



Stabilization of moduli in spacetime with nested warping and the UED

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Abstract

The absence, so far, of any graviton signatures at the LHC imposes severe constraints on the Randall–Sundrum scenario. Although a generalization to higher dimensions with nested warpings has been shown to avoid these constraints, apart from incorporating several other phenomenologically interesting features, moduli stabilization in such models has been an open question. We demonstrate here how both the moduli involved can be stabilized, employing slightly different mechanisms for the two branches of the theory. This also offers a dynamical mechanism to generate and stabilise the scale for the Universal Extra Dimensions, another long-standing issue.

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1. Introduction

The discovery of the Higgs boson [1,2], while seemingly completing the jigsaw that the Standard Model (SM) is, has also brought into sharp focus a long-standing puzzle that has plagued the SM. The very lightness of its being militates against the conventional wisdom that the mass of a fundamental scalar should flow, at the very least, to the next-higher scale in the theory. Several “resolutions” of this *hierarchy problem* have been proposed, some technically natural and others not so, most of these relying on some new dynamics and/or new states appearing at the

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few-TeV scale that would serve to nullify the largest of the quantum corrections accruing from within the SM. The continuing absence of any direct evidence of such states, though, bring into question many such explanations.

A particularly elegant resolution is proffered by higher-dimensional theories. While models with large extra dimensions [3,4] have been quite popular, these fail to truly solve this problem in that these proffer no mechanism to stabilize the corresponding moduli. Similar is the case with Universal Extra Dimensions (UED) [5] which, while proffering interesting phenomenological consequences, such as an origin of Dark Matter, flavour physics as well as collider signatures, again do not really solve the problem of large hierarchies. Quite the opposite is the case of theories with a warped geometry [6–8], wherein one assumes space-time to be a slice of AdS₅, bounded by two 3-branes, on one of which (the TeV brane) the SM fields are confined. There is but one fundamental scale (the scale of gravity M_5 , very close to the derived scale M_{Planck}) in the theory, and the smallness of the electroweak scale (with respect to M_5) is only an apparent one, caused by the non-trivial dependence of the background metric on our brane's location in the fifth (x_4) dimension, or rather its distance, r_c , from the other, and equally “end-of-the-world”, 3-brane (also termed the UV-brane). To be specific, one has, for the Higgs vacuum expectation value (and, similarly, for the mass), $v = \tilde{v} \exp(-\pi k_5 r_c)$, where $\tilde{v} = \mathcal{O}(M_5)$ and k_5 is a measure of the bulk curvature.

With the extent of the hierarchy now being determined by the modulus (r_c) of the compactified fifth dimension, the latter must be stabilized, an issue not addressed by the originators of the model. In other words, if the modulus is construed to be a dynamical field \mathcal{M} , then a mechanism that forces the field to settle (at $\langle \mathcal{M} \rangle = r_c$) should exist and be operative. As Ref. [9] showed, this could be achieved by introducing a new scalar field ϕ , with a non-vanishing potential, in the five-dimensional bulk. As ϕ interacts with \mathcal{M} through the metric, integrating out the former would result in an effective potential $V_{\text{eff}}(\mathcal{M})$. An apt and simple choice of the scalar-potential along with boundary conditions (without any discernible hierarchy) can, then, lead to a suitable form for $V_{\text{eff}}(\mathcal{M})$ and, thereby, an appropriate r_c .

With gravity percolating into the bulk, it is obvious that compactification would lead to a Kaluza–Klein (KK) tower of gravitons, with masses given by $m_n = x_n k_5 \exp(-\pi k_5 r_c)$ where x_n denote the roots of the Bessel function of order one. Both the applicability of semi-classical arguments (upon which the model hinges) as well as string theoretic arguments relating the D3 brane tension to the string scale (and, hence, to M_5 through Yang-Mills gauge couplings) restrict $k_5/M_5 \lesssim 0.15$ [10]. Thus, one expects the first KK-mode of the graviton, to be, at best, a few times heavier than the Higgs boson. Furthermore, the very warping that explains the hierarchy also concentrates the KK-modes (though, not the lowest and massless mode) near the TeV brane, thereby enhancing their couplings to the SM fields. Consequently, several search strategies at the LHC were designed [11–14] to detect their signatures in a multitude of channels. Negative results from the same viz. $m_1 \gtrsim 2.66$ TeV (at 95% C.L.) [15,16], thus, impose severe constraints on the model. It can be argued that, unless we allow a little hierarchy in the ratio \tilde{v}/M_5 , the RS scenario can be ruled out as a solution to the SM hierarchy problem.

The situation improves significantly if one were to consider an extension of the scenario to two extra dimensions with nested warpings [17]. The graviton spectrum, while now being enlarged to a tower of towers, is different from that in the five-dimensional case in two crucial aspects. For one, the change wrought in the graviton wave function results in the mass of the first KK-mode

being significantly higher than that of the corresponding mode in the (five-dimensional) RS case.¹ As this happens for natural values of the parameters, and does not need any fine-tuning, this feature, on its own, would imply a weakening of the aforementioned “little hierarchy” that the original RS scenario needed so as to explain the nonobservation of gravitons at the LHC [18]. More importantly, the large coupling (to the SM fields) enhancement that allowed for the graviton KK-modes to be extensively produced at the LHC, is now tempered to a great degree [18], a consequence, once again, of the double warping. Consequently, the graviton production rates are further suppressed and the scenario easily survives the current bounds from the LHC [18,19]. On the other hand, while the allowed parameter space of the model is still quite extensive, it can be probed well in the current run of LHC.

This, along with the fact that formulating the theory in a six-dimensional world has many other benefits, especially when the SM fields are also allowed into the bulk [20,21], renders this construction rather interesting. In particular, with the four-dimensional theory getting supplanted by a five-dimensional one at the lower of the two compactification scales, the infrared is effectively screened from modes traversing the far ultraviolet. This is also reflected by the explicit computations of the electroweak precision variables [21], which demonstrated that the little hierarchy is no longer a major issue. However, the very issue of stabilizing the moduli (two in the current case, as opposed to a single one in the 5-dimensional one) has not been addressed so far. This assumes particular significance in that the structure formulated in Ref. [17] does not boast of a conformally flat geometry. Furthermore, the branes are not necessarily flat and this introduces its own set of complications. In this paper, we aim to rectify this situation and develop two related, but distinct, stabilisation mechanisms, somewhat analogous to those in Refs. [9,22–24]. This would also be seen to offer a stabilization mechanism for the modulus in a UED theory, thereby addressing a long-standing general lacuna in this otherwise attractive scenario.

Before we venture into the actual stabilization mechanism or even a detailed discussion of the scenario, we wish to clarify certain issues. Naively, it might be argued that having the gravitons to be heavier than in the RS would result in a worse fine tuning for the electroweak scale. As we have already mentioned, this moderate heaviness is but a consequence of there being two extra dimensions. To appreciate this, let us consider a sequence of unrelated scenarios. The first example would be an ADD [4]-like scenario with two extra-dimensions being compactified toroidally, with radii (possibly different) only somewhat larger than M_6^{-1} , namely $R_i = \theta_i M_6^{-1}$ with $\theta_i \gtrsim 1$. This would have meant $M_{\text{Pl}}^2 = M_6^2 \theta_1 \theta_2$. In the analogous five-dimensional theory, one would have, instead, $M_{\text{Pl}}^2 = M_5^2 \theta_1$. Thus, for the theory with the larger number of extra dimensions, one would have a smaller hierarchy between the fundamental M_5 , M_6 etc. as the case may be and the electroweak scale. Consider, as the next example, a simplistic generalization of the RS scenario to a slice of AdS_6 bounded by two 4-branes, such that the apparent scale on the IR-brane is a few TeVs. This particular (multi-TeV) scale would, then, be protected with the graviton KK-modes now lying at the same scale. If the 5-dimensional world be further compactified (or, even, orbifolded) over a circle of small radius, there would extend an additional factor in the relation between M_6 and M_{Pl} , thereby further ameliorating the hierarchy problem. As we shall see, much the same happens in the present case.

¹ This is easy to understand once one realizes that the resolution of the hierarchy between the electroweak scale and the fundamental scale is now shared between two warpings. Consequently, the extent of the individual warpings is smaller here than required in the RS case.

On a related note, the cutoff of the effective four-dimensional theory needs to be identified too. For an effective theory, this is often described as the scale at which the loop contributions (often very large) are to be cut off, for the new physics beyond this scale would naturally regulate such contributions. Nonetheless, with the ultraviolet completion of the present theory being unknown (in the absence of any quantum theory of gravity), this cancellation cannot be demonstrated exactly. However, within the five-dimensional context, it has been argued that the addition of the Planck-brane and/or the TeV-brane allows for a holographic interpretation, with the former acting as a regulator leading to a UV cutoff, of the order of the inverse of the modulus, on the corresponding conformal field theory [25–27]. A similar conclusion also holds for theories with gauge fields extended in to the warped bulk [10,28,29]. While no such duality has been explicitly constructed for the six-dimensional case, one such would obviously exist, for, in a certain limit, the bulk is indeed AdS₆-like. Thus, the branes would provide a regulator, albeit in a deformed CFT. In particular, the cutoff for the four-dimensional quantum field theory is set not by M_6 , but the inverse of the larger of the two moduli. At such a scale, the higher-dimensional nature of the theory becomes quite apparent, and the four-dimensional effective theory (including the graviton KK-modes) is no longer an apt language. And while the compactification mechanism is not specified here (or within the RS theory), the physics responsible for it must be incorporated in any description that reaches beyond this scale.

2. The 6D warped model

The space-time of interest is a six-dimensional one with successive (nested) warpings along the two compactified dimensions. The uncompactified directions support four-dimensional (x^μ) Lorentz symmetry while the compactified directions are individually Z_2 -orbifolded. In other words, we have $M^{1,5} \rightarrow [M^{1,3} \times S^1/Z_2] \times S^1/Z_2$. Representing the compact directions by the angular coordinates $x_{4,5} \in [0, \pi]$ with R_y and r_z being the corresponding moduli, the line element is, thus, given by [17]

$$ds_6^2 = b^2(x_5)[a^2(x_4)\eta_{\mu\nu}dx^\mu dx^\nu + R_y^2 dx_4^2] + r_z^2 dx_5^2, \tag{1}$$

where $\eta_{\mu\nu}$ is the flat metric on the four-dimensional slice of spacetime. As in the RS case, orbifolding, in the presence of nontrivial warp factors, necessitates the presence of localized energy densities at the orbifold fixed points, and in the present case, these appear in the form of tensions associated with the four end-of-the-world 4-branes.

Denoting the natural (quantum gravity) scale in six dimensions by M_6 and the negative (six dimensional) bulk cosmological constant by Λ_6 , the total bulk-brane action is, thus,

$$\begin{aligned} \mathcal{S} &= \mathcal{S}_6 + \mathcal{S}_5 \\ \mathcal{S}_6 &= \int d^4x dx_4 dx_5 \sqrt{-g_6} (M_6^4 R_6 - \Lambda_6) \\ \mathcal{S}_5 &= \int d^4x dx_4 dx_5 \sqrt{-g_5} [V_1(x_5) \delta(x_4) + V_2(x_5) \delta(x_4 - \pi)] \\ &\quad + \int d^4x dx_4 dx_5 \sqrt{-\tilde{g}_5} [V_3(x_4) \delta(x_5) + V_4(x_4) \delta(x_5 - \pi)]. \end{aligned} \tag{2}$$

The five-dimensional metrics in \mathcal{S}_5 are those induced on the appropriate 4-branes which accord a rectangular box shape to the space. Furthermore, the SM (and other) fields may be localized on additional 3-branes located at the four corners of the box, viz.

$$S_4 = \sum_{y_i, z_i=0, \pi} \int d^4x dx_4 dx_5 \sqrt{-g_4} \mathcal{L}_i \delta(x_4 - y_i) \delta(x_5 - z_i).$$

Since S_4 is not relevant to the discussions of this paper, we shall not discuss it any further.

Rather than limit ourselves to the solutions to the Einstein equations presented in Ref. [17], we consider, here, a more general class. To motivate it, let us recollect that, in such models, the presence of bent branes is due to a “lower-dimensional cosmological constant” induced on the brane. For example, the four dimensional components of the Einstein equations, in the presence of such a term Ω would read

$$a^2 \left[\frac{3}{R_y^2} \left(\frac{a''}{a} + \frac{a'^2}{a^2} \right) + \frac{2}{r_z^2} \left(3\dot{b}^2 + 2b\ddot{b} + \frac{\Lambda_6 r_z^2}{2M_6^4} b^2 \right) \right] = \frac{\Omega}{r_z^2},$$

where primes (dots) denote derivatives with respect to x_4 (x_5). Introducing a constant of separation $\tilde{\Omega}$, we have

$$3\dot{b}^2 + 2b\ddot{b} + \frac{\Lambda_6 r_z^2}{2M_6^4} b^2 = \tilde{\Omega} \tag{3}$$

and

$$\frac{3}{R_y^2} \left(\frac{a''}{a} + \frac{a'^2}{a^2} \right) + \frac{2}{r_z^2} \tilde{\Omega} = \frac{\Omega}{r_z^2 a^2}. \tag{4}$$

The first equation has the solution

$$b(x_5) = b_1 \cosh(k|x_5| + b_2), \quad b_1 = \sqrt{\frac{-\tilde{\Omega}}{3k^2}} = \text{sech}(k\pi + b_2),$$

$$k = r_z \sqrt{\frac{-\Lambda_6}{10M_6^4}} \equiv r_z M_6 \epsilon \tag{5}$$

assuming² $\tilde{\Omega} < 0$. While Ref. [17] had considered only the special case of $b_2 = 0$, we shall admit the more general solution. As we shall see below, a nonzero b_2 would have very important consequences. Physically, $\tilde{\Omega}$ (or equivalently b_2) is related to the induced cosmological constant on a five dimensional hypersurface³ along the constant x_5 direction. Differing values of $\tilde{\Omega}$, thus, correspond to inequivalent extent of bending of the four-brane, and, hence, lead to different physical outcomes. We will demonstrate this shortly using widely different (in essence, limiting) values for $\tilde{\Omega}$. However, while the quantitative results do differ, qualitatively they turn out to be quite similar, with certain aspects essentially not changing at all. This was to be expected as many of the measurables (and certainly the most important ones) are only slowly varying functions of $\tilde{\Omega}$. Consequently, the physical consequences (and the exact stabilization potential) of any

² For $\tilde{\Omega} > 0$, one would, instead, have $b(x_5) = \sqrt{\tilde{\Omega}/2k^2} \sinh(k|x_5| + b_2)$. Not much would change materially, except for the fact that $b_2 = 0$ would no longer be allowed unless one is willing to admit a vanishing metric, albeit only at a given slice of space-time. It is intriguing to note that the notion of a degenerate spacetime has received recent attention from a different standpoint [30].

³ It should be realized that a five-dimensional cosmological constant is very different from a four-dimensional one. Indeed, even for a large value of the former, one could be left with a vanishing value for the latter, as would be the case here.

arbitrary intermediate value of $\tilde{\Omega}$ can be trivially obtained by effecting a simple interpolation between the results for the extremal values.

For future convenience, we have also introduced the dimensionless combination ϵ . Clearly, for a semi-classical approach to be valid, the curvature must be significantly smaller than the mass scale of the theory. In other words, ϵ must be small,⁴ namely $\epsilon \lesssim 0.15$. On the other hand, as we shall see below, too small an ϵ would either invalidate the resolution of the hierarchy problem, or, in the process, introduce a new (but smaller) hierarchy.

The solution to eqn. (4) for a nonzero Ω is given in terms of hyperbolic functions. While it is possible to work with the general solution, the consequent algebra is exceedingly complicated and the exercise does not proffer any extra insight that a simplifying choice does not. As a nonzero Ω results in a nonzero cosmological constant in the four-dimensional world, and as the observed cosmological constant in our world is infinitesimally small, we disregard it altogether and consider only⁵ $\Omega = 0$. We do not claim to offer any rationale for this choice but for the fact that it simplifies the algebra for the rest of the article without losing any of the essence. In this limit, the solution can be expressed as⁶

$$a(x_4) = a_1 e^{-c|x_4|} \quad c \equiv b_1 k \frac{R_y}{r_z} . \tag{6}$$

Normalizing the warp factors, at their maximum values, through $a(0) = 1$ and $b(\pi) = 1$, and imposing the orbifolding conditions, we have

$$b(x_5) = \frac{\cosh(k|x_5| + b_2)}{\cosh(k\pi + b_2)} , \quad a(x_4) = \exp(-c|x_4|) . \tag{7}$$

The brane potentials are determined by the junction conditions. The ones at $x_5 = 0, \pi$ are simple and are given by

$$V_3 = \frac{-8M_6^4 k}{r_z} \tanh(b_2) , \quad V_4 = \frac{8M_6^4 k}{r_z} \tanh(k\pi + b_2) , \tag{8}$$

whereas the ones at $x_4 = 0, \pi$ have x_5 -dependent tensions

$$V_1(x_5) = -V_2(x_5) = \frac{8M_6^4 c}{R_y b(x_5)} = \frac{8M_6^4 k}{r_z} \operatorname{sech}(k|x_5| + b_2) . \tag{9}$$

It should be noted that the Israel junction condition $V_1 = -V_2$ is necessitated only by our focus on $\Omega = 0$, or, in other words, a configuration wherein the four-dimensional cosmological constant vanishes exactly. Had we admitted $\Omega \neq 0$, this equality of magnitude would neither have been necessary nor would it have held. This, of course, is exactly as in the RS case. The dependence of $V_{1,2}$ on x_5 is easy to understand. Each slice of x_5 could, potentially, host a 4-brane, with distinct $(3 + 1)$ -dimensional worlds at the ends. Only if the potentials localized at the end of the branes are equal and opposite and related to the “overall size” of the 5-dimensional metric

⁴ While a slightly larger ϵ can be admitted, say by arguments relating the brane tension to the scale of some underlying string theory (or even to M_6) [10], the applicability of the semiclassical approximation grows progressively worse. On the other hand, $\epsilon \lesssim 0.15$ automatically ensures that the curvature in the x_4 -direction is sufficiently small.

⁵ While this may be perceived as a fine-tuning, it is, at worst, exactly the same as that in the RS model. Indeed, $\Omega = 0$ is not a special solution, and the same argument could be made against any finite value for Ω . On the other hand, $\Omega = 0$ could, in principle, be the result of some as yet unspecified symmetry [31].

⁶ Once again, we omit the second solution, viz. e^{cx_4} for reasons analogous to those operative for $b(x_5)$.

in that slice (just as in the RS case), would these hypothetical $(3 + 1)$ -dimensional worlds be associated with a vanishing cosmological constant. As has been demonstrated in Refs. [17,21], such a x_5 -dependent potential could be occasioned by a brane-localized scalar field, such as a kink solution corresponding to a quartic potential, or in a theory with a non-trivial kinetic term.

It should be noted, though, that with these particular forms for $V_{1,2}$ are *not strict requirements* for the model. Such a choice only helps to reduce the algebra. Indeed, as long as eqn. (9) holds at $x_5 = 0$ (with no restrictions for $x_5 \neq 0$), the vanishing of the four-dimensional cosmological constant is guaranteed. However, the relaxation of eqn. (9) does not add anything qualitatively different to either the phenomenology (whether in the graviton sector [18,19] or in the SM sector [20,21]) or to the main thrust of this paper, namely the stability of the scenario.

We now turn to the consequences of choosing a particular value for b_2 (this choice, as we shall see later, also serves to determine c). Rather than discuss the generic case (which does not afford closed-form analytical solutions), we illustrate the situations for two extreme limits. Physically, one of the limits corresponds to a vanishing five-dimensional cosmological constant (equivalently, straight, or unbent, four-branes at the ends of the world). The opposite limit corresponds to the case wherein the four-branes suffer the maximum possible bending commensurate with a semiclassical analysis (or, in other words, a five-dimensional cosmological constant comparable to the fundamental scale). The low energy phenomenology, naturally, would turn out to be quite different in the two cases. Clearly, any intermediate value of b_2 would correspond to an intermediate value of the five-dimensional cosmological constant and, similarly, for the low-energy phenomenology.

Case 1: The situation of $b_2 = 0$ recovers the results of Ref. [17] and we have

$$\begin{aligned} c &= \frac{R_y k}{r_z} \operatorname{sech}(k\pi) \\ V_1(x_5) &= -V_2(x_5) = \frac{8M_6^4 k}{r_z} \operatorname{sech}(k|x_5|), \\ V_3 &= 0 \\ V_4 &= \frac{8M_6^4 k}{r_z} \tanh(k\pi). \end{aligned} \quad (10)$$

This, obviously, corresponds to a bent brane scenario with nonvanishing induced five-dimensional cosmological constants on the hypersurfaces at $x_5 = 0, \pi$. This could easily be seen by observing that the induced metric on the $x_5 = 0$ surface, apart from an overall $b(0)$ factor, is given by

$$ds_5^2 = e^{-2c|x_4|} \eta_{\mu\nu} dx^\mu dx^\nu + R_y^2 dx_4^2,$$

or, in other words, the induced geometry is AdS_5 -like.

Case 2: In the opposite limit, viz. $b_2 \rightarrow \infty$, we have $b(x_5) \approx (b_1/2) \exp(k|x_5| + b_2)$ and, hence, the normalization of the warp factor would imply $b_1 \approx 2 \exp(-k\pi - b_2) \rightarrow 0$. Consequently, one is forced to $c \ll k$, unless one were to admit a large, and unpleasant, hierarchy between R_y and r_z . This situation should be contrasted to the previous case, where the limit was realizable for both branches of the theory, viz. $c \ll k$ as well as a moderate $c > k$.

With $c \rightarrow 0$ the brane potentials now read

$$\begin{aligned}
 V_1 &= -V_2 \approx 0 \\
 V_3 &\approx \frac{-8M_6^4 k}{r_z} \approx -V_4
 \end{aligned}
 \tag{11}$$

The fact of $V_3 \approx -V_4$ reveals the near vanishing of brane-induced cosmological constant. As for the line element, in this limit,

$$\begin{aligned}
 ds^2 &= e^{2k(|x_5|-\pi)} \left(e^{-2c|x_4|} \eta_{\mu\nu} dx^\mu dx^\nu + R_y^2 dx_4^2 \right) + r_z^2 dx_5^2 \\
 &\approx e^{2k(|x_5|-\pi)} \left(\eta_{\mu\nu} dx^\mu dx^\nu + R_y^2 dx_4^2 \right) + r_z^2 dx_5^2
 \end{aligned}$$

or that the metric is nearly conformally flat. It should be realized, though, that the approximate conformal flatness would have followed as long as $c \ll k$ (i.e., for $k \gtrsim 10$) and did not need $b_2 \rightarrow \infty$. However, a finite value of b_2 would have translated to unequal brane tensions and, consequently, nonvanishing induced cosmological constants.

The two opposing limits of b_2 are not special, but only serve to simplify the algebra. Any intermediate value of b_2 would only lead to phenomenological situations that interpolate between those listed above. In the following, we shall detail not only the stabilization of the radii, but also that of b_2 .

3. Radii stabilization

While it may seem, at first sight, that moduli stabilization in this (6D) framework can proceed in a fashion identical to that in the RS paradigm, there are certain crucial differences. In particular (and, as we shall see below), if we attempt a naive GW [9]-like mechanism, only one combination of the two moduli can be stabilized. This is but a reflection of the well-known fact that, for a multidimensional hidden compact space, it is easier to stabilize the shape rather than the volume. It should be realized, though, that had we been interested in a different compactification (such as, for example, $M^{(1,3)} \times S^2$ with an appropriate orbifolding), a single-field GW-like mechanism would indeed be enough. This is as expected, for in such a case there would, but, be only one modulus to stabilize. However, such a compactification is not favoured phenomenologically as, on the one hand, it requires extra fields to counterbalance the curvature of S^2 , while, on the other, if the SM fields are extended in to the bulk (so as to fully exploit the advantages of the 6D construction), the resultant spectrum cannot, easily, be made consonant with low energy observations.

While the same mechanism would work irrespective of the choice for induced cosmological constant, the algebraic simplification is significant in the two limits discussed in the preceding section. Similarly, treating the two distinct regimes (viz small k and large k) separately brings forth an appreciation of both the overall mechanism, as well as the subtle differences in the implementation thereof.

Before we do this, though, let us reexamine some potentially confusing features of this scenario, in particular the roles of the brane localized potentials V_i , the separation constant $\tilde{\Omega}$ and the constant b_2 . At first glance, the “choices” might seem to associated with fine-tunings. We begin by showing that not all of them are independent and, then, explore the stabilization of the truly independent.

To begin with, it should be realized that the special case of $\tilde{\Omega} = 0$ would have led to a generic solution of the form

$$b(x_5) = \beta_1 \cosh^{2/5} \left[\frac{5kx_5}{2} + \beta_2 \right]$$

where $\beta_{1,2}$ are the constants of integration, with β_1 to be fixed by our normalization of $b(x_5 = \pi) = 1$. This special solution is unique to $\tilde{\Omega} = 0$ and untenable for $\tilde{\Omega} \neq 0$, when only the solution of eqn. (5) applies. More importantly, the two solutions differ by at most 50% (almost independent of the value of $\tilde{\Omega}$). For large k (~ 8 , as would be the case for preferred solution for the hierarchy problem), the warp factors are very nearly indistinguishable, throughout the bulk, with the difference being noticeable only very close to the IR brane. In other words, the conclusions that we would draw are not very sensitive to the exact value of $\tilde{\Omega}$. Put differently, there is no severe fine-tuning associated with $\tilde{\Omega}$.

Note further that eqn. (5) also implies

$$\tilde{\Omega} = -3b_1^2 k^2 = -3k^2 \operatorname{sech}^2(k\pi + b_2)$$

whereas eqn. (8)

$$V_3 = -8 \sqrt{\frac{-\Lambda_6 M_6^4}{10}} \tanh b_2, \quad \frac{-V_4}{V_3} = \frac{\tanh(k\pi + b_2)}{\tanh b_2}.$$

In other words, there is a one-to-one relation between (V_3, V_4) and (k, b_2) or, equivalently, $(k, \tilde{\Omega})$. Stabilizing one set automatically stabilizes the others. While we propose below a mechanism to stabilize the last (or, equivalently, the first) set, note that we have already seen that the dependence of physical observables on $\tilde{\Omega}$ is a suppressed one. Thus, stabilizing k would be enough.

3.1. Small k and large c

As we have discussed in Sec. 2, in this regime, the metric cannot be approximated by a conformally flat one, and, of the two limits discussed therein, only Case I can be applicable. Rather than work with the general solution, we shall work in this limit, for it simplifies the algebra considerably without altering the physical essence.

As we have also explained earlier, starting with a single canonically quantized scalar field, it is not possible to stabilize both the moduli. Consequently, we postulate two such scalar fields. In order to minimize the number of effective four-dimensional fields (on KK reduction), we incorporate one scalar field $\phi_1(x_\mu, x_4, x_5)$, permeating the entire bulk, that would serve to stabilize r_z (or, equivalently, the dimensionless quantity k). A second field $\phi_2(x_\mu, x_4)$, introduced (localized) only on the $x_5 = 0$ brane, would, similarly, stabilize the length (R_y) of the brane. Given the box structure and the orbifolding, together, they stabilize both the moduli.

The Lagrangians for these scalars are given by

$$\mathcal{L}_6 = \sqrt{-g} \left(-\frac{1}{2} g^{MN} \partial_M \phi_1 \partial_N \phi_1 - \frac{1}{2} m^2 \phi_1^2 \right) + \sqrt{-g_5} \left(U_1(\phi_1) \delta(x_5) + U_2(\phi_1) \delta(x_5 - \pi) \right) \tag{12}$$

and

$$\mathcal{L}_5 = \sqrt{-g_5} \left(-\frac{1}{2} g^{\bar{M}\bar{N}} \partial_{\bar{M}} \phi_2 \partial_{\bar{N}} \phi_2 - \frac{1}{2} m^2 \phi_2^2 \right) \delta(x_5) + \sqrt{-g_4} \left(U_3(\phi_2) \delta(x_4) + U_4(\phi_2) \delta(x_4 - \pi) \right) \delta(x_5) \tag{13}$$

respectively. Here, $g = \det(g_{MN}) = -a^4 b^5 R_y r_z$, whereas, for the induced metrics, we have $g_5 = \det(g_{\bar{M}\bar{N}}) = -R_y a^4 b^5$ and $g_4 = \det(g_{\mu\nu}) = -a^4 b^4$.

In particular, the 5d metric, apart from the constant $b^2(0)$, induced on the $x_5 = 0$ brane is

$$ds_5^2 = e^{-2c|x_4|} \eta_{\mu\nu} dx^\mu dx^\nu + R_y^2 dx_4^2.$$

Given this AdS_5 geometry and the form of \mathcal{L}_5 , it is clear that the stabilization of R_y can proceed exactly as in the GW mechanism [9] or its variants [22] using the classical configuration of ϕ_2 . Since this technique is well-known, we, for the sake of brevity, will eschew any details here, assuming that R_y can be stabilized. Indeed, with an appropriately modified five-dimensional potential for ϕ_2 , it is also possible to take into account the back reaction and achieve an exact solution [23]. We will come back to a generalized version of this.

Unlike the 4-brane at $x_5 = 0$ (hereafter called the 4_0 brane) itself, the x_5 -direction possesses a non-zero induced cosmological constant, as shown in section 2. Stabilization of this direction, thus, requires a more careful analysis which we proceed to now.

The effective potential for r_z (equivalently, k) can be obtained, starting from the Lagrangian of eq. (12). While the classical configuration of ϕ could, in principle, have nontrivial dependences on both x_4 and x_5 (and, yet, maintain the requisite Lorentz symmetry), such a general consequence would have required complicating the boundary-localized terms and does not add anything qualitatively different to the system. Since we are primarily interested in the effective potential in the x_5 -direction, for brevity's sake, we restrict our discussion to the case where ϕ has a nontrivial dependence only along x_5 and denoting it

$$\langle \phi_1(x_\mu, x_4, x_5) \rangle = \frac{\phi(x_5)}{\sqrt{R_y r_z}},$$

the effective one-dimensional Lagrangian for $\phi(x_5)$ is given by

$$\widehat{\mathcal{L}}_6 = \frac{a^4 b^5}{2} \left[-r_z^{-2} (\partial_5 \phi)^2 - m^2 \phi^2 \right] + a^4 b^5 R_y [U_1(\phi_1) \delta(x_5) + U_2(\phi_1) \delta(x_5 - \pi)]. \quad (14)$$

Understandably, $a(x_4)$ appears only as an overall multiplicative factor and plays no dynamical role. The corresponding equation of motion is

$$\partial_5 (b^5 \partial_5 \phi) - b^5 m^2 r_z^2 \phi + R_y r_z^2 b^5 \left(\frac{\partial U_1(\phi_1)}{\partial \phi} \delta(x_5) + \frac{\partial U_2(\phi_1)}{\partial \phi} \delta(x_5 - \pi) \right) = 0. \quad (15)$$

The solution, in the bulk, is given in terms of associated Legendre functions, viz.

$$\phi = \text{sech}^{5/2}(kx_5) \left[c_1 P_{3/2}^v(\tanh(kx_5)) + c_2 Q_{3/2}^v(\tanh(kx_5)) \right], \quad (16)$$

where $c_{1,2}$ are the constants of integration and

$$v = \frac{5}{2} \sqrt{1 + \frac{4\mu^2}{25}}, \quad \mu \equiv \frac{mr_z}{k} = \frac{m}{M_6 \epsilon}. \quad (17)$$

The constants $c_{1,2}$ can be determined once boundary conditions are imposed. To do this, we turn to the brane-localized potentials $U_{1,2}(\phi_1)$ which, until now, were unspecified. We are not sensitive to the exact form of $U_{1,2}(\phi_1)$ as long as they admit nonzero minima at $\phi = v_{1,2}$ respectively. Noting that the cutoff scale on this brane is given by R_y^{-1} , such minima, for example, can be

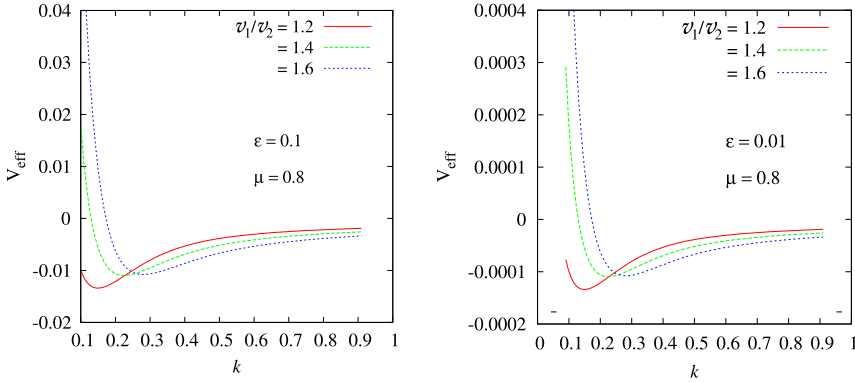


Fig. 1. The effective potential $V_{eff}(k)$ for different values of the ratio v_2/v_1 of the classical values of the field ϕ on the two constant- x_5 branes. The left (right) panels correspond to $\epsilon = 0.1$ (0.01).

easily achieved if one were to consider $U_{1,2}(\phi_1) = V_{3,4} + R_y^{-1} \lambda_{1,2} (\phi^2 - v_{1,2}^2)^2$ with $\lambda_{1,2}$ being dimensionless constants and $V_{3,4}$ being defined as in eq. (2).⁷ This immediately leads to

$$c_2 = \frac{v_1 P_{3/2}^v(\tau_\pi) - v_2 \cosh^{5/2}(k\pi) P_{3/2}^v(0)}{Q_{3/2}^v(0) P_{3/2}^v(\tau_\pi) - Q_{3/2}^v(\tau_\pi) P_{3/2}^v(0)}$$

$$c_1 = \frac{1}{P_{3/2}^v(0)} (v_1 - c_2 Q_{3/2}^v(0))$$

$$\tau_\pi \equiv \tanh(k\pi) .$$

Putting the solution back in the effective Lagrangian 14, we have

$$\widehat{\mathcal{L}}_6 = \frac{k^2 a^4}{2r_z^2} \operatorname{sech}^5(k\pi) \left[\frac{(5 - 2v)^2}{4} (c_1 P_{5/2}^v(\tanh(kx_5)) + c_2 Q_{5/2}^v(\tanh(kx_5)))^2 - \mu^2 (c_1 P_{3/2}^v(\tanh(kx_5)) + c_2 Q_{3/2}^v(\tanh(kx_5)))^2 \right].$$

Eliminating the irrelevant factor $a^4(x_4)$ and integrating $\widehat{\mathcal{L}}_6$ over x_5 , we would obtain an effective potential for k , defined, in dimensionless form, as

$$V_{eff}(k) \equiv \frac{1}{M_6^2 v_1^2} \int dx_5 \frac{\widehat{\mathcal{L}}_6}{a^4(x_4)} . \tag{18}$$

Since $k = r_z M_6 \epsilon$, with the last two quantities being fixed parameters of the theory, V_{eff} is, thus, equivalently, a potential for r_z . As a closed form expression for V_{eff} is not possible, and even a good approximate form complicated enough, we present it, instead, only in a graphical form.

⁷ On the boundary, once the scalar field ϕ settles down to the vacuum $v_{1,2}$, the brane localized potential becomes $U_{1,2} = V_{3,4}$ and one recovers the action given in eq. (2). While this unifies the explanation of the brane-tensions $V_{3,4}$ with the stabilization mechanism, truthfully, it, of course, does not yet explain their values. On the other hand, as we have explained earlier, with Einstein’s equations and matter equation of motion being coupled, only certain values of $V_{3,4}$ can be consistent with the metric and the orbifolding. The stabilization of k , though, would imply the stabilization of $V_{3,4}$ too.

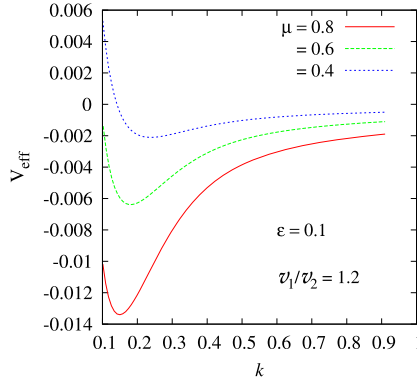


Fig. 2. The effective potential $V_{eff}(k)$ for different values of the mass of the bulk scalar ϕ_1 .

In Fig. 1, we display $V_{eff}(k)$ for a fixed value of the mass parameter μ (equivalently, m). As is obvious, depending on the ratio v_2/v_1 , minima exist for $0.1 \lesssim k \lesssim 0.6$, the range that is of particular interest not only to explain the non-observation (so far) of the KK-graviton at the LHC [18,19], but also for scenarios wherein the SM fields are extended in to the bulk [20, 21]. What is particularly encouraging is that *such minima arise for very natural values of the parameters* and are not overly sensitive to their precise values. Indeed, the strongest dependence, of the stabilized value of the modulus r_z , is on the ratio v_1/v_2 of the classical values.

It is also instructive to examine the dependence of $V_{eff}(k)$ on ϵ (as depicted in the two panels of Fig. 1) and μ (as shown in Fig. 2). As can be readily ascertained, while the size of the potential has a strong dependence on μ (understandable, since it is μ that allows for a nontrivial V_{eff}), the position of the minimum has only a muted dependence.

Until now, we have neglected the back reaction on the metric due to the scalar field. While we could, in principle, attempt this, as we shall indeed do for the other regime (namely, large k) in the next section. However, in the current context, the presence of a non-negligible induced cosmological constant Λ_{ind} queers the pitch. In the absence of Λ_{ind} , the change in the warp factor due to back-reaction can be computed exactly by integrating three first order equations (namely, the one for the scalar field, the warp factor and the bulk potential). For a non-zero Λ_{ind} , additional nonlinearities, that couple these three equations in a non-trivial manner, emerge [23], and a closed-form analytic solution is not possible. Of course, the equations can still be solved numerically. However, in view of the fact that such solutions can be easily obtained by deforming the solutions presented above, we eschew a detailed discussion for the sake of brevity. The neglect of the back-reaction is eminently justified, for the largest value of the scalar field mass that we have used is sufficiently smaller than $\Lambda_6^{1/6}$ (and, certainly M_6). Consequently, the energy content in the ϕ -field is rather subdominant to that due to the bulk cosmological constant and the back-reaction in the bulk is not much of a worry. Furthermore, with the hierarchy, for the most part, being dictated by the warp factor in the x_4 -direction, even moderate changes in $b(x_5)$, as would be introduced by the back-reaction, have relatively little bearing on the phenomenology.

3.2. Large k and small c

In this regime, the metric is nearly conformally flat. With c being infinitesimally small, neglecting the c -dependence of the metric would not introduce a decipherable difference in the

analysis. To simplify the algebra, we will take recourse⁸ to **Case 2** of section 2, whence the metric reduces to

$$ds^2 = e^{2A(x_5)} \left(\eta_{\mu\nu} dx^\mu dx^\nu + R_y^2 dx_4^2 \right) + r_z^2 dx_5^2, \tag{19}$$

where in the absence of backreaction $A(x_5) = k|x_5|$. With the 4₀-brane-localized induced cosmological constant being infinitesimally small, it is possible to obtain an almost exact solution incorporating the back reaction as well and we now attempt this. Introducing a scalar field ϕ in the bulk, the entire action is given by⁹

$$S = \int d^6x \sqrt{-g} \left[M_6^4 R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right]. \tag{20}$$

The corresponding equations of motion are

$$\begin{aligned} \ddot{\phi} + 5\dot{\phi}\dot{A} &= r_z^2 \frac{\partial V}{\partial \phi} \\ 5\dot{A} + 2\ddot{A} &= \frac{-r_z^2}{2M_6^4} \left[\frac{\dot{\phi}^2}{2r_z^2} + V(\phi) \right] \\ \dot{A}^2 &= \frac{r_z^2}{10M_6^4} \left[\frac{\dot{\phi}^2}{2r_z^2} - V(\phi) \right]. \end{aligned} \tag{21}$$

For a scalar with a localized potential on the x_5 -constant 4-branes $V(\phi)$ could be written as

$$V(\phi) = V_{\text{bulk}}(\phi) + r_z^{-1} \left[f_0(\phi(0)) \delta(x_5) + f_\pi(\phi(\pi)) \delta(x_5 - \pi) \right]$$

where $V_{\text{bulk}}(\phi)$ is the bulk potential and $f_{0,\pi}(\phi(x_5))$ are some as-yet undetermined functions of the scalar field. Integrating eqn. (21) across the 4-brane locations ($\alpha \equiv x_5 = 0, \pi$), we have

$$\begin{aligned} \dot{A} \Big|_{\alpha-\epsilon}^{\alpha+\epsilon} &= \frac{-1}{4M_6^4} f_\alpha(\phi(\alpha)) \\ \dot{\phi} \Big|_{\alpha-\epsilon}^{\alpha+\epsilon} &= r_z \frac{\partial f_\alpha}{\partial \phi}(\phi(\alpha)), \end{aligned}$$

which provide the junction conditions. An exact closed-form solution to eqns. (21) can be obtained only for particular bulk potentials. Borrowing from techniques of supersymmetric quantum mechanics, we assume the bulk potential can be expressed as

$$V_{\text{bulk}} = \frac{1}{2} \left(\frac{\partial W}{\partial \phi} \right)^2 - \frac{5}{2M_6^4} W^2, \tag{22}$$

where $W(\phi)$ can be thought of as a superpotential. This, immediately leads to

$$\begin{aligned} \dot{A} &= \frac{r_z}{2M_6^4} W \\ \dot{\phi} &= -2r_z \frac{\partial W}{\partial \phi} \end{aligned} \tag{23}$$

as long as $W(\phi)$ satisfies the junction conditions

⁸ Once again, the choice of Case 2 does not represent a fine-tuning. Rather, it only serves to simplify the algebra permitting an analytics closed-form solution.

⁹ We do not write Λ_6 explicitly, preferring to include it in $V(\phi)$.

$$W \Big|_{\alpha-\epsilon}^{\alpha+\epsilon} = \frac{1}{2} \frac{1}{r_z} f_\alpha(\phi(\alpha))$$

$$\frac{\partial W}{\partial \phi} \Big|_{\alpha-\epsilon}^{\alpha+\epsilon} = -\frac{1}{2} \frac{\partial f_\alpha(\phi(\alpha))}{\partial \phi} .$$

Each choice for $W(\phi)$ gives a different V_{bulk} , but an analytic closed-form solution can be found for only some. An explicit example is afforded by a quadratic superpotential [23,24], namely

$$W(\phi) = 2 M_6^5 \epsilon - \frac{1}{4} u M_6 \phi^2 ,$$

where $u \lesssim 0.1$ is a constant, parameterizing not only the mass of ϕ , but its (quartic) self-interaction as well. The corresponding brane localized potentials read

$$f_0(\phi) = \frac{1}{2 r_z} W(\phi) - \frac{1}{2} \frac{\partial W}{\partial \phi} (\phi - v_0) + \gamma_0^2 (\phi - v_0)^2$$

$$f_\pi(\phi) = \frac{1}{2 r_z} W(\phi) - \frac{1}{2} \frac{\partial W}{\partial \phi} (\phi - v_\pi) + \gamma_\pi^2 (\phi - v_\pi)^2 ,$$

where $\gamma_{\pi,0}$ are arbitrary positive constants that ensure that $\phi(x_5)$ assumes values $v_{\pi,0}$ on the Planck (TeV) branes. The solutions to eqns. (23) are given by

$$\phi(x_5) = \phi_0 \exp(u M_6 r_z |x_5|)$$

$$A(x_5) = k |x_5| - \frac{v_0^4}{8 M_6^4} \exp(2 u M_6 r_z |x_5|) . \tag{24}$$

Note that the warp factor has changed from the simple exponential form that it had in the absence of ϕ .

It is worthwhile to reflect on the difference between this analysis and that presented in the preceding subsection. While we could have adopted the same procedure, namely substitute eqn. (24) in eqn. (20) and integrate over x_5 to yield an effective potential $V_{\text{eff}}(r_z)$, it is not necessary to do so. Rather, note that the very structure of the solution (eqn. 24), along with the boundary-localized potential ensures that

$$r_z = \frac{1}{u \pi M_6} \ln \frac{v_\pi^2}{v_0^2} . \tag{25}$$

No other value for r_z would admit a solution, consistent with the boundary conditions, to the system of coupled nonlinear differential equations that we are endowed with. It is also worthwhile to note that a natural set of values for v_π/v_0 and u can reproduce the required r_z (and, hence, the correct warping) without any fine-tuning being needed.

We now turn to the stabilization of R_y . If the approximation of eqn. (19) were truly exact, R_y cannot be stabilized. On the other hand, it need not be, at least in the context of hierarchy stabilization, for it really does not play a role in defining the overall warp-factor. Apparently, thus, the primary constraints would be those on the ADD scenario [4], such as deviation from Newton’s law or the fast cooling of a supernova. And while it might be argued that such an extremely large value for R_y reintroduces a hierarchy, it is not obvious that this is a problem (far less a serious one), given that R_y plays only a subservient role in defining the gap between M_6 and the electroweak scale. Indeed, well before R_y becomes so large (the sub-millimeter range), c becomes quite non-negligible. This not only invalidates the approximation of eqn. (19), but also carries the seed for the stabilization of R_y .

The latter can proceed, for example, in a fashion exactly analogous to the GW mechanism as defined for the original RS scenario. Consider, for example, a second scalar ϕ_2 (of mass $m_2 \lesssim M_6$) confined to the 4-brane at $x_5 = \pi$. Assume that the only self-interactions are localized at the boundaries, viz. at $(x_4, x_5) = (0, \pi)$ and (π, π) which, in turn, force $\phi_2(0, \pi) = v_3$ and $\phi_2(\pi, \pi) = v_4$. Clearly, this would lead to a stabilized $R_y^{-1} \sim \mathcal{O}(m_2 \ln(v_3/v_4))$, and, consequently, to a moderate R_y/r_z and a small c (as desired).

A more interesting option would be to locate ϕ_2 on the 4_0 brane instead, with the boundaries now corresponding to $(x_4, x_5) = (0, 0)$ and $(\pi, 0)$ respectively. With m_2 now suffering a large warping (due to $b(x_5)$), the stabilized value for R_y^{-1} would, naturally, be in the TeV range. This, immediately, raises the intriguing possibility that new physics at a few-TeV scale could indeed be stabilized by the SM Higgs itself (or a cousin of its). Even more intriguingly, if one allows the SM fields to percolate into the x_4 direction, the setup under discussion would provide a dynamical justification for the scale in a Universal Extra Dimension-like scenario [5].

4. Conclusion

The six dimensional warped scenario provides a cure for various ailments of Randall–Sundrum model. Nevertheless, the problem of modulus stabilization, which was quite simple for Randall–Sundrum scenario, had, until now, not been executed for either of the two moduli in the nested warped model, largely on account of the fact that the model’s space-time is neither conformally flat, nor are the end-of-the world branes flat. Consequently, the stabilization mechanism presents a technical challenge, and this is the issue that we have addressed in this paper. To this end, we begin by exploring the metric for nested warping, showing that the solutions for each of the two regimes allowed to the theory can be generalized beyond what was considered earlier.

In the small k (equivalently, large c) regime of the theory, the induced geometries on the x_5 -constant 4-branes are AdS₅-like, and hence a Goldberger–Wise mechanism (or even one incorporating back-reaction) involving a brane-localized scalar field trivially stabilizes the corresponding modulus R_y . The second modulus r_z cannot be stabilized by the same scalar field. It is intriguing to consider leaving it unstabilized, especially since the corresponding warping is minor, and a slow temporal variation would have very interesting cosmological ramifications. However, r_z can also be stabilized by a six-dimensional analogue of the GW mechanism, as we have demonstrated here. Although the form of the effective potential for r_z is much more complicated than that in the minimal RS scenario, a numerical analysis shows that a minimum does exist and reproduces the desired hierarchy without the need for any fine tuning. Indeed, the phenomenologically acceptable domain in the parameter space of the theory is more extensive than that in the RS model. As for the back-reaction, while it can be incorporated, closed-form analytic solutions are not possible owing to the non-zero induced cosmological constants on the constant- x_5 hypersurfaces. However, numerical solutions are indeed possible.

In the other regime of the theory, characterized by a vanishingly small induced cosmological constant, the scenario changes dramatically, with the bulk tending to become conformally flat (and the warp factor nearly exponential). With the induced cosmological constant on the branes being infinitesimally small,¹⁰ a closed form solution can be found even on the inclusion of the backreaction. This allows us to stabilize r_z without taking recourse to any unnatural values of the

¹⁰ Note that a non-zero value for the five-dimensional cosmological constant does not preclude a vanishing four-dimensional cosmological constant (witness the original RS model), and, indeed, we do obtain the latter even in the

parameters. And while the aforementioned exact solution is achievable for only certain specific potentials, deviations thereof still lead to stabilization (with backreaction taken into account) with the only difference being that the solutions can be expressed only in terms of complicated integrals.

The situation with the corresponding R_y is more intriguing. With c now being infinitesimally small, it is tempting to consider the possibility of a rolling R_y , especially since a slowly varying R_y would have very interesting and attractive cosmological consequences. On the other hand, R_y can indeed be stabilized, by introducing a second scalar on one of the two 4-branes, viz. at $x_5 = \pi$ or at $x_5 = 0$. The first alternative naturally leads to R_y being stabilized to a value of the order of r_z . The second alternative, on the other hand, leads to a situation whereby a scalar field of apparently TeV-range mass (on account of the warping) leads to R_y being stabilized at a scale somewhat higher than the electroweak one. If the SM fields were considered to be five-dimensional ones, defined on the entire brane at $x_5 = 0$, this immediately leads to a UED-like scenario with the TeV-scale protected naturally. The orbifolding inherent to the system would not only eliminate unwanted modes, but also introduce for a KK-parity that, in turn, provides for a Dark Matter candidate on the one hand and eliminates many contributions to rare decays and precision variables on the other, thereby improving agreement with observed phenomenology. It might be argued, though, that with c being different from zero, the KK-parity is not exact. This is indeed so, but with the extent of Z_2 -breaking being determined by the (vanishingly small) induced cosmological constant $\tilde{\Omega}$, the lifetime of such DM-candidates would be exceedingly long.

Before ending, we revisit the question of fine tuning in such models. Both the original formulation [17] as well as the extended version (Sec. 2) seemed to be dependent on the presence of particular values of brane tensions. Exactly analogous to the original RS model, this could be interpreted as a fine-tuning endemic to this class of models. Naively, the choice of values for b_2 and the constant of separation $\tilde{\Omega}$ represent additional fine-tunings. We have shown here, though, this is not so. The exact value of $\tilde{\Omega}$ (equivalently, b_2) has relatively little bearing on the phenomenology. Indeed, for any two values of b_2 , in the range $0 \leq b_2 < \infty$, the difference between the resultant warp factors differs by at most 50%, and that too, only for small k . For large k , on the other hand, the warp factors are virtually indistinguishable except for very close to the IR brane. With the physical observables being differentiable functions of b_2 (and, hence, $\tilde{\Omega}$), the (small) differences due to finite values of b_2 can be easily worked out by interpolating between the results for $b_2 = 0$ and $b_2 \rightarrow \infty$ respectively. We have, consequently, chosen to demonstrate the results in these two limits as they admit simple analytical solutions whereas the general b_2 would need numerical methods to be employed.

Having argued that a specific value of $\tilde{\Omega}$ (or, equivalently, b_2) does not imply any fine-tuning over and above that endemic to RS models, we now turn to the latter, or more specifically, to the analogue thereof. As we have argued earlier, within the original RS model (sans modulus stabilization), the brane tension had to be just so, for the bulk solution and the orbifolding to be valid simultaneously. Furthermore, these were not related to the modulus. Here too, a similar situation seems to hold (see eq. (8)), with the recognition that the ratio k/r_z is determined entirely by fundamental scale M_6 and the bulk cosmological constant Λ_6 . Indeed, it has parallels with the RS model wherein the branes were allowed to have a nonvanishing cosmological constant.

general case. Furthermore, a vanishing value of the former is not a requirement for our analysis, and serves only to simplify the algebra.

On introducing the stabilization mechanism the brane tensions $V_{3,4}$ were identified with the stabilized values of the brane-localized potentials $U_{1,2}(\phi)$ of the bulk scalar ϕ .

A deeper understanding is afforded if one considers possible quantum corrections to the bulk Einstein–Hilbert action. While no such actual calculation is available, these, presumably, would appear as diffeomorphism-invariant higher derivative terms. Assuming that these could be parametrized as a polynomial in R , Ref. [32] considers, for example, a 5-dimensional bulk theory defined in the Jordan frame by

$$f(R) = R + a_1 \frac{R^2}{M^2} + a_2 \frac{R^3}{M^4} ,$$

with the constants $a_i \sim \mathcal{O}(10^{-1})$ and M the cut-off scale. In the Einstein frame (obtained from the Jordan frame through a conformal transformation), the extra degree of freedom associated with the higher-derivatives can be recast in terms of a scalar field with a very nontrivial potential. Most interestingly, this degree of freedom can play the role of the Goldberger–Wise scalar, thereby allowing for a “geometric stabilization” of the modulus. A similar stabilization can occur in six-dimensions too with the field ϕ_1 parametrizing such higher derivative terms appearing in the bulk action. Additional possibilities arise in the shape of brane-localized $f(R)$ -terms (since the branes are characterized by matter fields, the quantum corrections to the Einstein–Hilbert action would, in general, be different on them). These extra terms would play the role of the brane-localized fields (masquerading as our ϕ_2) with their own potentials. Being very steep [32], these would enforce the system being in the vacuum state, thereby according a quantum origin to $V_{3,4}$. Of course, once again, this entire paradigm depends upon the exact form of $f(R)$, including the coefficients, for both the bulk and the branes. However, the conjecture that the entire stabilization process is but a consequence of an effective geometric action born of quantum corrections, is, undoubtedly, a very interesting one, especially in the quest to understand the fine-tuning problem (such as that associated with choosing $\Omega = 0$).

It is also worthwhile to consider extending the formalism developed herein to still higher dimensions. For example, it has been shown [33–35] that a six-dimensional UED model not only suppresses proton decay through a higher dimensional operator, but also gives a topological origin for the number of chiral fermion generations. The extension of the formalism presented here to seven dimensional nested warping [17] would accord a dynamical origin to the scale of the model. These and other issues are currently under investigation.

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References

- [1] G. Aad, et al., ATLAS Collaboration, Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC, *Phys. Lett. B* 716 (2012) 1, arXiv:1207.7214 [hep-ex];
G. Aad, et al., ATLAS Collaboration, Combined search for the Standard Model Higgs boson in pp collisions at $\sqrt{s} = 7$ TeV with the ATLAS detector, *Phys. Rev. D* 86 (2012) 032003, arXiv:1207.0319 [hep-ex].

- [2] S. Chatrchyan, et al., CMS Collaboration, Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC, *Phys. Lett. B* 716 (2012) 30, arXiv:1207.7235 [hep-ex].
- [3] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, New dimensions at a millimeter to a Fermi and superstrings at a TeV, *Phys. Lett. B* 436 (1998) 257, [http://dx.doi.org/10.1016/S0370-2693\(98\)00860-0](http://dx.doi.org/10.1016/S0370-2693(98)00860-0), arXiv:hep-ph/9804398.
- [4] N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali, The hierarchy problem and new dimensions at a millimeter, *Phys. Lett. B* 429 (1998) 263, [http://dx.doi.org/10.1016/S0370-2693\(98\)00466-3](http://dx.doi.org/10.1016/S0370-2693(98)00466-3), arXiv:hep-ph/9803315.
- [5] T. Appelquist, H.C. Cheng, B.A. Dobrescu, Bounds on universal extra dimensions, *Phys. Rev. D* 64 (2001) 035002, <http://dx.doi.org/10.1103/PhysRevD.64.035002>, arXiv:hep-ph/0012100.
- [6] M. Gogberashvili, Hierarchy problem in the shell universe model, *Int. J. Mod. Phys. D* 11 (2002) 1635, arXiv:hep-ph/9812296.
- [7] M. Gogberashvili, Our world as an expanding shell, *Europhys. Lett.* 49 (2000) 396, arXiv:hep-ph/9812365.
- [8] L. Randall, R. Sundrum, A large mass hierarchy from a small extra dimension, *Phys. Rev. Lett.* 83 (1999) 3370, arXiv:hep-ph/9905221.
- [9] W.D. Goldberger, M.B. Wise, Modulus stabilization with bulk fields, *Phys. Rev. Lett.* 83 (1999) 4922, <http://dx.doi.org/10.1103/PhysRevLett.83.4922>, arXiv:hep-ph/9907447.
- [10] H. Davoudiasl, J.L. Hewett, T.G. Rizzo, Phenomenology of the Randall–Sundrum Gauge Hierarchy Model, *Phys. Rev. Lett.* 84 (2000) 2080, arXiv:hep-ph/9909255.
- [11] G. Aad, et al., ATLAS Collaboration, Search for Extra Dimensions using diphoton events in 7 TeV proton–proton collisions with the ATLAS detector, *Phys. Lett. B* 710 (2012) 538, arXiv:1112.2194 [hep-ex].
- [12] G. Aad, et al., ATLAS Collaboration, Search for Extra Dimensions in diphoton events using proton-proton collisions recorded at $\sqrt{s} = 7$ TeV with the ATLAS detector at the LHC, *New J. Phys.* 15 (2013) 043007, arXiv:1210.8389 [hep-ex].
- [13] ATLAS Collaboration, Search for high-mass dilepton resonances in 20 fb⁻¹ of *pp* collisions at $\sqrt{s} = 8$ TeV with the ATLAS experiment, ATLAS-CONF-2013-017, ATLAS-COM-CONF-2013-010.
- [14] V. Khachatryan, et al., CMS Collaboration, Search for massive resonances decaying into pairs of boosted bosons in semi-leptonic final states at $\sqrt{s} = 8$ TeV, *J. High Energy Phys.* 1408 (2014) 174, arXiv:1405.3447 [hep-ex].
- [15] G. Aad, et al., ATLAS Collaboration, Search for high-mass diphoton resonances in *pp* collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector, *Phys. Rev. D* 92 (2015) 032004, arXiv:1504.05511 [hep-ex].
- [16] CMS Collaboration, Search for high-mass diphoton resonances in *pp* collisions at $\sqrt{s} = 8$ TeV with the CMS detector, CMS-PAS-EXO-12-045.
- [17] D. Choudhury, S. SenGupta, Living on the edge in a spacetime with multiple warping, *Phys. Rev. D* 76 (2007) 064030, arXiv:hep-th/0612246.
- [18] M.T. Arun, D. Choudhury, A. Das, S. SenGupta, Graviton modes in multiply warped geometry, *Phys. Lett. B* 746 (2015) 266, arXiv:1410.5591 [hep-ph].
- [19] M.T. Arun, P. Saha, Gravitons in multiply warped scenarios – at 750 GeV and beyond, *Pramana* 88 (6) (2017) 93, <http://dx.doi.org/10.1007/s12043-017-1387-y>, arXiv:1512.06335 [hep-ph].
- [20] M.T. Arun, D. Choudhury, Bulk gauge and matter fields in nested warping: I. The formalism, *J. High Energy Phys.* 1509 (2015) 202, [http://dx.doi.org/10.1007/JHEP09\(2015\)202](http://dx.doi.org/10.1007/JHEP09(2015)202), arXiv:1501.06118 [hep-th].
- [21] M.T. Arun, D. Choudhury, Bulk gauge and matter fields in nested warping: II. Symmetry breaking and phenomenological consequences, *J. High Energy Phys.* 1604 (2016) 133, [http://dx.doi.org/10.1007/JHEP04\(2016\)133](http://dx.doi.org/10.1007/JHEP04(2016)133), arXiv:1601.02321 [hep-ph].
- [22] J.M. Cline, H. Firouzjahi, Brane world cosmology of modulus stabilization with a bulk scalar field, *Phys. Rev. D* 64 (2001) 023505, arXiv:hep-ph/0005235;
D. Choudhury, D.P. Jatkar, U. Mahanta, S. Sur, On stability of the three 3-brane model, *J. High Energy Phys.* 0009 (2000) 021, arXiv:hep-ph/0004233.
- [23] O. DeWolfe, D.Z. Freedman, S.S. Gubser, A. Karch, Modeling the fifth-dimension with scalars and gravity, *Phys. Rev. D* 62 (2000) 046008, arXiv:hep-th/9909134.
- [24] C. Csaki, M.L. Graesser, G.D. Kribs, Radion dynamics and electroweak physics, *Phys. Rev. D* 63 (2001) 065002, arXiv:hep-th/0008151;
C. Csaki, M. Graesser, L. Randall, J. Terning, Cosmology of brane models with radion stabilization, *Phys. Rev. D* 62 (2000) 045015, arXiv:hep-ph/9911406;
D. Maity, S. SenGupta, S. Sur, Stability analysis of the Randall–Sundrum braneworld in presence of bulk scalar, *Phys. Lett. B* 643 (2006) 348, arXiv:hep-th/0604195.
- [25] N. Arkani-Hamed, M. Porrati, L. Randall, Holography and phenomenology, *J. High Energy Phys.* 0108 (2001) 017, <http://dx.doi.org/10.1088/1126-6708/2001/08/017>, arXiv:hep-th/0012148.

- [26] R. Rattazzi, A. Zaffaroni, Comments on the holographic picture of the Randall–Sundrum model, *J. High Energy Phys.* 0104 (2001) 021, <http://dx.doi.org/10.1088/1126-6708/2001/04/021>, arXiv:hep-th/0012248.
- [27] M. Perez-Victoria, Randall–Sundrum models and the regularized AdS / CFT correspondence, *J. High Energy Phys.* 0105 (2001) 064, <http://dx.doi.org/10.1088/1126-6708/2001/05/064>, arXiv:hep-th/0105048.
- [28] A. Pomarol, Gauge bosons in a five-dimensional theory with localized gravity, *Phys. Lett. B* 486 (2000) 153, [http://dx.doi.org/10.1016/S0370-2693\(00\)00737-1](http://dx.doi.org/10.1016/S0370-2693(00)00737-1), arXiv:hep-ph/9911294.
- [29] K. Agashe, A. Delgado, A note on CFT dual of RS model with gauge fields in bulk, *Phys. Rev. D* 67 (2003) 046003, <http://dx.doi.org/10.1103/PhysRevD.67.046003>, arXiv:hep-th/0209212.
- [30] R.K. Kaul, S. Sengupta, Degenerate spacetimes in first order gravity, arXiv:1602.04559 [gr-qc].
- [31] S. Alexander, Y. Ling, L. Smolin, A thermal instability for positive brane cosmological constant in the Randall–Sundrum cosmologies, *Phys. Rev. D* 65 (2002) 083503, <http://dx.doi.org/10.1103/PhysRevD.65.083503>, arXiv:hep-th/0106097.
- [32] S. Anand, D. Choudhury, A.A. Sen, S. SenGupta, A geometric approach to modulus stabilization, *Phys. Rev. D* 92 (2) (2015) 026008, <http://dx.doi.org/10.1103/PhysRevD.92.026008>, arXiv:1411.5120 [hep-th].
- [33] B.A. Dobrescu, E. Poppitz, Number of fermion generations derived from anomaly cancellation, *Phys. Rev. Lett.* 87 (2001) 031801, <http://dx.doi.org/10.1103/PhysRevLett.87.031801>, arXiv:hep-ph/0102010.
- [34] G. Burdman, B.A. Dobrescu, E. Ponton, Resonances from two universal extra dimensions, *Phys. Rev. D* 74 (2006) 075008, <http://dx.doi.org/10.1103/PhysRevD.74.075008>, arXiv:hep-ph/0601186.
- [35] T. Appelquist, B.A. Dobrescu, E. Ponton, H.U. Yee, Proton stability in six-dimensions, *Phys. Rev. Lett.* 87 (2001) 181802, <http://dx.doi.org/10.1103/PhysRevLett.87.181802>, arXiv:hep-ph/0107056.