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Procedia Engineering 173 (2017) 1642 – 1649

Procedia Engineering

www.elsevier.com/locate/procedia

11th International Symposium on Plasticity and Impact Mechanics, Implast 2016

Failure Analysis of Civil Engineering Composite Shell Roofs

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Abstract

Light weight laminated composite materials have seen dramatic increase in civil, aerospace, marine and other weight sensitive engineering applications due to their design versatility and high stiffness-to-weight and strength-to-weight ratios. Naturally different behavioural aspects like failure of composites gained importance as research areas for confident use of these materials. Failure in composites starts with the failure of the weakest lamina and may propagate in different directions due to reduction of stiffness thus introduced leading to progressive failure of the structure. In the present study composite shell structures are solved for failure using an eight noded isoparametric shell bending element. Failure behaviour of cross and angle ply, symmetrically laminated graphite-epoxy composite shell roofs are reported.

Peer-review under responsibility of the organizing committee of Implast 2016

Keywords: Laminated composite; First ply failure; Shell roofs; Finite element method.

1. Introduction

A shell is defined as an arbitrarily curved thin structural surface, which resists any externally superimposed load by combined in-plane thrusts and bending of the surface. This coupling of in-plane forces and bending moments renders the shell forms with high strength. Thin shells as a structural entity occupy a leadership position in engineering and, in particular, in civil, mechanical, architectural, aeronautical and marine engineering. In civil engineering, it is advantageous to use thin shells instead of plates to cover large column free open spaces as one sees in airports, car parking lots, auditoriums and shopping malls. A skewed hypar shell apart from being good looking is a doubly ruled anticlastic surface and hence easy to cast and fabricate. No wonder that this form is preferred by

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Peer-review under responsibility of the organizing committee of Implast 2016
doi:10.1016/j.proeng.2016.12.258

practicing engineers. Most of the researchers have done their research work on composite plates and some of them have worked on cylindrical and spherical shells only. A close look through the literature shows that the industrially important skewed hypar shells need more in-depth study although some important aspects of these shell forms were studied by Nayak and Bandyopadhyay [1] and Sahoo and Chakravorty [2, 3].

The vast spread of composite materials in civil, aerospace, marine and other weight-sensitive engineering applications is due to its improved properties, such as high strength-to-weight and high stiffness-to-weight ratios, long fatigue life, good corrosion resistance and dimensional stability during large temperature change in space. Researchers like Pal and Ray [4], Pal and Bhattacharyya [5], Prusty [6], Chang and Chiang [7] studied the first and progressive failure analysis of composite materials under static loading for both symmetrically and anti-symmetrically built laminated plates. Progressive failure analysis of composite plates by layer wise B-spline finite strip method was elaborated by Zhang et al [8]. The progressive failure model and analysis of composite plates was also studied by some researchers like Philippidis and Antoniou [9], Cardenas et al [10] and Ellul et al [11]. Adali and Cagdas [12] reported linear first ply failure loads of laminated composite singly and doubly curved shell panels subjected to static load. Gohari et al [13] worked on first ply failure of an internally pressurized spherical shell.

The study of the literature reveals that research reports on failure of laminated composite plates are available abundantly, but similar work on shells is much less in number in general and absolutely missing for industrially important skewed hypar shell form which is basically the hyperbolic paraboloid shape bounded by straight lines. To supplement the volume of information available on these shells the present study intends to study the failure behaviour of composite skewed hypar shell roofs with clamped and simply supported boundary conditions. The results are interpreted from practical engineering point of view.

2. Mathematical Formulation

An eight noded isoparametric curved quadratic shell element having five degrees of freedom (*u*, *v* and *w* are the displacements along *X*, *Y* and *Z* axes respectively and α and β are the rotations along *X* and *Y* axes respectively) is used for the present skewed hypar shell analysis. The element displacement field $\{d\}$ is described as the following,

$$\{d\} = \{u \quad v \quad w \quad \alpha \quad \beta\}^T \tag{1}$$

The shape functions derived from a cubic interpolation polynomial are:

$$N_i = (1+\xi\xi_i)(1+\eta\eta_i)(\xi\xi_i+\eta\eta_i-1)/4, \qquad \text{for } i = 1, 2, 3, 4 \tag{2}$$

$$N_i = (1+\xi\xi_i)(1-\eta^2)/2, \qquad \text{for } i = 5, 7 \tag{3}$$

$$N_i = (1+\eta\eta_i)(1-\xi^2)/2, \qquad \text{for } i = 6, 8 \tag{4}$$

A laminated composite hypar shell of uniform thickness *h* and twist radius of curvature R_{xy} is considered. Keeping the total thickness the same, the thickness may consist of any number of thin laminae each of which may be arbitrarily oriented at an angle θ with reference to the *X* axis of the coordinate system. The surface equation of this shell is:

$$z = \frac{4c}{ab}(x-a/2)(y-b/2) \tag{5}$$

The strain-displacement matrix, laminate constitutive relationship matrix and the systematic development of the stiffness matrix as presented by Das and Chakravorty [14] are taken as components of the finite element tool used here. In the present approach the laminate stress resultant vectors are expressed as the following (Fig. 1).

$$\begin{aligned}
\{N\} &= \sum_{k=1}^{n}[Q_{ij}]\left\{\int_{z_{k-1}}^{z_k}\{\varepsilon^0\}dz + \int_{z_{k-1}}^{z_k}\{\kappa\}z\,dz\right\}; i, j = 1,2,6 \\
\{M\} &= \sum_{k=1}^{n}[Q_{ij}]\left\{\int_{z_{k-1}}^{z_k}\{\varepsilon^0\}z\,dz + \int_{z_{k-1}}^{z_k}\{\kappa\}z^2 dz\right\}; i, j = 1,2,6 \\
\{Q\} &= \sum_{k=1}^{n}[Q_{ij}]\left\{\int_{z_{k-1}}^{z_k}\{\gamma\}dz\right\}; i, j = 4,5
\end{aligned} \tag{6}$$

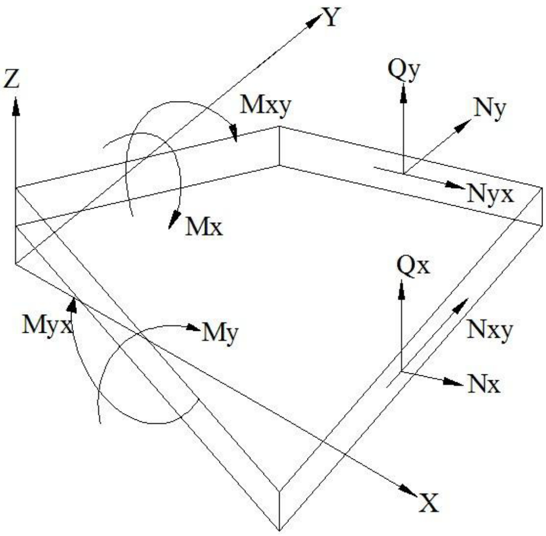

Fig. 1. Generalized force and moment resultants.

In-plane strain components for a lamina situated at a distance z from the lamina mid-plane are evaluated in global axes as: $\varepsilon_x = \varepsilon_x^0 + zk_x$, $\varepsilon_y = \varepsilon_y^0 + zk_y$ and $\gamma_{xy} = \gamma_{xy}^0 + zk_{xy}$ (7)

Lamina strains are transformed from the global axes of the shell to the local axes of the lamina using transformation matrix,

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \dfrac{\varepsilon_6}{2} \end{Bmatrix} = \begin{bmatrix} \sin^2\theta & \cos^2\theta & \sin 2\theta \\ \cos^2\theta & \sin^2\theta & -\sin 2\theta \\ -\dfrac{\sin 2\theta}{2} & \dfrac{\sin 2\theta}{2} & \sin^2\theta - \cos^2\theta \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \dfrac{\gamma_{xy}}{2} \end{Bmatrix} \tag{8}$$

Lamina stresses are obtained using the constitutive relation of the lamina,

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} \dfrac{E_{11}}{1-\nu_{12}\nu_{21}} & \dfrac{E_{11}\nu_{21}}{1-\nu_{12}\nu_{21}} & 0 \\ \dfrac{E_{11}\nu_{21}}{1-\nu_{12}\nu_{21}} & \dfrac{E_{22}}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix} \tag{9}$$

So once the force-strain relationship is known, the element stiffness matrix can be found. The element stiffness matrix and element load vectors are assembled to get the global stiffness matrix [*K*] and global load vectors {*F*} respectively. The basic problem of static equilibrium takes the form, [*K*]{*d*} = {*F*} (10)

The equation is solved to get the unknown displacements. After calculating all the displacements the strains and hence the stress resultants are evaluated at the Gauss points (2 X 2). Lamina stresses are used in well accepted failure theories like maximum stress, Tsai-Hill, Tsai-Wu and Hoffman failure criterion to evaluate the first ply and ultimate ply failure loads of the composite skewed hypar shells under present study. A composite lamina may fail in different manner. The stiffness of a laminated composite hypar shell will decrease when failure of plies occurs. In the present failure analysis, the hypar shell structure will be considered to have completely failed when it is no longer capable of carrying any further load.

3. Numerical Investigations

Numerical examples are analyzed using an 8 X 8 division finite element mesh. First ply failure loads evaluated using the present formulation and are compared with the linear failure loads reported by Kam et al [15] to establish the precision of this formulation (Table 1). Also, the results displayed in Table 2 show a good match with the results obtained by Reddy [16] using exact method.

Material properties of the graphite-epoxy composite to fabricate the hypar shells under present study are taken from Kam et al [15] and its geometric dimensions and the stacking sequences of the shell combinations taken up by the authors are furnished in Table 3. The shells are subjected to uniformly distributed transverse static loading under clamped and simply supported boundary conditions.

Table 1. Comparison of first ply failure loads in Newton for a $\left(0_2^0/90^0\right)_s$ plate.

Failure criteria	Side/thickness	First ply failure loads [15]	First ply failure loads [present formulation]
Maximum stress		108.26	112.14
Hoffman	105.26	106.45	104.40
Tsai-Wu		112.77	110.50
Tsai-Hill		107.06	104.40

Table 2. Non-dimensional central deflections (w_1 x10^3) of simply supported composite spherical shell under uniformly distributed load.

Lamination	Reddy [16]	Present approach
$0^0/90^0$	16.980	17.010
$0^0/90^0/0^0$	6.697	6.701
$0^0/90^0/90^0/0^0$	6.833	6.836

Note: $E_{11}/E_{22}=25$, $G_{12}=G_{13}=0.5E_{22}$, $G_{23}=0.2E_{22}$, $\nu_{12}=0.25$, a/b =1, a/h=100, $E_{22}=10^6$N/cm^2, R/a=10^{30}

Table 3. Geometric dimensions and stacking sequences.

Hypar shell dimensions	Values (mm)	Lamina name	Stacking orders
Length (*a*)	1000	Cross Ply	$0^0/90^0/0^0$
Width (*b*)	1000		
Thickness (*h*)	10	Angle Ply	$45^0/-45^0/45^0$
Rise of hypar shell (c)	200		

4. Results and Discussions

4.1. First ply and ultimate ply failure load for different boundary condition

From all the first ply failure load values corresponding to different stress based failure criterion, the one which is least (shown in italic in Table 4) is accepted as the first ply failure load value. When the first ply failure load and the ultimate ply failure load values are compared for the different boundary conditions it is found from Table 4 that the clamped shells having the maximum number of support constraints yield the highest load values and the simply supported ones where the maximum number of boundary movements are released exhibit the lowest values.The first ply failure load of cross ply is lesser than that of angle ply laminate while the cross ply laminate gives the highest magnitude of ultimate ply failure load. The general performance of the angle ply shell is better than another one. It is also interesting to note that the ultimate ply failure load is by far higher than the first ply failure load. From practical civil engineering perspective, besides strength, serviceability is also an important factor in design. A permissible deflection of span/250 which is 4 mm here (a = b = 1000mm) is considered in the present study. In case of angle ply shells when the failure progress is carefully noticed it is observed that over a wide range of load values one of the plies remains almost undamaged and this accounts for the fact that the deflections remain arrested below 4 mm even when the first ply failure load is exceeded. The geometry of a skewed hypar shell is such that the shell tends to transfer the loads in the diagonal directions. In the angle ply laminates that are selected here the fiber directions are aligned along the diagonal directions too. So angle ply shell is in general stiffer than the cross ply ones and deflects less even after getting partly damaged. The location of the first ply failure point is extremely important to be known to a practising civil engineer because any instrumentation needed for hidden flaw detection should start from that point. In other words if it is found that the point prone to first ply failure damage is free of any hidden flaw it can safely be concluded that the shell surface is free of any damage caused due to overloading.

Table 4. First ply and ultimate ply failure distributed load (MPa).

Type of lamina & boundary condition	Failure criteria	First ply failure				Ultimate ply failure load	Load for 4mm deflection where such load is less than first ply failure load
		Failure load	Failed ply	Failed Gauss point	Failed element		
Cross ply (clamped)	Maximum stress	1.0869	3	(1,2)	44	*492.7712*	0.4193
	Hoffman	*1.0348*	3	(1,2)	13	504.5858	
	Tsai–Hill	1.0851	3	(1,2)	44	504.5696	
	Tsai–Wu	1.0405	3	(1,2)	13	508.0282	
Angle ply (clamped)	Maximum stress	19.0772	3	(1,2)	56	510.3336	
	Hoffman	1.8177	1	(1,2)	8	*468.5668*	
	Tsai–Hill	1.9166	1	(1,2)	8	500.4260	
	Tsai–Wu	*1.7506*	1	(1,2)	8	491.4502	
Cross ply (Simply supported)	Maximum stress	0.7980	1	(2,1)	8	265.7825	0.1870
	Hoffman	*0.7244*	2	(2,1)	8	296.7450	
	Tsai–Hill	0.7922	1	(2,1)	8	*265.3874*	
	Tsai–Wu	0.7249	2	(2,1)	8	300.7547	
Angle ply (Simply supported)	Maximum stress	25.6408	1	(2,1)	57	504.5590	
	Hoffman	1.3737	1	(1,2)	8	356.7432	
	Tsai–Hill	1.5189	1	(2,1)	57	418.6368	
	Tsai–Wu	*1.2917*	1	(1,2)	8	*328.4164*	

4.2. Evaluation of working load and values of partial safety factor (PSF) and load factor (LF)

The study carried out so far indicates that for the cross ply laminates the first ply failure load value is more than the load value corresponds to 4 mm of deflection. In these cases the load corresponding to the permissible deflection of 4 mm may be adopted as the working load. In these cases the first ply failure load may be divided with the load value corresponding to 4 mm deflection to get partial safety factor (PSF). In the case of angle ply laminate, however, the first ply failure occurs when the deflection is less than 4 mm. So the first ply failure loads in these cases are adopted as the working values of the load. Naturally, the partial safety factors are unity in these cases. For both cross and angle ply shells the ultimate ply failure load value is divided by the working load to obtain the corresponding load factors (LF). The partial safety factor (PSF) and the load factor (LF) values as explained here are furnished in Table 5.

Table 5. Partial safety factor (PSF) and load factor (LF) values.

Boundary conditions	Lamination	PSF	LF
Clamped	Cross ply	2.4679	1175.2235
	Angle ply	1.0000	267.6607
Simply supported	Cross ply	3.8738	1419.1840
	Angle ply	1.0000	254.2513

4.3. Concept of tailored laminates and their performances

A close inspection of the failure propagation through individual lamina shows that some of the lamina remains undamaged or slightly damaged even up to high load value. Although not elaborately reported here such studies are carried out for the different laminations and boundary conditions to propose new composite laminates made up of the relatively undamaged / less damaged laminae with an expectation of improved performance. Such results are furnished in Table 6. In fact the gross behaviour of a laminate depends on the individual behaviour of its laminae and such behaviour interacts in a very complex way with the boundary condition to determine the first ply failure load value.

Table 6. Comparing the first ply failure load (MPa) between regular and tailored laminates.

Boundary conditions	Regular laminate and first ply failure load	Tailored laminate and first ply failure load
CCCC	$45^0/-45^0/45^0$, 1.7506	$0^0/-45^0/0^0$, 2.2003
SSSS	$45^0/-45^0/45^0$, 1.2917	$0^0/-45^0/45^0$, 1.0297

4.4. Correlating between deflection and area of damage

Figs. 2 to 5 represent the variation of central deflection with the area of damage for two laminations and two boundary conditions. The general trend which is observed is that the deflection varies nonlinearly with the area of damage, for small and extensive damages while in between the variation is somewhat linear. So, explicit equations may be suggested correlating the deflections with area of damages when the damage is between certain limits. Such correlations are given in Table 7 for different laminations. It is easy to measure deflections by setting up simple dial gauges and the capacity of predicting the extent of damage through deflection measurement is expected to be a very helpful tool to a practising civil engineer.

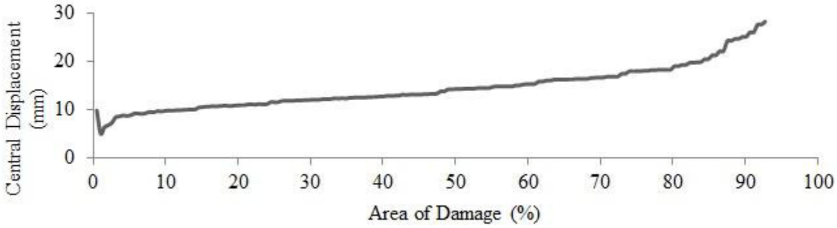

Fig. 2. Boundary condition: clamped; lamina type: cross ply.

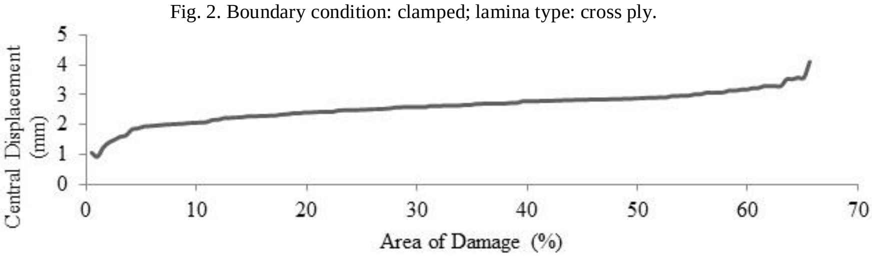

Fig. 3. Boundary condition: clamped; lamina type: angle ply.

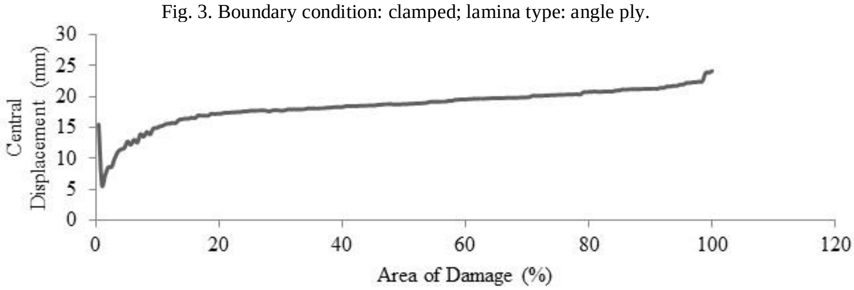

Fig. 4. Boundary condition: simply supported; lamina type: cross ply.

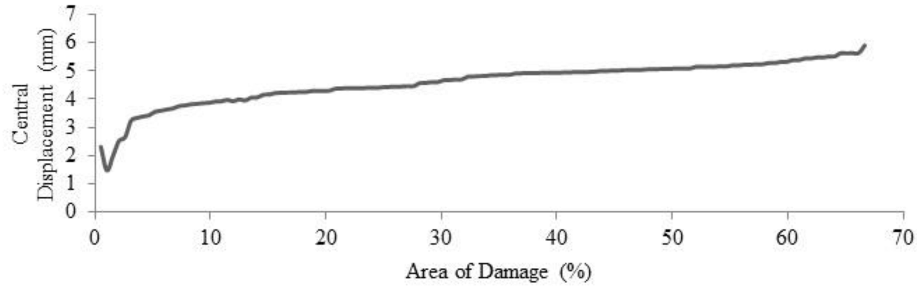

Fig. 5. Boundary condition: simply supported; lamina type: angle ply.

Table 7. Explicit equations between central deflections (C) and percentage of total area that is damaged (A).

Boundary Conditions	Lamination	Range of area of damage for quasi – linear relation between C and A	Equations connecting central deflections in mm (C) and percentage of total area that is damaged (A)
Clamped	Cross ply	20% to 80%	2A – 15C +110 = 0
	Angle ply	10% to 60%	A – 50C +100 = 0
Simply supported	Cross ply	20% to 80%	A – 20 C + 340 = 0
	Angle ply	10% to 60%	9A – 250C +860 = 0

5. Conclusions

The following conclusions may be drawn from the present study.

- The finite element code applied here can be accepted as a successful tool to explore the first and ultimate ply failure aspects of composite hypar shells. Solutions obtained for the benchmark problem using the present method indicate this fact.
- By virtue of the geometry of a skewed hypar shell, majority of the loads and moments are transferred along the diagonal directions. This is why the angle ply laminates taken up here which have their fibres oriented along the diagonals prove to be better than the cross ply ones in terms of first ply failure.
- The lamina wise failure investigation and using the information to evolve tailored laminates may be utilised as design guidelines to fabricate stiff shell surfaces for a given material consumption.
- The fact that the damaged area of a skewed hypar shell may be explicitly correlated through an equation with a visible measurable gross response of the shell that is deflection, may be utilised by practicing civil engineers in health monitoring of the shell without going into detailed non-destructive testing programme.
- The values of load factors are extremely high and indicate that the laminated composites have substantial amount of load resisting capacity beyond the working load and hence these materials are expected to exhibit a ductile failure. This virtue of these materials makes them appropriate to be used in earthquake prone zones.

Acknowledgements

The first author gratefully acknowledges the financial assistance of Technical Education Quality Improvement Programme, Phase-II (A World Bank aided project of Govt. of India) of Jadavpur University, Kolkata-700032, India.

Nomenclature

ξ, η	natural coordinates of isoparametric elements
{N}, {M}, {Q}	force, moment and transverse shear force vectors
$\{Q_{ij}\}$	elastic constant matrix
$\{\kappa\}$	curvature vectors
$\{\gamma\}$	transverse shear strain vectors
Z_k, Z_{k-1}	top and bottom distance of the k^{th} ply from mid-plane of a laminate

ε_x, ε_y	in-plane strain components along X and Y axis
γ_{xy}	in-plane shear strain components in XY plane
$\{\varepsilon^0\}$	in-plane strain vectors at the mid-surface
ε_1, ε_2	in-plane normal strains along 1 and 2 axes of a lamina respectively
ε_6	in-plane shear strain in 1-2 plane of a lamina
σ_1, σ_2, σ_6	in-plane lamina stresses
1, 2, 3	local coordinates of a lamina
ν_{ij}	Poisson's ratio
E_{11}, E_{22}, E_{33}	elastic moduli
G_{12}, G_{13}, G_{23}	shear moduli
q	transverse loading intensity
w	transverse displacement in cm
w_1	non-dimensional transverse displacement of shell $=[wE_{22}h^3/(qa^4)]$

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