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Procedia Engineering 173 (2017) 1619 – 1626

Procedia Engineering

www.elsevier.com/locate/procedia

11th International Symposium on Plasticity and Impact Mechanics, Implast 2016

Geometrically Nonlinear First Ply Failure Loads of Laminated Composite Conoidal Shells

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Abstract

Review of the existing literature on shells indicates that first ply failure of laminated conoidal shells, particularly using geometric nonlinearity was not reported by any researcher. Conoidal shell is an aesthetically appealing, stiff, easy to cast surface which is popular to cover large column free open spaces found in shopping malls, auditoriums and aircraft hangers. Lightweight laminated composites are often used to fabricate them. Laminated composite is weak in transverse shear and may initiate failure at any inner lamina. Such latent damages progress gradually within the laminate and lead to sudden catastrophic collapse under service condition. Thus, the first ply failure load has to be known to a practicing engineer. This paper reports first ply failure of conoidal shells using geometrically nonlinear isoparametric finite element formulation. The results are presented for different parametric variations and the study concludes with set of meaningful engineering design guidelines keepingalso the serviceability criterion in mind.

Peer-review under responsibility of the organizing committee of Implast 2016

*Keywords:*First ply failure; geometric nonlinearity; conoidal shells; finite element formulation; serviceability criterion.

1. Introduction

The advanced materials like laminated composites gained popularity over conventional reinforced cement concrete from second half of the last century. The advantages of composite materials like high strength/stiffness to weight ratio, superior fatigue resistance, less susceptible to be damaged by weathering actions and the flexibility to tailor its stiffness

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Peer-review under responsibility of the organizing committee of Implast 2016
doi:10.1016/j.proeng.2016.12.261

and strength properties by varying fiber orientations and lamina stacking sequences made these materials a lucrative option in weight sensitive engineering applications like in civil, marine and aerospace structures.

The civil engineering shell structures are stiffer than the flat plates by virtue of the curved geometry. The doubly curved shells are rigid than the singly curved ones. Among different doubly curved shell forms available in the literature, the conoidal shell enjoys special attention from practicing engineers due to its aesthetically appealing and singly ruled geometry. A ruled surface is easy to cast and fabricate.

Conoidal shells are used in practical civil engineering to cover large column free open spaces as one may found in shopping malls, car parking lots, aircraft hangers and in auditoriums. Moreover, these structural units allow entry of daylight and natural air which is preferred in chemical, medicine and food processing units. There is no wonder that a number of researchers studied static bending and dynamic behaviors of laminated conoids for different industrially important parametric variations. Free vibration of laminated stiffened conoidal shells was reported by Nayak and Bandyopadhyay for clamped [1] and simply supported boundary conditions [2]. The authors worked on forced vibration of isotropic stiffened shells also [3]. Das and Chakravorty studied static bending of laminated conoids [4]. The authors [5] later reported fundamental frequencies of those shells. Static bending of delaminated shells was investigated by Kumari and Chakravorty [6] while Pradyumna and Bandyopadhyay [7] studied and instability of laminated conoids.

The static and dynamic behaviors of laminated conoids are now well known since a good number of researchers worked on these aspects. The laminated composite is weak in transverse shear and may fail within the laminate. Such latent damages if remain undetected and unattended may lead to sudden catastrophic collapse of the shell under service. Thus, the load carrying capacity of the composite laminate is an imperative knowledge for a practicing engineer. The laminated composite fails progressively as reported by Ganesan and Liu [8]. The initiation of failure is designated as first ply failure and the corresponding load is termed as first ply failure load. Reddy and Reddy [9] worked on first ply failure of laminated composite plates using geometrically linear and nonlinear strains. Kam et al. [10] reported experimental and theoretical first ply failure loads of laminated plates. The authors adopted finite element method. Prusty et al. [11] studied first ply failure of singly and doubly curved shell panels using geometrically linear formulation. Chang and Chiang [12] worked on first and ultimate ply failures of antisymmetrically laminated composite plates using experimental and theoretical techniques. Stochastic nonlinear failure analysis of laminated composite plates under compressive transverse loading was reported by Lal et al. [13]. First ply failure prediction of an internally pressurized spherical shell was studied by Gohari et al. [14].

The review of literature clearly shows that first ply failure study of laminated conoidal shells is absolutely missing using geometrically nonlinear formulation. Hence, the present study aims to report nonlinear first ply failure loads of laminated conoidal shells. The failure loads are furnished for varying boundary conditions, laminations and stacking sequences of the shell.

2. Mathematical Formulation

An isoparametric finite element code is formulated using Sander's nonlinear shell kinematics and first order shear deformation theory to study first ply failure of laminated composite conoidal shell (Fig. 1). The shell has radii of curvatures, R_{yy} and R_{xy}, and uniform thickness ,*h*, which may consists of any number of thin laminae oriented at an angle 'θ' with respect to the global 'x' axis. The shell has sides 'a' and 'b' respectively measured at the mid-surface.

The governing differential equation is derived based on the minimization of total potential energy. The total potential energy (π) is expressed as a function of strain energy (U) and work done by the external load (W) as was indicated by Bakshi and Chakravorty [15].

The constitutive relationship of the composite shell is the same as it was reported by Das and Chakravorty [4]. The laminate elasticity matrix [D] and proper shear correction factors are adopted from Das and Chakravorty [4] while obtaining the shear stress resultants.

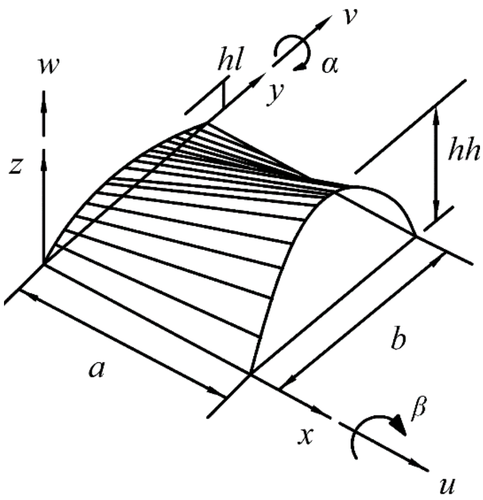

Fig. 1. A typical conoidal shell

The isoparametric formulation uses eight noded doubly curved elements with C^0 continuity. Five degrees of freedom u, v, w, α and β (Fig. 1) are assigned at each node of the element. The mid-surface strain of the shell (Equation 1) is derived by a combination of Sanders' nonlinear strain – displacement relations and first order shear deformation theory.

$$\{\varepsilon\} = \left\{\varepsilon_x^0 \quad \varepsilon_y^0 \quad \gamma_{xy}^0 \quad k_x \quad k_y \quad k_{xy} \quad \gamma_{xz}^0 \quad \gamma_{yz}^0\right\} \tag{1}$$

The mid-surface strain of the shell $\{\varepsilon\}$ is expressed as a combination of linear $\{\varepsilon\}^L$ and nonlinear $\{\varepsilon\}^{NL}$ strains as indicated below,

$$\{\varepsilon\} = \{\varepsilon\}^L + \{\varepsilon\}^{NL} \tag{2}$$

where $\{\varepsilon\}^L = [B]^L\{d\}$, $\{\varepsilon\}^{NL} = [B]^{NL}\{d\}$. $\{d\}$ =Mid-surface displacements of the shell [4].

$[B]^L$ is the linear strain - displacement relationship of the shell and adopted from Das and Chakravorty [4].

The nonlinear strain - displacement relationship $[B]^{NL}$ is given as follows,

$$\{\varepsilon\}^{NL} = \left\{ \begin{array}{c} \frac{1}{2}\left(\frac{\partial w_0}{\partial x} - \frac{u}{R_{xx}}\right)^2 \\ \frac{1}{2}\left(\frac{\partial w_0}{\partial y} - \frac{v}{R_{yy}}\right)^2 \\ \left(\frac{\partial w_0}{\partial x} - \frac{u}{R_{xx}}\right)\left(\frac{\partial w_0}{\partial y} - \frac{v}{R_{yy}}\right) \\ 0 \\ 0 \end{array} \right\} \text{ and } [B]^{NL} = [A][G]$$

where

$$[A]=\begin{bmatrix} \frac{\partial w_0}{\partial x}-\frac{u}{R_x} & 0 \\ 0 & \frac{\partial w_0}{\partial y}-\frac{v}{R_y} \\ \frac{\partial w_0}{\partial y}-\frac{v}{R_y} & \frac{\partial w_0}{\partial x}-\frac{u}{R_x} \end{bmatrix},\ [G]=\begin{bmatrix} -\frac{N_i}{R_x} & 0 & \frac{\partial N_i}{\partial x} & 0 & 0 \\ 0 & -\frac{N_i}{R_y} & \frac{\partial N_i}{\partial y} & 0 & 0 \end{bmatrix} \quad i=1 \text{ to } 8$$

The nonlinear equilibrium equation is solved by using the Newton-Raphson iterative approach as reported by Chattopadhyay et al. [16]. The partial derivative of the total potential energy yields the tangent stiffness matrix $[K_T]$ of the shell as depicted below,

$$[K_T]=\frac{\partial\{\psi(d_n)\}}{\partial\{d_n\}} \tag{3}$$

The tangent stiffness matrix $[K_T]$ is given as follows:

$$[K_T]=\iint_A([B]^L)^T[D][B]^L dxdy+\iint_A([B]^L)^T[D][B]^{NL} dxdy+\iint_A([B]^{NL})^T[D][B]^L dxdy+\iint_A([B]^{NL})^T[D][B]^{NL} dxdy$$
$$+\iint_A[G]_n{}^T\begin{bmatrix} N_1 & N_6 \\ N_6 & N_2 \end{bmatrix}[G]_n dxdy$$

N_1, N_2 and N_6 are inplane stress resultants [4].

The internal force vector is given by, $\iint_A[\overline{B}]^T[D]\{\varepsilon\}dxdy=[K_s]\{d_n\}$

$$[K_s]=\iint_A([B]^L)^T[D][B]^L dxdy+\frac{1}{2}\iint_A([B]^L)^T[D][B]^{NL} dxdy+\iint_A([B]^{NL})^T[D][B]^L dxdy+\frac{1}{2}\iint_A([B]^{NL})^T[D][B]^{NL} dxdydy$$

The stiffness matrices ($[K_T]$ and $[K_S]$) and external and internal load vectors are calculated by numerical integration method using 2×2 Gauss quadrature rule. Global stiffness matrices ($[K_T]$ and $[K_S]$) and load vectors are obtained by assembling the element matrices with proper transformations due to the curved geometry of the shell [15].The convergence of the Newton – Raphson iterative process is checked using the guidelines proposed by Chattopadhyay et al. [16].The converged displacements of the conoid are used to obtain the lamina stresses and strains [15]. The first ply failure loads are obtained by applying those stresses and strains in well-known failure theories like maximum stress, maximum strain, Tsai-Wu, Tsai-Hill and Hoffman failure criterion. The failure theories are adopted from Reddy and Reddy [9].

3. Numerical Problems

The correctness of the conoidal shell formulation is confirmed by mutually comparing the static displacements of an isotropic conoid obtained using the proposed code and the result reported by Hadid [17]. Equal values of elastic moduli, shear moduli and Poission's ratio are assigned in the code developed for composite shell to model the isotropic shell as a special case. Figure 2 reports the comparison of static displacements along with which the material properties of the isotropic shell are also furnished.

The first ply failure loads of a partially clamped plate obtained from the proposed code are compared with the values reported by Kam et al. [10] to confirm the accuracy of the proposed nonlinear first ply failure formulation. The comparison is furnished in Table 1 which establishes the correctness of the proposed code. The geometric dimensions of the plate are furnished with Table 1. The graphite-epoxy used by Kam et al [10] is the same as the authors adopted here and are presented in Table 2.

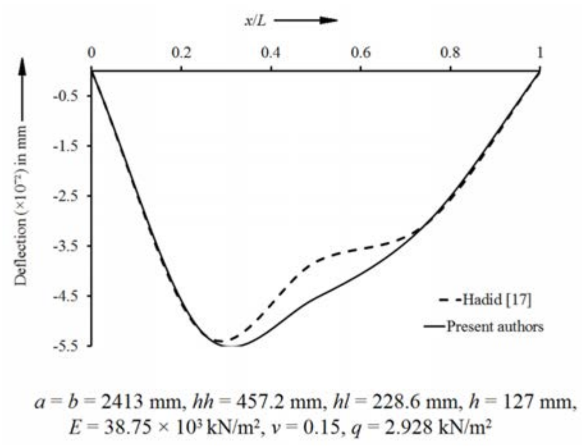

Fig. 2.Deflection profile of isotropic conoid under uniformly distributed load along $\overline{y}$ =0.5

Table 1: Comparison of nonlinear first ply failure loads in Newton for a $(0^0_2/90^0)_s$ plate

Failure criteria	Side/ thickness	Failure loads (Kam et al. [10])	Experimental failure load (Kam et al. [10])	Failure loads (present formulation)
Maximum stress		147.61		135.94
Maximum strain		185.31		218.10
Hoffman	105.26	143.15	157.34	133.21
Tsai-Wu		144.42		134.50
Tsai-Hill		157.58		134.91

[1]Length=100mm, ply thickness=0.155mm, load details=central point load.

Table 2: Material properties of Q-1115 graphite-epoxy composite

Material constants			Strengths				
E_{11}	142.50	GPa	X_T	2193.50	MPa	$X_{\varepsilon t}$	0.01539
E_{22}	9.79	GPa	X_C	2457.0	MPa	$X_{\varepsilon c}$	0.01724
E_{33}	9.79	GPa	Y_T=Z_T	41.30	MPa	$Y_{\varepsilon t}$=$Z_{\varepsilon t}$	0.00412
G_{12}=G_{13}	4.72	GPa	Y_C=Z_C	206.80	MPa	$Y_{\varepsilon c}$=$Z_{\varepsilon c}$	0.02112
G_{23}	1.192	GPa	R	61.28	MPa	R_ε	0.05141
ν_{12}=ν_{13}	0.27		S	78.78	MPa	S_ε	0.01669
ν_{23}	0.25		T	78.78	MPa	T_ε	0.01669

Once the accuracy of the proposed code is confirmed, it is further used to study first ply failure loads of shallow, thin and multi-layered composite conoidal shells subjected to concentrated load at the centre. The failure loads are reported for two practical boundary conditions which are depicted in Figure 3. The laminations involve symmetric and antisymmetric stacking sequences of cross and angle-ply laminates. The failure load is achieved by continuous increment of the external pressure and by continuously checking the developed stresses and strains against the failure criteria. The external pressure is modified following the guidelines of Pandey and Reddy [18]. First ply failure loads for different boundary conditions are reported in Table 3. Along with the failure loads, the failure locations on the shell surface and failed ply number are furnished in Table 4. The tables also contain the geometric dimensions of the conoid and the material properties are furnished in Table 2.

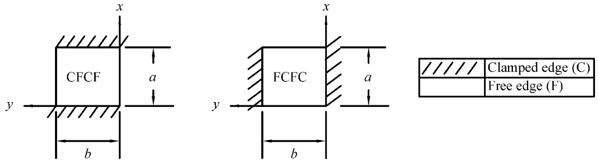

Fig. 3.Support conditions

4. Results and Discussion

The first ply failure loads of the conoidal shell are presented in Table 3 for different boundary conditions and laminations. Five different failure theories are adopted here to obtain the failure loads for a given lamination. Among of those failure loads, the engineering factor of safety should be applied on the minimum load to obtain the working load. The minimum failure load of a lamination is highlighted using bold letters in Tables 3 and 4. The governing failure, exhibiting the minimum failure load, for a given shell option reported in Table 3 is obtained from the Hoffman failure theory except the $45^0/-45^0$ shell for which the Tsai - Wu theory yielded the minimum failure load. It is interesting to observe from Table 3 that for a given boundary condition, the cross-ply laminations fail at convincingly greater loads than the angle-ply ones when the number of laminae in the laminate kept constant. The conoidal shells taken up in this study are supported along a set of parallel faces only and subjected to transverse loads. Thus, the shells deform along the plan directions only. The elastic modulus of an individual lamina is higher along its on-axis compared to its off-axis elastic modulus. The cross-ply shells have fibers oriented to its plan directions while in angle-ply ones, the fibers are not orientated along the x or y-axis. This is why the cross-ply shells shows higher failure load values compared to the angle-ply ones. It is also important to note from Table 3 that for both the boundary conditions the symmetric stacking sequences of cross and angle-ply laminations show greater failure load values than the antisymmetric ones.

The boundary conditions considered in Table 3 have a set of parallel edges clamped only. The arch directions of the shell are clamped and straight beam edges are free in CFCF boundary condition and the arch directions are free and straight beam edges are clamped in FCFC boundary condition. Naturally, a practicing civil engineer may be interested to study relative failure performances of these shells as both of them have the same numbers of support movements locked but arranged differently.

In order to study the effect of fiber orientation on the overall shell stiffness, a number of idealized unidirectional single layered laminates are used to obtain the failure loads. The single layered shells are fabricated using identical material properties and geometric dimensions indicated in Tables 2 and 3. The failure loads of single layered 0^0, 45^0 and 90^0 shells are 37.4 kN, 29.56 and 38.7 kN for CFCF boundary condition and 52.4 kN, 25.1 kN and 15.9 kN for FCFC boundary condition. The conoidal shell surface is curved along its y – axis and ruled along the x – axis. The arch direction of the shell is relatively stiffer than the beam direction by virtue of the curved geometry. The 90^0 fibers, being oriented to the arch direction, additionally stiffen this direction by the on-axis elastic modulus. The beam direction of the CFCF shells is stiff since the degrees of freedom at both the ends of the ruled direction are locked. Thus, the CFCF shell achieves a balance of stiffness along both the plan directions by utilizing the 90^0 fibers. As a result, the 90^0 fiber exhibits the maximum load carrying capacity among all the single layered CFCF shells. Unlike to the CFCF shells, the FCFC ones are supported at the ends of the arch directions only. Thus the FCFC shells lack stiffness along the beam direction while 90^0 fibers are adopted. The result is the minimum load carrying capacity among the single layered laminates. The governing failure of all the shell options taken up in Table 3 initiates through the bottommost fiber. Thus, it is not be incorrect to expect that CFCF shell shows greater load carrying capacity than the FCFC one while the bottommost lamina is fabricated using the 90^0 fibers. The $0^0/90^0$ shell in Table 3 agree exactly with this expectation and truly it fails at higher load for CFCF boundary condition.

The 0^0 fibers run along the beam direction of the conoid and stiffen its relatively weaker side. Thus, the FCFC shell achieves the balance of stiffness along both its plan direction by using the 0^0 fibers. The output is the greatest load carrying capacity among all the single layered laminates adopted here. The CFCF single layered 0^0 shell shows a comparable performance with the CFCF single layered 90^0 one, however, the failure load reduces marginally by 3.5%. This is why the FCFC shell in Table 3 showed higher failure load than the CFCF one when the $0^0/90^0/0^0$ lamination is adopted.

The angle-ply laminations, being not oriented along any of the plan directions, the governing stresses come directly to the matrix which has equal components along both arch and beam directions of the conoid. Thus, both the single layered 45^0 shells show comparable performances for FCFC and CFCF support conditions, however, still the CFCF shell exhibits the higher failure load value. Perfectly in tune to this observation, the symmetric and antisymmetric multi-layered angle-ply units considered in Table 3 fail at greater loads for CFCF support condition.

The CFCF shell is typically selected for further failure study (Table 4). The results include maximum deflections of the shell at failure, failure locations on the shell surface and first failed ply number. Plies are numbered from the top of the laminate downwards i.e. the topmost ply is numbered one and bottommost ply has the highest number. Table 4 shows that all the governing failures initiate beneath the concentrated load and at the bottommost fiber. This indicates tensile failure of the lamina due to stress concentration under the load.

Table 3: First ply failure loads of multi-layered conoidal shells

Lamination (degree)	Failure theory	Failure load in Newton	
		CFCF	FCFC
0/90	Maximum stress	26192	22206
	Maximum strain	24914	21535
	Hoffman	**21245**	**20552**
	Tsai-Hill	23276	22206
	Tsai-Wu	24175	22154
0/90/0	Maximum stress	26989	28975
	Maximum strain	44031	45673
	Hoffman	**23679**	**25665**
	Tsai-Hill	25034	27039
	Tsai-Wu	26267	28270
45/-45	Maximum stress	9714	9088
	Maximum strain	11379	10678
	Hoffman	9720	9091
	Tsai-Hill	9678	9056
	Tsai-Wu	**9468**	**8865**
45/-45/45	Maximum stress	17086	15833
	Maximum strain	17629	16415
	Hoffman	**16985**	**15726**
	Tsai-Hill	17067	15812
	Tsai-Wu	17265	16014

a =1000 mm, a/h = 100, a/b=1, hl/hh = 0.25

Table 4: First ply failure loads, deflections, locations and modes/tendencies of CFCF shell

Lamination (degree)	Failure theory	Failure load (N)	Downward Deflection (mm)	Location (x,y) (m,m)	First failed ply
0/90	Maximum stress	26192	16.373	(0.5,0.38)	1
	Maximum strain	24914	15.641	(0.5,0.38)	1
	Hoffman	**21245**	13.673	(0.5,0.5)	2
	Tsai-Hill	23276	14.956	(0.5,0.5)	2
	Tsai-Wu	24175	15.556	(0.5,0.5)	2
0/90/0	Maximum stress	26989	12.164	(0.5,0.5)	3
	Maximum strain	44031	20.026	(0.57,0.5)	2
	Hoffman	**23679**	10.563	(0.5,0.5)	3
	Tsai-Hill	25034	11.219	(0.5,0.5)	3
	Tsai-Wu	26267	11.816	(0.5,0.5)	3
45/-45	Maximum stress	9714	4.344	(0.5,0.5)	2
	Maximum strain	11379	5.318	(0.5,0.5)	2
	Hoffman	9720	4.345	(0.5,0.5)	2
	Tsai-Hill	9678	4.324	(0.5,0.5)	2
	Tsai-Wu	**9468**	4.215	(0.5,0.5)	2
45/-45/45	Maximum stress	17086	5.902	(0.5,0.5)	3
	Maximum strain	17629	6.792	(0.5,0.5)	3
	Hoffman	**16985**	5.861	(0.5,0.5)	3
	Tsai-Hill	17067	5.894	(0.5,0.5)	3
	Tsai-Wu	17265	5.982	(0.5,0.5)	3

a =1000 mm, a/h = 100, a/b=1, hl/hh = 0.25

While subjected to service loads, a civil engineering shell has to maintain the developed stresses and strains within the permissible limits. Moreover, the deflections of the shell should also be under the serviceable limits. Table 4 indicates that before the material failure, the conoidal shell violated the serviceability criterion if the limiting deflection is set to span/250, which is 4 mm here. Hence, further exercise is carried out to obtain the nonlinear loads corresponding to 4 mm deflection. These load values are termed as working loads and are furnished in Table 5.

Table 5: Working loads and FOS of CFCF shell

Lamination (degree)	Load in Newton corresponding to 4 mm deflection	Factors of safety (FOS)	Modified FOS values to be applied
0/90	7150	2.971	3.00
0/90/0	9699	2.441	2.50
45/-45	9085	1.042	1.10
45/-45/45	12408	1.368	1.50

The factor of safety (FOS) values which are to be applied on failure loads to obtain their working values are reported in Table 5. The FOS values are evaluated by using the working loads as denominator. Since the FOS values are prescribed in design codes of practices as integers or half the integers, the values recommended in Table 5 are modified for a given lamination.

5. Conclusion

The study may lead to the following conclusions:

a) The proposed code is capable to provide accurate non linear first ply failure loads of composite conoidal shells.
b) The cross-ply laminations are relatively better options to fabricate the conoidal shell surfaces than the angle-ply ones.
c) Among the cross-ply laminations adopted in this study, the $0^0/90^0/0^0$ lamination shows the best performance by yielding the maximum value of failure load.
d) The FCFC boundary condition should be adopted for the $0^0/90^0/0^0$ shell to maximize its failure load.

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