.e.,

$$\int_{t_c}^{t_h} dt\ p(t') \frac{4\pi}{3} v_s^3 (t - t') = 1 \quad . \tag{10}$$

Substituting Eq. (7) in Eq. (10) and carrying out the integration one finds the equation determining η_h to be,

$$x^6 \exp(x) = 2.21 \times 10^{-2} \left(\frac{M_{Pl}\, \gamma^2}{T_c} \right)^4 \quad , \tag{11}$$

where

$$x = \frac{16\pi}{3} \frac{\gamma^2}{\eta_h^2}$$

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is now known[15] that near T_c there are strong non-perturbative effects, they will affect Eq. (3) by a factor of O(1) and have therefore been ignored.

(We have put $C = 1$.). An approximate solution of Eq. (11) given in ref. 10 is $\eta_h = 0.4\gamma$. Note that the RHS of Eq. (11) is totally dominated by the very large ratio of the Planck scale and the QCD scale, which is typically $\sim 10^{19}$. Thus, a large variation of γ, over several orders of magnitude but still sufficiently small compared to the ratio, changes the solution very little, making it almost independent of the details of the QCD thermodynamics.

One of the parameters important for primordial nucleosynthesis is the mean separation per nucleation site, $l \simeq N_n^{-1/3}$, which can be obtained from Eq. (6) by evaluating the integral using Eqs. (7)-(8) and the solution to Eq. (11) given above. One obtains

$$l = (8\pi)^{1/3} \frac{v_s}{\alpha} \quad .$$

A more detailed calculation by Meyer et al [10] gives

$$l = 4.38 \frac{v_s}{\alpha} \quad . \tag{12}$$

During the subsequent evolution of the universe, when the hadronization of the quark-gluon plasma is complete, the mean free paths of neutrons and protons in the surrounding medium of hadrons, electrons, photons and neutrinos become different leading to a segregation such that high-density proton-rich and low-density neutron-rich regions develop. Nucleosynthesis then takes place among these inhomogeneously distributed nucleons [3]. It is clear that for this picture of nucleosynthesis to be valid l should be greater than the comoving diffusion length of the protons, which is ~ 0.5 m. The fluctuations would otherwise damp out entirely before the onset of nucleosynthesis.

It is apparent from Eq. (8), (9) and (12) that the degree of supercooling and hence l depend only on the bulk thermodynamic properties, namely, v_s, σ, L and T_c of the strongly interacting matter. One knows, however, from finite size scaling theory [16] that the degree of metastability is also decided by the size of the system relative to its correlation length, $\xi(T_c)$ at the transition point. Indeed , the width, ΔT of the metastable region for a first order phase transition goes to zero as the volume, V, increases:

$$\frac{\Delta T}{T_c} \equiv \frac{T}{T_c} - 1 = \frac{A}{V}, \tag{13}$$

where A is a constant of $O(1)$ if V is measured in units of $\xi(T_c)$.

In order to proceed further and see the consequences of Eq. (13) we shall assume that the world of quenched quantum chromodynamics (QCD) is a good approximation to ours. In particular, we wish to use the precise information available from lattice QCD in this approximation. Furthermore, we shall assume that asymptotic scaling is valid so that we can relate the lattice results to the continuum theory. We shall return to the discussion of the effects of these assumptions later. The best estimates from lattice QCD for the quantities required are as follows [15, 17, 18]:

$$\begin{aligned} L/T_c^4 &= 1.8 \pm 0.1 \\ \sigma/T_c^3 &= 0.024 \pm 0.004 \quad . \\ T_c &= 235 \pm 42 \text{ MeV} \end{aligned} \tag{14}$$

Note that the transition temperature is much larger here than the value used in Ref. 4 from the MIT bag model. The other parameters are roughly the same. In fact, using T_c as a scale, one sees that it is really the latent heat,L, which is much smaller here (by almost a factor of 8), while the surface tension is within the range used in Ref. 4. Using these values in Eqs.(9) and (12), one obtains $\gamma = 2.066 \times 10^{-3}$ and $l = 2.84 \pm 1.25$ cm, which is much less than what is required even if we ignore the question of the size of the universe at $T = T_c$.

If we now ask how much supercooling we can expect by taking size of the universe into account the situation becomes considerably worse. We know from lattice QCD [19] that *i*) the physical correlation length in the deconfined phase is $\xi(T_c)T_c = 1.38 \pm 0.24$ and *ii*) the width of the metastable region in the QCD-coupling space for a system of volume V is given by

$$\begin{aligned} \Delta\beta &= 0.29 \left(\frac{\xi^3}{V}\right)^{0.984 \pm 0.039} , \\ &= 0.75 \left(VT_c^3\right)^{-0.984 \pm 0.039} \end{aligned} \quad . \tag{15}$$

Further, by assuming the validity of the asymptotic scaling relation, the width in the coupling β can be related to the width of the metastable region in temperature: $\Delta T/T_c = 1.122\Delta\beta$. Using the expression for the age of the universe, Eq. (3), the volume of the universe at temperature T_c is found to be

$$VT_c^3 = 7.1 \times 10^{55} \left(\frac{200MeV}{T_c}\right)^3 \quad . \tag{16}$$

Substituting in Eq. (15), one obtains the degree of supercooling possible in the early universe to be

$$\frac{\Delta T}{T_c} \equiv \eta = 9.28 \times 10^{-56 \pm 2.2} \left(\frac{T_c}{200MeV}\right)^{2.952 \pm 0.117} \quad . \tag{17}$$

Thus for any value of T_c in the range given in Eq. (14) the universe practically never supercools at all. Even the miniscule amount indicated by Eq. (17) will not be sufficient for any bubble nucleation, since the nucleation probability goes to zero as $\exp\left(-1/\eta^2\right)$ as $\eta \to 0$.

Let us now discuss whether our assumptions made earlier could have led to the negative result. Two crucial inputs were the validity of the asymptotic scaling relation and the finite size scaling relation Eq. (14). Due to its better precision, we used above the lattice data obtained on lattices with four points in the temporal direction. It is expected [15] for quenched QCD that scaling may have set in for such lattices but the asymptotic scaling is violated by $\sim$ 40-50 %, causing at most a factor of 2 change in Eq. (17). The systematic error in the finite size scaling relation of Eq. (15) due to this is more difficult to estimate but one does find similar results for larger temporal lattices than those used in ref. 19, suggesting again changes by factors of O(1). Indeed, within the quenched QCD approximation it appears difficult to get away from the fact that the size of the universe at T_c measured in the units of the physical correlation length is essentially infinite, yielding an almost null result for the supercooling parameter.

The real universe of course has dynamical fermions and one must therefore ask what happens in the case of full QCD. Unfortunately, in this case one has to worry about more input parameters in addition to the usual technical problem of putting fermions on the lattice. Assuming the current state of the art results, at best a very weak first order phase transition may be indicated for the realistic two light flavours and one heavier flavour case [20]. No signs of a first order transition were found on a lattice of linear size $4/T_c$, suggesting a still larger correlation length at T_c, if finite at all. A larger correlation length at T_c will increase the possibility of appreciable supercooling since a weaker phase transition will presumably yield broader region of metastability. Furthermore, a weaker phase transition will have lower latent heat and surface tension. These, in turn, may reduce γ and therefore also the required degree of supercooling, η_h, to achieve the desired mean free path. Thus, the scenario proposed in refs. [1-4] becomes more feasible for a very weak first order phase transition. However, it still looks rather improbable that the supercooling necessary for hadronic bubble creation will be achieved unless γ for the full QCD is lower by several orders of magnitude from its value for the quenched QCD used above. Whether future results from simulations of full QCD will still allow a small probability for homogeneous nucleation thus remains to be seen. The alternative and more likely possibility is heterogeneous nucleation caused by 'impurities' left over from an earlier epoch.

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