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Quark number susceptibilities from lattice QCD[1]

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Abstract: Results from our recent investigations of quark number susceptibilities in both quenched and 2-flavour QCD are presented as a function of valence quark mass and temperature. A strong reduction (∼40%) is seen in the strange quark susceptibility above T_c in both the cases. A comparison of our isospin susceptibility results with the corresponding weak coupling expansion reveals once again the non-perturbative nature of the plasma up to $3T_c$. Evidence relating the susceptibility to another non-perturbative phenomena, pionic screening lengths, is presented.

1 Introduction

Quantum chromodynamics (QCD) is the theory of interactions of quarks and gluons which are the basic constituents of strongly interacting particles such as protons, neutrons and pions. A complete lack of any experimental observation of a free quark or gluon led to the hypothesis of their permanent confinement in the observable particles. As pointed out by Satz [3] in his talk, relativistic heavy ion collision experiments at BNL, New York, and CERN, Geneva offer the possibility of counting the number of degrees of freedom of strongly interacting matter at high temperatures, thereby providing a strong argument for Heisenberg to accept the physical reality of quarks and gluons in spite of their confinement at lower temperatures. The basic idea here is to look for the production of quark-gluon plasma (QGP), predicted by QCD to exist beyond a transition temperature T_c, in the heavy ion collisions and when found, determine its thermodynamical properties, such as, *e.g.*, its energy density. At very high temperatures, these properties can be computed theoretically and are seen to be directly proportional to the number of quarks and gluons. In order to test this idea, however, one has to face the fact that the temperatures reached in the current, and near future, experiments are likely to lie below about 5-10 T_c. The current best theoretical estimates for thermodynamic observables in this temperature range are provided by numerical simulations of QCD defined on a discrete space-time lattice [4]. It seems therefore prudent to evaluate as many independent observables as possible and compare them with both experiments and approximate analytical methods. Quark number susceptibilities constitute a useful independent set of observables for testing this basic idea of counting the degrees of freedom of strongly interacting matter.

[1]Based on work done [1, 2] with Sourendu Gupta and Pushan Majumdar.

Investigations of quark number susceptibilities from first principles can have direct experimental consequences as well since quark flavours such as electric charge, strangeness or baryon number can provide diagnostic tools for the production of flavourless quark-gluon plasma in the central region of heavy ion collisions. It has been pointed out recently [5, 6] on the basis of simple models for the hadronic and QGP phases that the fluctuations of such conserved charges can be very different in these two phases and thus can act as probes of quark deconfinement. Indeed, excess strangeness production has been suggested as a signal of quark-gluon plasma almost two decades ago [7]. Lattice QCD can provide a very reliable and robust estimate for these quantities in *both* the phases since in thermal equilibrium they are related to corresponding susceptibilities by the fluctuation-dissipation theorem:

$$\langle \delta Q^2 \rangle \propto \frac{T}{V} \frac{\partial^2 \log Z}{\partial \mu_Q^2} = \chi_Q(T, \mu_Q = 0) \ . \tag{1}$$

Here μ_Q is the chemical potential for a conserved charge Q, and Z is the partition function of strongly interacting matter in volume V at temperature T. Unfortunately, the fermion determinant in QCD becomes complex for any nonzero chemical potential for most quantum numbers including those mentioned above. Consequently, Lattice QCD is unable to handle finite chemical potential satisfactorily at present, and cannot thus yield any reliable estimates of any number density. However, the susceptibility above, i.e., the first derivative of the number density at zero chemical potential, can be obtained reasonably well using conventional simulation techniques, facilitating thereby a nontrivial extension of our theoretical knowledge in the nonzero chemical potential direction.

Quark number susceptibilities also constitute an independent set of observables to probe whether quark-gluon plasma is weakly coupled in the temperature regime accessible to the current and future planned heavy ion experiments (say, $1 \leq T/T_c \leq 10$). A lot of the phenomenological analysis of the heavy ion collisions data is usually carried out assuming a weakly interacting plasma although many lattice QCD results suggest otherwise. It has been suggested [8] that resummations of the finite temperature perturbation theory may provide a bridge between phenomenology and the lattice QCD by explaining the lattice results starting from a few T_c. As we will see below, quark number susceptibilities can act as a cross-check of the various resummation schemes. Earlier work on susceptibilities [9] did not attempt to address this issue and were mostly restricted to temperatures very close to T_c. Furthermore, the quark mass was chosen there to vary with temperature linearly. We improve upon them by holding quark mass fixed in physical units ($m/T_c =$ constant). We also cover a larger range of temperature up to 3 T_c and the accepted range of strange quark mass in our simulations.

2 Formalism

After integrating the quarks out, the partition function Z for QCD at finite temperature and density is given by

$$Z = \int \mathcal{D}U \mathrm{e}^{-S_g} \det M(m_u, \mu_u) \det M(m_d, \mu_d) \det M(m_s, \mu_s). \tag{2}$$

Here $\{U_\nu(x)\}$, $\nu = 0$–3, denote the gauge variables and S_g is the gluon action, taken to be the standard [4] Wilson action in our simulations. Due to the well-known "fermion doubling problem" [4], one has to face the choice of fermion action with either exact chiral symmetry or violations of flavour symmetry. The Dirac matrices M depend on this choice. Since we employ staggered fermions, the matrices, M, are of dimensions $3N_s^3N_t$, with $N_s(N_t)$ denoting the number of lattice sites in spatial(temporal) direction. These fermions preserve some chiral symmetry at the expense of flavour violation. Although, they are strictly defined for four flavours, a prescription exists to employ them for arbitrary number of flavours which we shall use. m_f and μ_f are quark mass and chemical potential (both in lattice units) for flavour f, denoting up(u), down(d), and strange(s) in eq.(2). The chemical potential needs to be introduced on lattice as a function $g(\mu)$ and $g(-\mu)$ multiplying the gauge variables in the positive and negative time directions respectively, such that [10] i) $g(\mu) \cdot g(-\mu) = 1$ and ii) the correct continuum limit is ensured. While many such functions g can be constructed, $\exp(\mu)$ being a popular choice, the results for susceptibilities at $\mu = 0$ can easily be shown to be independent of the choice of g even for finite lattice spacing a. From the Z in eq. (2), the quark number densities and the corresponding susceptibilities are defined as

$$n_f \equiv \frac{T}{V}\frac{\partial \ln Z}{\partial \mu_f} \qquad \chi_{ff'} \equiv \frac{\partial n_f}{\partial \mu_{f'}} = \frac{T}{V}\left[\frac{1}{Z}\frac{\partial^2 Z}{\partial \mu_f \partial \mu_{f'}} - \frac{1}{Z}\frac{\partial Z}{\partial \mu_f}\frac{1}{Z}\frac{\partial Z}{\partial \mu_{f'}}\right] , \tag{3}$$

where the volume $V = N_s^3 a^3$ and the temperature $T = (N_t a)^{-1}$. To lighten the notation, we shall put only one subscript on the diagonal parts of χ.

In order to obtain information for quark-gluon plasma in the central region, we evaluate the susceptibilities at the point $\mu_f = 0$ for all f. In this case, each n_f vanishes, a fact that we utilize as a check on our numerical evaluation. Moreover, the product of the single derivative terms in eq. (3) vanishes, since each is proportional to a number density. We set $m_u = m_d < m_s$. Noting that staggered quarks have four flavours by default, $N_f = 4$, and defining $\mu_3 = \mu_u - \mu_d$, one finds from eq. (3) that the isotriplet and strangeness susceptibilities are given by

$$\chi_3 = \frac{T}{2V}\mathcal{O}_1(m_u), \qquad \chi_s = \frac{T}{4V}[\mathcal{O}_1(m_s) + \frac{1}{4}\mathcal{O}_2(m_s)] , \tag{4}$$

where $\mathcal{O}_1 = \langle \mathrm{Tr}(M''M^{-1} - M'M^{-1}M'M^{-1})\rangle$, $\mathcal{O}_2 = \langle(\mathrm{Tr}M'M^{-1})^2\rangle$, $M' = \partial M/\partial\mu$ and $M'' = \partial^2 M/\partial \mu^2$. The angular brackets denote averaging with respect to the Z in eq. (2). One can similarly define baryon number and charge susceptibilities. We refer the reader for more details on them to Ref. [2].

In the discussion above, quark mass appears as an argument of $\mathcal{O}_i$ and implicitly in the Boltzmann factor of Z. Let us denote it by m_{val} and m_{sea} respectively. While the two should ideally be equal, we evaluated the expressions above in steps of improving approximations (and increasing computer costs) by first setting $m_{sea} = \infty$ for all flavours (quenched approximation [1]) and then simulating two light dynamical flavours, by setting $m_{sea}/T_c = 0.1$ (2-flavour QCD [2]). In each case we varied m_{val} over a wide range to cover both light u,d quarks as well as the heavier strange quark. Details of our simulations as well as the technical information on how the thermal expectation values of $\mathcal{O}_i$ were evaluated are in Refs. [1, 2].

3 Results

Based on our tests [1] of volume dependence, made by varying N_s from 8 to 16, we chose $N_s = 12$, as the susceptibilities differed very little from those obtained on an $N_s = 16$ lattice. Choosing $N_t = 4$, the temperature is $T = (4a)^{-1}$, where the lattice spacing a depends on the gauge coupling β. Using the known $\beta_c(N_t')$ for $N_t' = 6, 8, 12$, where β_c is

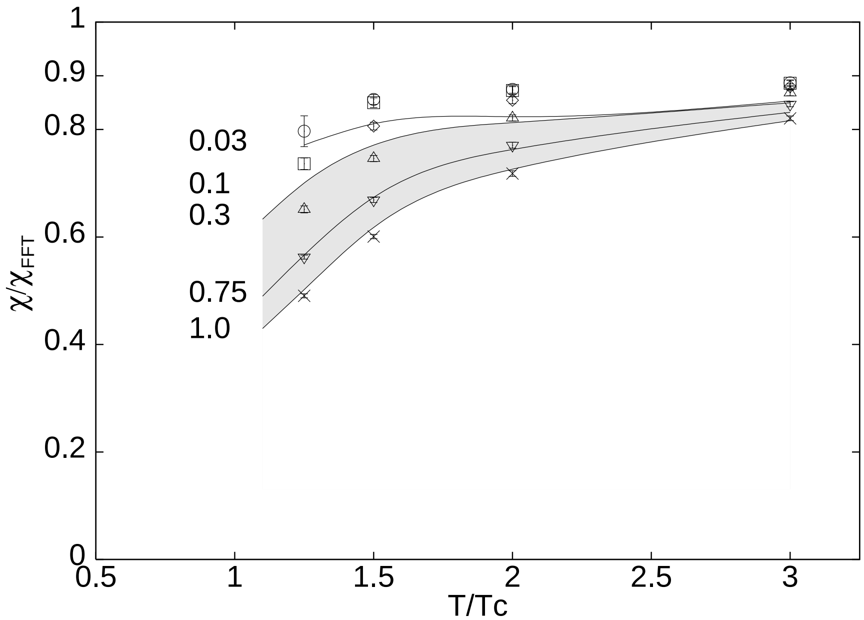

Figure 1: χ/χ_{FFT} as a function of temperature for various valence quark masses.

the gauge coupling at which chiral (deconfinement) transition/cross-over takes place for lattices with temporal extent N_t', we obtained results at $T/T_c = N_t'/N_t$ =1.5, 2 and 3 in both the quenched approximation and the 2-flavour QCD. From the existing estimates [11] of T_c for 2-flavour QCD, one finds that the sea quark mass in our dynamical simulations corresponds to 14-17 MeV. Fig. 1 shows our results, normalized to the free field values on the same size lattice, χ_{FFT}. These can be computed by setting gauge fields on all links to unity: $U_\nu(x) = 1$ for all ν and x. Note that due this choice of our normalization, the overall factor N_f for degenerate flavours cancels out, permitting us to exhibit both χ_3 and χ_s of eq.(4) on the same scale in Fig. 1. The continuous lines in Fig. 1 were obtained from the interpolation of the data obtained in the quenched approximation (the data are not shown for the sake of clarity), while the results for two light dynamical flavours are shown by the data points. In each case the value of m_{val}/T_c is indicated on the left. Due to the fact [2] that the contribution of $\mathcal{O}_2$ to χ_s turns out to be negligibly small for $T > T_c$, χ_3 and χ_s appear coincident in Fig. 1. For the real world QCD, the low valence quark mass results are relevant for χ_3 and those for moderate valence quark masses are for χ_s.

Although T_c differs in the quenched and 2-flavour QCD by a factor of 1.6-1.7, the respective susceptibilities shown in Fig. 1 as a function of the dimensionless variable T/T_c change by at most 5-10% for any m_{val}/T. Thus the effect of "unquenching", i.e., making 2 flavours of quarks (u and d) light enough to include the contribution of the corresponding quark loops, appears to be primarily a change of scale set by T_c. Since the strange quark is a lot heavier than the up and down quarks, this suggests further that including its loop contributions, i.e., including a dynamical but heavier strange quark, may not change the results in Fig. 1 significantly. For a wide range for strange quark mass of 75 to 170 MeV, the strangeness susceptibility can be read off from the shaded region. It is smaller by about 40% compared to its ideal gas value near T_c and the suppression shows a strong temperature dependence. This has implications for the phenomenology of particle abundances, where an ideal gas model without such a suppression of strangeness is employed and could therefore result in underestimates of the temperatures reached.

For the smallest m_{val}/T and highest temperature we studied, the ratio χ/χ_{FFT} in Fig. 1 is seen to be 0.88 (0.85) for 2-flavour (quenched) QCD, with a mild temperature

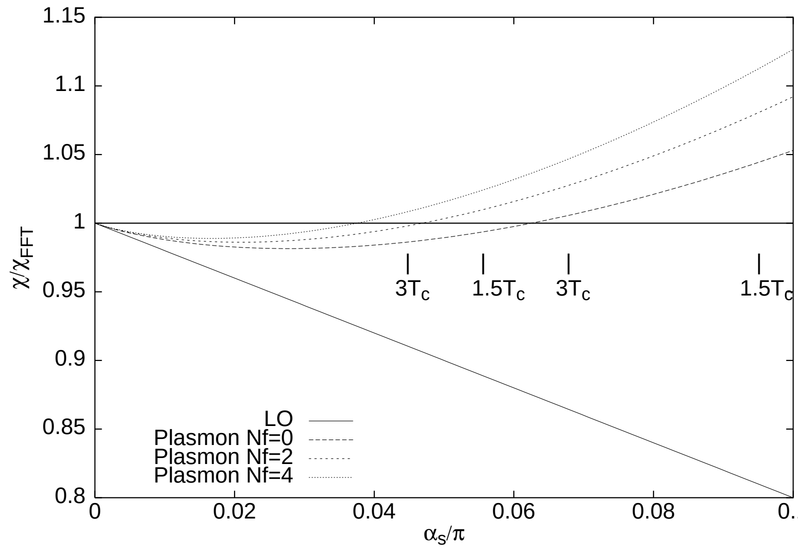

Figure 2: χ/χ_{FFT} as a function of α_s/π for N_f dynamical massless quarks.

variation in the large T-region. Since its variation with valence quark mass is negligibly small for small m_{val}/T, one can assume the results for massless valence quarks to be essentially the same as those for $m_{val}/T = 0.03$ in Fig. 1. In order to check whether the degrees of freedom of QGP can really be counted using these susceptibilities, one needs to know whether the deviation from unity can be explained in ordinary perturbation theory or its improved/resummed versions. Usual weak coupling expansion [12] yields $\chi/\chi_{FFT} = 1 - 2\frac{\alpha_s}{\pi}[1 - 4\sqrt{\frac{\alpha_s}{\pi}(1 + \frac{N_f}{6})}]$. Fig. 2 shows these predictions for various N_f along with the leading order N_f-independent prediction. Using a scale $2\pi T$ for the running coupling and $T_c/\Lambda_{\overline{MS}} = 0.49(1.15)$ for the $N_f = 2(0)$ theory [11], the values T/T_c =1.5 and 3 are marked on the figure as the second (first) set. As one can read off from the Fig. 2, the ratio decreases with temperature in *both* the cases in the range up to $3T_c$ whereas our results in Fig. 1 display an increase. Furthermore, the perturbative results lie significantly above in each case, being in the range 1.027–1.08 for 2-flavour QCD and

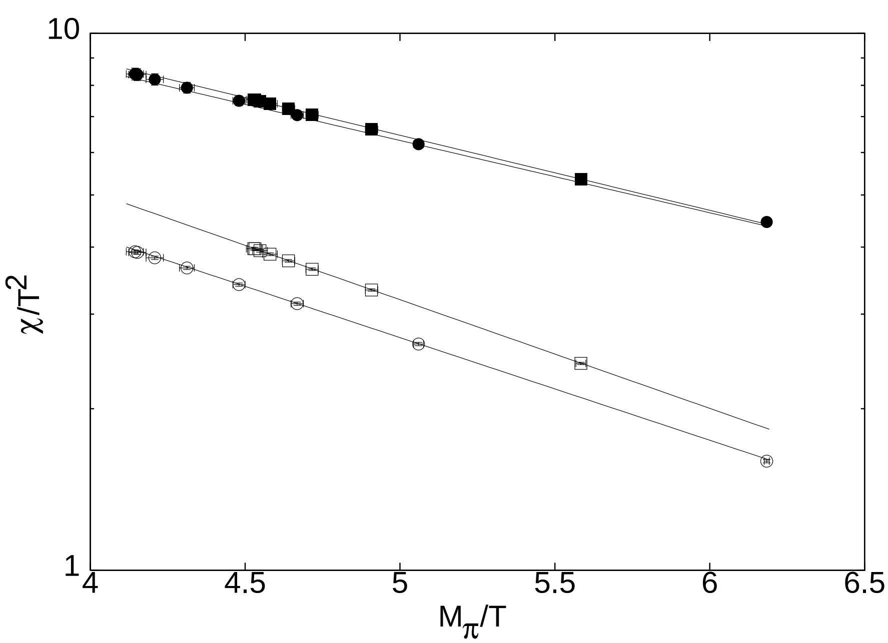

Figure 3: $4\chi_3/T^2$ (open symbols) and $\chi_\pi/10T^2$ (filled symbols) as a function of M_π/T at $2T_c$ (circles) and $3T_c$ (boxes).

0.986–0.994 for quenched QCD. Although the order of magnitude of the degrees of freedom can be gauged from these results and their eventual comparison with experiments, they do call for clever resummations of perturbation theory for a more convincing and precise count.

Alternatively, the deviations from free field theory could stem from non-perturbative physics. One known indicator of non-perturbative physics in the plasma phase is the screening length in the channel with quantum numbers of pion. While it exhibits chiral symmetry restoration above T_c by being degenerate with the corresponding scalar screening length, its value is known to be much smaller than the free field value unlike that for other screening lengths. Fig. 3 shows our results for χ_3 and χ_π (defined as a sum of the pion correlator over the entire lattice) as a function of the inverse pionic screening length, M_π/T. It suggests the non-perturbative physics in the two cases to be closely related, if not identical.

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