Impact of Z_2 monopoles and vortices on the deconfinement transition

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Suppressing Z_2-monopoles shifts the line of deconfinement transitions in the coupling plane of the $SU(2)$ lattice gauge theory with a mixed Villain form of action but it still continues to behave also like the bulk transition line. Separate deconfinement and bulk phase transitions are found on the same lattice, suggesting the two to be indeed coincident at higher adjoint couplings. Universality is restored when both monopoles and vortices are suppressed.

1. UNIVERSALITY

Since most numerical lattice field theory investigations are necessarily carried out for a finite value of the lattice cut-off a, it seems imperative that universality of the results so obtained is verified by employing other forms of lattice actions. A study of the universality of the deconfinement transition for SU(2) gauge theory, which has already been extensively investigated for the Wilson action[1], was made in Ref. [2], for the Bhanot-Creutz action [3]

$$S = \sum_p \left[\beta_f \left(1 - \frac{\mathrm{Tr}_f U_p}{2} \right) + \beta_a \left(1 - \frac{\mathrm{Tr}_a U_p}{3} \right) \right] \quad (1)$$

Here the summation runs over all the plaquettes of the lattice and the subscript $a(f)$ indicates that the trace is taken in the adjoint (fundamental) representation. Later, a Villain form of action[4], defined by

$$S = \sum_p \left[\beta_f + \beta_v - \frac{(\beta_f + \beta_v \sigma_p) \, \mathrm{Tr}_f U_p}{2} \right] \quad (2)$$

was also used[5] for similar studies with essentially similar results, where σ_p are auxiliary Z_2-variables defined on the plaquettes and the partition function has an additional sum over all possible values of the σ_p variables as well. The Wilson action corresponds to setting β_a or β_v to zero above. Simulations[2,5] on $N_\sigma^3 \times N_\tau$ lattices with $N_\tau = 4$ showed surprising results for the above actions. While the second order deconfinement

transition point for the Wilson action entered the coupling plane as a line of second order transitions, the transition turned first order for large enough β_a or β_v. The order parameter for the deconfinement transition acquired a nonzero value discontinuously there and the exponent of the corresponding susceptibility changed from the Ising model value of about 1.97 to 3. If the change of the order of the deconfinement transition were to persist at a finite β_a with increasing temporal lattice size N_τ, i.e., in the continuum limit, it would be a serious violation of universality. On the other hand, the line of deconfinement transitions was found to coincide with the known bulk transition lines[3,4]. Additional studies with varying N_τ further revealed that the line scarcely moves in the region where a strong first order deconfinement transition is observed[6].

Inspired by the results[7] for the $SO(3)$ lattice gauge theory, we investigated the finite temperature phase diagram of the mixed action (2) with suppression of the Z_2-monopoles and vortices by addition of chemical potentials for them. These terms are irrelevant in the naive continuum limit.

2. MONOPOLES AND VORTICES

The Z_2-monopoles are suppressed[7] by the addition of a chemical potential term, $\lambda \sum_c (1 - \sigma_c)$, to the mixed Villain action (2). The summation runs over all the elementary 3-cubes of the lattice, and $\sigma_c = \prod_{p \in \partial c} \sigma_p$. Note that in the classi-

cal continuum limit one still obtains the same λ-independent continuum relation, $4g^{-2} = \beta_f + \beta_v$. Following the $SO(3)$ results, we took $\lambda = 1$ for our simulations in the entire β_f - β_v plane. Our each iteration consisted of heatbath sweeps for all the gauge links, followed by those for the Z_2- variables. A fraction of the links (arbitrarily chosen to be $\frac{1}{4}$) were Z_2-rotated subject to a probability determined by the β_f term at the end of each iteration to reduce the otherwise enormous autocorrelations for large λ simulations. Measurements were made after every iteration. Using hysteresis runs of 15000 iterations per point we mapped out the phase diagram on an $8^3 \times 4$ lattice.
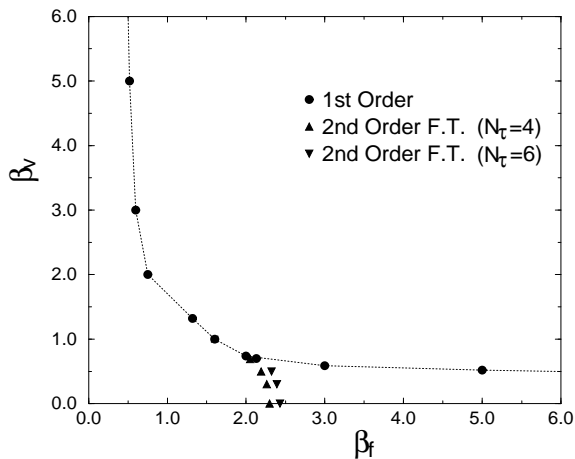

Figure 1. The phase diagram for the action (2) with monopole suppression on an $8^3 \times 4$ lattice.

For $\beta_v > \beta_f$, first order transitions, with discontinuities in both the average plaquette $P = \langle \frac{1}{2} \mathrm{Tr}_f U_p \rangle$ and the fundamental Polyakov loop $\langle |L_f| \rangle$, were observed. These transition points are shown by filled circles in Fig. 1. Since $\langle |L_f| \rangle$ becomes nonzero discontinuously at these couplings, it clearly indicates a first order deconfinement phase transition. A second order deconfinement phase transition coincident with a first order bulk phase transition, signaled by the discontinuity in the average plaquette at the same location, is also possible. This general behavior is very similar to what was observed for $\lambda = 0$ [5]. In contrast to that case, however, one now also has a first order phase transition for $\beta_v < \beta_f$, as shown in Fig. 1. Here the observable $P_a = \langle \frac{1}{2} \sigma_p . \mathrm{Tr}_f U_p \rangle$

displays a sizeable discontinuity. A qualitatively new feature of the phase diagram in Fig. 1 thus is the absence of any end point for the transition line because of the new line of transitions coming from the large β_f side along which the deconfinement order parameter is nonzero on both sides of the transition.

As a continuation of the deconfinement transition on the Wilson axis, we looked for a deconfinement transition at $\beta_v = 0.3$, 0.5 and 0.7. The transition point was located approximately from the sharp but continuous rise of $\langle |L_f| \rangle$. From the peak heights of the $|L_f|$ - susceptibility, $\chi_{|L_f|}$, on $N_\sigma^3 \times 4$ lattices for $N_\sigma = 8$, 12 and 16, its critical exponent was obtained at each β_v. A linear fit to $\ln (\chi_{|L_f|})_{max} = \omega \ln N_\sigma$ gave $\omega = 1.91 \pm 0.02, 1.87 \pm 0.05$ and 1.92 ± 0.05 for $\beta_v = 0.7$, 0.5 and 0.3, respectively. They indicate second order transitions and are in agreement with the $\beta_v = 0$ exponent and the Ising model exponent. The average plaquette $\langle P \rangle$ from these runs was smooth everywhere, and the corresponding susceptibility peaks did not sharpen with N_σ at all, indicating a lack of bulk transition at these points. These transition points, shown in Fig. 1 by triangles, are therefore pure finite temperature transitions. Similar results for $N_\tau = 6$ are also shown.
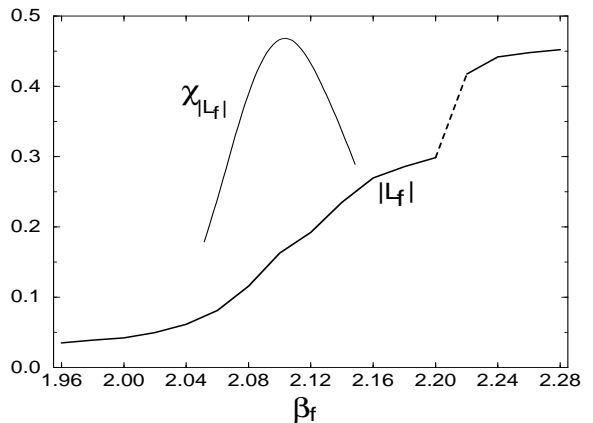

Figure 2. $\langle |L_f| \rangle$ and its susceptibility as a function of β_f at $\beta_v = 0.7$.

The $\lambda = 1$ simulations for the mixed Villain action thus lend a strong credibility to the hy-

pothesis that the deconfinement transition line for possibly a large range of N_τ merges with the bulk transition line. Since the latter branches out in this case and exhibits no end point, the merger is easy to observe numerically: the small β_v region has only a deconfinement transition line and the large β_f region has only a bulk transition line, while they seem to be coincident for $\beta_v \geq 0.7$.
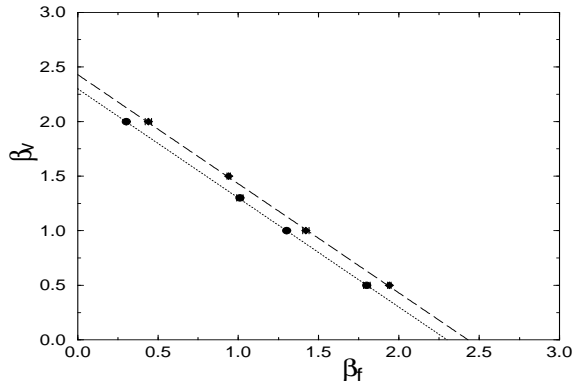

Figure 3. The deconfinement transition points in the (β_f, β_v) plane for $N_\tau = 4$ (circles) and 6 (diamonds) lattices.

A possible litmus test of the coincidence scenario is to see two separate transitions on the same lattice. Fig. 2 shows the results of such a test at $\beta_v = 0.7$ on $8^3 \times 4$ lattices. It shows the susceptibility, $\chi_{|L_f|}$, obtained from the longer run mentioned above along with the order parameter $\langle |L_f| \rangle$ obtained from a hysteresis run from a hot start. It clearly shows a second order deconfinement transition taking place first at $\beta_f \sim 2.1$, followed by a bulk phase transition later at $\beta_f \sim 2.2$.

In order to suppress the Z_2-electric vortices in addition to the magnetic monopoles, we added to the action (2) another irrelevant term, $\gamma \sum_l (1 - \sigma_e)$, where $\sigma_e = \prod_{p \in \hat{\partial} l} \sigma_p$. For sufficiently large γ, one expects that the bulk transition line in Fig. 1, caused presumably by the condensation of electric loops, will also be suppressed. As $\lambda \to \infty$, the monopole term is frozen and the plaquette variables σ_p are replaced by products over corresponding Z_2-link variables.

Following the same procedure as above, and using a heat-bath algorithm for both the gauge and Z_2-variables, we studied the phase diagram on $N_\tau = 4$ and 6 lattices for $(\lambda, \gamma) = (\infty, 5.)$. Fig. 3 shows the only transition points found, which are Ising-like second order deconfinement transitions and obey $\beta_f + \beta_v \approx \beta_c^W$, where β_c^W is the deconfinement transition point for the Wilson action. The transition lines are also consistent with the expected continuum limit behavior of this action.

3. CONCLUSIONS

Our numerical simulations for the monopole-suppressed action showed an interesting phase diagram which was different from that of the original theory. Nevertheless, it too had the paradoxical coincidence of bulk and deconfinement transitions. The bulk transition line in this case had no end point and the change of the order of the deconfinement phase transition occurred as the two lines merged. We showed the presence of two phase transitions on the same finite lattice in the vicinity of the point of merger (See Fig. 2).

A further suppression of the Z_2 electric vortices got rid of the bulk transitions completely and yielded only lines of second order deconfinement transitions, in agreement with universality. Since the terms added to the action in the process do not contribute in the naive continuum limit, one can formally attribute the anomalous behavior of the deconfinement transition lines for both the mixed actions to the presence of bulk transitions.

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