

Story of the Developments in Statistical Physics of Fracture, Breakdown and Earthquake: A Personal Account

Bikas K. Chakrabarti

*Condensed Matter Physics Division
Saha Institute of Nuclear Physics
1/AF Bidhannagar Kolkata 700064, India
bikask.chakrabarti@saha.ac.in*

Received 25 October 2017

Accepted 29 November 2017

Published 4 January 2018

We review the developments of the statistical physics of fracture and earthquake over the last four decades. We argue that major progress has been made in this field and that the key concepts should now become integral part of the (under-) graduate level text books in condensed matter physics. For arguing in favor of this, we compare the development (citations) with the same for some other related topics in condensed matter, for which Nobel prizes have already been awarded.

Keywords: Fracture; earthquake; brittle crack nucleation; ductile failure avalanche statistics; self-organised criticality; fiber-bundle model; Burridge-Knopoff model; fractal-overlap model.

1. Introduction

When we decided in 1981, working from Kolkata, to investigate the statistical physics of fracture in disordered solids, our colleagues in statistical physics had not been very kind to us. Studies on renormalization group theory of critical phenomena were at their peak (Nobel prize to Keneth Wilson the following year), while the mechanical engineering friends took pity on us as, though not complete, most were assumed to be reasonably understood (from continuum mechanics discussed in standard engineering text books).

Our motivation had been somewhat ambitious. Skal and Shklovskii¹ in 1975 and de Gennes² the following year had already forwarded their node-link-blob model of percolation clusters in disordered solids and analyzed the scaling behavior of their (classical) linear responses like electrical conductivity or elasticity. We intended to extend these studies for nonlinear (and irreversible) responses, like fuse, (dielectric)

This is an Open Access article published by World Scientific Publishing Company. It is distributed under the terms of the Creative Commons Attribution 4.0 (CC-BY) License. Further distribution of this work is permitted, provided the original work is properly cited.

breakdown or fracture of disordered solids. We of course realized the major difficulty of the problem to come from the extreme nature of the breakdown statistics: Unlike the linear responses which get affected by all the defects (with self averaging statistics), the breakdown phenomena get nucleated around the “weakest” defect (inducing extreme statistics).

Extending the study of the statistical physics of fracture to earthquake statistics had been natural, though more involved and formidable. Detailed reviews of these developments have been published in review papers and books, some of which have been referred to in this account as well as in appropriate places. This account tries to capture those developments since early 1980s, in which my colleagues and I have been involved.

2. Short Story of the Past: From da Vinci to de Gennes and Mott

Fracture or breakdown studies might be the oldest physical science studies, which still remain intriguing and very much alive. Leonardo da Vinci, more than 500 years ago, observed that the tensile strengths of nominally identical specimens of iron wire decrease with increasing length of the wires (see e.g., Ref. 3 for a recent discussion). This is the manifestation of the extreme statistics of failure (bigger sample volume can have larger defects due to cumulative fluctuations where failures nucleate and induce lower strength of the sample). Similar observations were made by Galileo Galilei more than 400 years ago: “From what has already been demonstrated, you can plainly see the impossibility of increasing the size of structures to vast dimensions either in art or in nature; likewise, the impossibility of building ships, palaces, or temples of enormous size in such a way that their oars, yards, beams, iron-bolts, and, in short, all their other parts will hold together; nor can nature produce trees of extraordinary size because the branches would break down under their own weight; so also it would be impossible to build up the bony structures of men, horses, or other animals so as to hold together and perform their normal functions if these animals were to be increased enormously in height; for this increase in height can be accomplished only by employing a material which is harder and stronger than usual, or by enlarging the size of the bones, thus changing their shape until the form and appearance of the animals suggest a monstrosity”.⁴

For the next 300 years, we did not see major developments in the context of such problems. In 1921, Alan Arnold Griffith of the Royal Aircraft Establishment (UK) estimated how the crack nucleation stress for an otherwise pure material decreases with the dimension of the single defect in the brittle limit (when the stress-strain relationship remains linear until breaking; inducing the elastic energy density to grow with the square of the stress).⁵ This energy balance theory for brittle crack nucleation, obtained by equating the lost elastic energy (proportional to the crack volume) with the surface energy (proportional to the crack surface area) of the additional surface created by further opening up of the defect or micro-crack, led to a precise estimation of the breaking strength or stress of the brittle solid, decreasing

with inverse square root of the size or length of the crack or defect in the direction perpendicular to the stress. This led to a major development in the study of the mechanics of brittle fracture.

Subsequently, in 1926, Frederick Thomas Pierce⁶ from the British Cotton Industry Research Association in Manchester discovered what is known today as the Fiber Bundle model, a fantastically rich and elegant model to capture the fracture dynamics in composite materials. In this model, a large number of parallel Hooke-springs or fibers are clamped between two horizontal platforms; the upper one helps the hanging bundle while the load hangs from the lower one. The springs/fibers are assumed to have identical spring constants though their breaking strengths are assumed to be different. Once the load per fiber exceeds its own threshold, it fails and this extra load is shared by the surviving fibers. If the platforms are rigid, there is no local deformation around a failed fiber (and no stress concentration around the defect created) and load is shared equally by all the surviving fibers. Obviously, such a fluctuation-less model allows several features of its failure dynamics analytically. This was first indicated⁷ by Henry Daniels from the Wool Industries Research Association in Leeds, in 1945. Study of these models led to important developments, though they were practically confined to structural engineers for fitting the material failure data. Physicists did not notice, rather were unaware of, these models until late '80s or early '90s.

2.1. Fracture propagation

Nevill Mott⁸ of the Cambridge University, in 1948, extended the energy balance method of Griffith to include the crack propagation energy. This energy (kinetic energy of propagation), along with the energy of the newly opened up surfaces, should balance the elastic energy lost due to the crack propagation. The crack velocity, which had been zero in Griffith theory, starts growing with the length of the crack and approaches the sound velocity in the solid corresponding to elastic modulus of the released elastic energy. This led to an extensive literature on the growth of brittle cracks. Particularly, the morphology of the crack surfaces (out of plane) was claimed to be universal and the crack dynamics was characterized as a dynamical critical phenomena (see Ref. 9 for an early review). Much was studied later on the in-plane growth of the crack, starting with the nice experiment from the Oslo group¹⁰ (see also Ref. 11).

2.2. Extreme statistics & distributions

It is natural to expect that for randomly disordered solids, the linear response to stresses or fields, like those given by the elastic moduli, or the electrical conductivity (of random resistor networks), will have self-averaging property ensuring that the (configurationally) averaged elastic moduli and conductivity are defined in the thermodynamic limit (unlike in quantum cases; e.g., the nonself-averaging conductivity due to Anderson localization). It is obvious, however, that the same would not be true for (even classical) nonlinear and irreversible breakdown properties of

disordered solids. The stressed solid sample would survive (not break or fail) only if all the microscopic defects (due to disorder) survive under the stress, indicating that the fracture or breakdown strength of the solid would be determined by the weakest or extremely vulnerable defect in it.

As indicated already, the above-mentioned studies following Griffith-like energy balance concept, had limitations on several counts. The assumption of brittleness of the solid, or linearity in stress–strain relation up to the breaking point, had been one. More serious had been the assumption of a single or dominant defect in the entire solid volume. We discussed earlier (in the context of Griffith law), the strength of a solid with one isolated defect (or a dominant defect in an otherwise elastically homogeneous solid, having nonoverlapping stress released regions of the other microscopic defects) decreases with the defect size (inversely with the square-root of the defect length in the direction perpendicular to the stress, in a brittle solid).

In presence of random generic defects in a solid, even brittle ones, the stress released regions of the defects overlap and do not allow a straightforward generalization. In a randomly disordered solid, therefore the probability of a larger defect due to configurational fluctuation increases with the volume of the sample. As the survival of the sample under stress means then survival of the weakest one in the sample, with increasing volume (with nominally identical microscopic defect concentration) the fracture or breakdown strength of the solid sample decreases.

Because of the possibility of the existence of bigger or weaker defects coming from statistical fluctuations of overlapping neighboring micro-defects, the effective strength of the solid decreases with increasing volume, even for nominally identical composition and elastic behavior. This cumulative growth of micro-defect fluctuations, as captured in the “distribution tail” argument of Ilya Lifshitz,¹² induces extreme statistics of the failure behavior of solids: the cumulative failure probability of such a solid increases to unity as the stress grows at a fixed volume or as the volume grows, at any fixed nonvanishing stress.

This nonself-averaging statistics of the breakdown of solids are well captured in different limits by the extreme statistics of Waloddi Weibull¹³ and Emil Gumbel.¹⁴ Microscopic derivations of these results came much later (see the next section) and phenomenologically they were fitted to the celebrated extreme statistics of Weibull and Gumbel (see Ref. 15, published in early 1985, and Ref. 16 for an approximate microscopic theory, using percolation statistics, to derive these extreme statistics of breakdown in solids, employing the fluctuation model sketched in the earlier para). Obviously, equating the failure probability to unity, one would get from both the distributions, fracture strength decreasing with the increasing volume of the sample. In early winter of 1985, the author was visiting Oxford and gave a talk essentially based on the report made in Ref. 15. Phil Duxbury, who just finished his Ph.D., and moved to the US for Post Doc, was among the audience. He immediately realized and asked for a more precise argument to justify the decreasing failure strength with sample volume. Soon, with Paul Leath and others, he made a beautiful argument using the Lifshitz tail argument to estimate the dominant defect size and the

consequent decrease in fracture strength with volume of the sample using the Griffith kind of argument to relate the decreasing strength of a solid with increasing defect size.¹⁷

3. Statistical Physics of Fracture: Mandelbrot and Others

3.1. *Fracture surface roughness*

As discussed in Sec. 2.1, Mott initiated the study of fracture propagation in solids and studied the propagation velocity (terminal value) in brittle solids. Such calculations assume that the excess of the released elastic energy over the crack surface energy (taking flat surface structure) goes to the velocity dependent kinetic energy of the crack-tip. However, the roughness of the crack surfaces were too prominent to neglect, and there were even conjectures that crack propagation is more like a turbulent motion (rather than streamline) and the fracture surface roughness captures this frozen turbulence in crack propagation.

Benoit Mandelbrot and colleagues first analyzed¹⁸ in 1984 the observed roughness of different fractured surfaces and suggested a scale-free fractal behavior. They measured the growth of out-of-plane fluctuation of the fractured surfaces for several steel samples, by defining the average fluctuation in the surface heights at different distances of separation on the fracture propagation plane, and found that on average the fluctuations in heights grow with the distance of separation along the plane and follow a power law (do not follow a scale-dependent functional form like exponential or similar) with an universal value of the power (exponent). This observation of universality, together with the later extensive ones, confirmed the existence of critical behavior and statistics in fracture and breakdown phenomena. This opened up the investigations of critical phenomena in fracture and breakdown.

3.2. *Fracture of disordered solids: Percolation models*

When Purusattam Ray joined me in 1984 for his Ph.D. research, I found him bold enough to take up the challenge of exploring the origin of extreme statistics of fracture and breakdown in lattice statistical percolation models of disordered solids. Though the nature of challenge was not realized immediately, the prospect of any success in the limited period of Ph.D. research was not clear and looked rather frightening! The idea was first to extend the percolation scaling theories of random resistor networks or elastic networks of Skal-Shklovskii¹ and de Gennes² for linear responses like conductivity and elastic moduli, to that for electrical (fuse or dielectric) breakdown and fracture of percolating networks. The next step of (off-lattice) molecular dynamic simulation of such elastic networks appeared already a formidable and distant goal, if at all achievable in any reasonable time frame with the computing facilities available that time to us! However, the spirit of Purusattam was indomitable and that encouraged us a lot.

As mentioned already, observation of Mandelbrot *et al.*¹⁸ encouraged the view supporting the existence of critical phenomena in breakdown dynamics. We therefore proceeded with the node–link–blob model of the incipient critical percolation cluster proposed by Skal-Shklovski and de Gennes (see e.g., Ref. 19) to estimate the scaling behavior of the fracture stress, as the percolation threshold is approached, of a fixed sized sample (large but finite, to avoid the failure at vanishing stress, due to the presence of the extremely weak defects in the sample). Here, one could assume that the vulnerable defect size would be given by the percolation correlation length, while the elastic modulus would have the power law behavior already established in the node–link–blob model.¹⁹ One could also utilize the fractal dimension of the percolating backbone to find the scaling behavior of the surface energy density for calculating the fracture stress in the Griffith model.^{15,16,20} Indeed, Purusattam achieved already the molecular dynamic simulation of Lennard–Jones systems of randomly dilute solid initially on square lattice and with interaction cut-off beyond a distance of 1.6 lattice constant and up to a modest system size of 400 atoms.²⁰ Though the general trend of decreasing fracture strength with increasing concentration of initial lattice (site) dilution could be seen, results for bigger system sizes were needed for any reasonable analysis.

The paper however attracted attention of several important groups. Dietrich Stauffer, in particular, invited us to extend this molecular dynamic study of fracture in disordered lattices near percolation threshold. Redefining on triangular lattices (to avoid the shear instabilities) and parallelizing the simulation program, we were allowed to utilize for more than seven/eight months the Vector computing facilities in Germany available to him that time (using remote log-in and job submissions, etc., through telephone from his office in Cologne!). The results of this study,²¹ for system size up to 4225 atoms, clearly demonstrated that, at fixed system size, the fracture stress monotonically decreases with increasing dilution concentration and tends to vanish at the percolation threshold. Also, at any fixed dilution concentration, the fracture stress decreases with increasing system size (as the consideration of extreme statistics would suggest). This confirmation was very intriguing and led to important investigations later. It was clear, however, the off-lattice molecular dynamic simulations for disordered elastic networks, undergoing large local deformations for the nucleation and propagation of fracture would soon become formidable as the system size is increased further to check the scaling behaviors.

Hans Herrmann from Cologne and his collaborators immediately introduced²² the random fuse networks, where the local failures or fuses of any lattice bond would induce modifications in the current distributions to keep the total current through the network conserved. This would induce further fuse at the hot spots and the breakdown would proceed. Since the lattice remains intact (no off-lattice simulations were required), the computations became much simpler and universalities in breakdown phenomena could be immediately checked. When we were struggling so hard with the molecular dynamic simulations to extract the universal features of the breakdown, the fuse model²² proposed by de Arcangelis *et al.* clearly indicated a

much softer way to proceed. The paper came to Stauffer for refereeing, and he made important comments (including some on the earlier studies) on the manuscript, which were accommodated in the published version. The model became an instant success in this field of investigating critical behavior of breakdown. It was like the success of the lattice-gas model over that of the extensive analytical and numerical (including molecular dynamic simulation) studies in the 1940–60s to establish the Ising universality class of the liquid–gas transition at the critical point. We were indeed awestruck, though chose to continue our molecular dynamic studies of fracture in randomly disordered solids for some more time! Later, my student Subhrangshu Sekhar Manna studied the statistical difference, if any, between the minimum gap (number of dielectric bonds) and the breakdown voltage (number of broken bonds on the breakdown path) in the case of dielectric breakdown in the lattice model of random conductor insulator mixtures.²³ Among others, this study also triggered several brilliant experimental investigations on the breakdown behavior of random resistor networks. In particular, Lucien Gilles Benguigui of Technion performed a series of experiments by employing light-emitting diodes for insulators in random conducting networks under large voltage gradient. The failure path could be made visible by the lighted diodes (e.g., Refs. 24 and 25, see also Ref. 16).

It is worth noting, however, that Purusattam and coauthor²⁶ showed that percolation-like mode of breaking (rather than nucleation-like breaking) dominates as one increases disorder. Recently, Shekhawat *et al.*²⁷ claimed from their renormalization group study that the avalanche behavior seen in the fuse model is unstable for finite disorder and flows to nucleating failure in large system size limit. A percolation-like failure mode can be seen for very high disorder limit.²⁸

Anyway, going back to late 1980s, on invitation from one of the editors, I wrote a mini-review²⁹ on these developments on fracture and breakdown in disordered solids. The journal itself broke down and quickly disappeared! However, when David Bergman (Tel Aviv) and David Stroud (Ohio) wrote their review on Physical Properties of Macroscopically Inhomogeneous Media in Vol. 46 of the *Solid State Physics* (Academic Press, 1992), they noted (in pp. 264–267) my mini-review as an “authoritative” one and suggested for a detailed one in the same series. I came to know of it much later, and then planned immediately and wrote together with Benguigui, the book *Statistical Physics of Fracture & Breakdown in Disordered Systems*¹⁶ which was published from Oxford University Press in 1997. Muhammad Sahimi (Southern California) developed further these scaling studies for disordered solids in a series of papers during this period and reviewed all these results in a major compendium³⁰ in 2003. Somehow, the choice of timing of the both these books was somehow wrong. The major developments in the statistical physics of fracture in Fiber Bundle models started getting settled a little later!

3.3. Fiber bundle model & its statistics

As mentioned in the Introduction (Sec. 1), the fiber bundle model was introduced by Pierce⁶ in 1926 as a model to understand the strength of composite materials. The model is deceptively simple: the bundle consists of a macroscopically large number of parallel hook springs of identical length and, for simplicity, each having identical spring constants. They have however different breaking stresses. All these springs hang, say, from a rigid horizontal platform. The load hangs from a lower horizontal platform, connected to the lower ends of the springs. This lower platform can be assumed to be absolutely rigid when the load at any point of time is shared equally, irrespective of how many fibers or springs have broken and where, by all the surviving fibers (equal load-sharing model). The lower platform can also be assumed to have finite rigidity, so that local deformation of the platform occurs wherever springs fail and the surviving neighbor fibers have to share larger fraction of that transferred from the failed fiber. Extreme case is that of local load-sharing model, where load of the failed spring or fiber is shared (usually equally) by the surviving nearest neighbor fibers. As may be guessed, the failure dynamics of the equal load-sharing model is easier to formulate and analyze. In fact, the strength of such a solid was first estimated by Daniels⁷ in 1945.

In spite the elegance of the model and many profound features, the model did not catch the attention of physicists until late '80s in the last century, when Didier Sornette noted some other attractive features of the equal load-sharing fiber bundle model.³¹ Later, when Purusattam explained to us in early 1998 about their intriguing mean-field study in Gene Stanley's group in Boston on the possible first order transition behavior of fractures in fiber bundle models,³² we were taken by surprise!

Starting a little earlier, when Srutarshi Pradhan joined me for his Ph.D. research, we started to explore some simple yet nontrivial versions of the equal load-sharing model. Though these versions were not of much practical interest, say, to the engineers, they were expected to allow us making more precise formulation and analysis of some universal features of its breaking dynamics. The simplest such fiber bundle model assumed that the strength of the fibers in the bundle are uniformly distributed, starting from zero to a normalized maximum. It was then easy to set up a simple recursive equation for the breaking dynamics: when the bundle is loaded with an external load, all the fibers having strength up to that value of the load per fiber break and surviving fraction would be given simply by the difference of this load per fiber from the strength of the strongest fiber (normalized to unity). However, due to the breaking of these fibers, the load per surviving fiber increase exactly by the inverse of the fiber fraction broken in the earlier step. This increased load per fiber will induce failure of a further fraction of bonds, and the surviving fraction of fibers at this stage will again be given by the difference of this (increased) load fraction per fiber from unity (normalized highest strength). This gives a simple nonlinear recursion relation for the surviving fraction of fibers at any stage or time (as the updated load per fiber at any time is given by the inverse of the surviving fiber

fraction of the earlier step or time). If there is a fixed point of the relation at any nonzero fraction of fibers, then the bundle does not fail under that load (initially hanged from the lower platform of the bundle), and the runaway dynamics otherwise indicates failure of the bundle. The model was straightforward and the calculations (even the naturally emerging critical behavior of its dynamics) was so simple that we first thought, this must be known already! Srutarshi made an extensive search and could not find. Just around that time, we received the acceptance of one of our paper on the numerical studies on precursors of criticality in some Self-Organized-Criticality Models in *Physical Review E*. We then made an odd request to the editor to allow us to accommodate a brief section giving some calculations in a fiber bundle model, where such precursors can also be seen analytically, and also added that in its title! Surprisingly, the editor readily agreed and we got the first publication of this model and its charmingly simple recursion relation capturing the breaking dynamics.³³ My student Pratip Bhattacharyya noted several intriguing features in the structure of the recursion relations in the model and a series of studies were made in the following years (see e.g., Ref. 34).

It was clearly demonstrated in a series of papers (starting with Ref. 33, see e.g., Ref. 35 for a review) that although there occurs a discontinuous jump in the value of the surviving fiber fraction across the critical load, it does not signify any first-order transition. This is because, the failure time, breakdown susceptibility (given by the ratio of the fraction of failed fibers and marginal increase in the external load), etc., diverge at the critical load on the bundle (with mean field like exponent values; due to suppression of load fluctuations among the fibers in this equal load-sharing model). The scaling forms of the relaxation time were later extensively studied in Ref. 36.

Unlike in the brittle fractures, where essentially a single (weakest) crack chooses to nucleate and propagate throughout the sample (as in the fiber bundle model with local load-sharing), incremental failures throughout the sample, giving rise to avalanches, occur in such equal load-sharing fiber bundle models. With uniform distribution of the fiber strengths, as discussed above, the power law exponent value for the size distribution of the avalanches was already argued precisely by Per Hemmer and Alex Hansen from Trondheim in their classic paper³⁷ in 1992. This universal value of the avalanche size distribution clearly fitted the critical nature of the breakdown statistics in the equal load-sharing fiber bundle model (see e.g., Refs. 35 and 38).

When Srutarshi joined the Trondheim group for his post-doctoral work, they together essentially established analytically the structures of the pre-failure and post-failure dynamics of the equal load-sharing fiber bundle models (mostly discussed in Ref. 35). Some of the intriguing signatures of dynamic precursors in the statistics of an over-loaded fiber bundle were discovered later (see e.g., Ref. 38 for discussions on them).

My student Amit Dutta, together with his student in the Indian Institute of Technology, Kanpur, studied the fiber bundle model with discontinuities in the

threshold distribution.³⁹ They found universal critical exponents, except for the avalanche sizes, which show nonuniversal statistics.

The fiber bundle model is a so-called toy model. Though it captures the essential dynamical feature of load sharing following a failure and the subsequent dynamics, it lacks many other realistic features of fracture in solids. It has therefore received more than its fair share of criticism in its early physics-entry stages mainly from the referees (who could eventually be overruled in most cases). However, it is worth noting that it is the simplicity of the model that gives rise to immense flexibility and hence could be applied to diverse topics such as power-grid networks, failures in ice blocks, traffic jams and of course fracture of disordered solids. Such advantages of flexibility and potential for diverse applications were exploited in many cases. Particularly, in Ref. 40, a limiting strength for the system under uniform loading but nonuniform load sharing was derived. Although not directly applicable to fracture, in other systems such as power grids, nonuniform load sharing could be interesting.

The main difference between the fiber bundle model and other models of fracture (e.g., fuse model) is that in fiber bundle model, the range of load redistribution is also a parameter at hand. While the critical behavior in the equal load-sharing model (mean field limit) was mostly studied, the local load-sharing rule,^{41,42} gives nucleation-driven extreme statistics (a crossover occurs near the percolation threshold, when these two rules are mixed; see discussions in Sec. 4.2 following the Ref. 56). In Ref. 43, a full-phase diagram in range of redistribution and strength of disorder was estimated and presented. This shows the various modes of failures observed in the model over the years in different parts of the phase diagram.

4. Statistical Physics of Earthquakes: Omori and Gutenberg–Richter

That earthquakes are large scale dynamical breaking phenomena, occurring due to the stick-slip kind of failure at the Earth-crust interface with the slowly moving tectonic plates, had been known for a long time. Two major power laws in the statistics of earthquakes had clearly indicated the possibility of criticality in their dynamics. Long ago in 1895, Fusakichi Omori of the Tokyo University suggested that the rate of the aftershock counts decreases inversely with time elapsed since the main shock at any epicentre.⁴⁴ Utsu later modified that law saying that the rate of aftershocks decrease inversely with a power of the time plus an adjustable constant and the power is close to unity⁴⁵ and varies in the range 0.7–1.5. Beno Gutenberg and Charles Francis Richter, both from Caltech, in 1956 proposed a law saying that the logarithm of the number of earthquakes of a particular magnitude or more, occurring in a given region and time period decreases linearly with that particular magnitude. Equating the log of the energy released in an earthquake linearly with its magnitude (as often confirmed in underground nuclear blasts of known energy and the consequent seismic magnitudes), one gets a power law relation between the earthquake frequency and the energy released. More specifically, the number of earthquake

events releasing a particular amount of energy or more, in any area or period, decreases with an inverse power of that particular energy.⁴⁶

4.1. *Burridge–Knopoff model and its statistics*

As mentioned earlier, these scale-free form (power laws with universal values of the exponents) of the earthquake statistics immediately indicated the possible role of the underlying critical phenomena in the dynamics. One of the earliest and so far the most successful model for earthquakes was proposed⁴⁷ by Robert Burridge (University of Cambridge) and Leon Knopoff (University of California, Los Angeles), in 1967 (see also Ref. 48 for a detailed discussion on the model). One takes a chain of a large number of wooden blocks connected by Hook springs placed on a rough horizontal table. One end of the chain is free and the other end is pulled horizontally by a motor. The rough surface contacts between the wooden blocks and the table top would mimic different portions of the Earth’s crust and the tectonic plates. The plate motion (in a reverse way) is captured essentially by the motion of the chain induced by the motor pull. Though the motor pull would be uniform, the chain would have stick-slip type motion; as the static friction force is higher than while in relative motion (essential source of nonlinearity in the dynamics of the otherwise harmonic chain), different number of blocks will slip (different amounts of elastic energy of the inter-block springs will be released) at different points of time. A motion picture of the block positions would allow calculations of elastic energies of the inter-block springs and thereby of the entire chain or “train” as the dynamics progresses from an initial “charging” state to a steady one. The decrease in the number of bursts with increase in the amount of energy released in those bursts clearly indicated a power law, as suggested by the Gutenberg–Richter law.

James Langer (University of California, Santa Barbara) and collaborators, in a series of papers published over a decade starting mid ’80s, formulated a simple version of the Burridge–Knopoff model using numerical tricks. Here, the equation of motion of each block has a part of the forces coming from the relative displacements of the neighboring blocks connected by Hook springs and a nonlinear part depending on the relative velocity of the block compared to the table top. Extensive simulations indeed showed the Gutenberg–Richter like behavior of the (elastic) energy burst statistics. A summary of their results were published⁴⁹ in a nice review in 1994. Hikaru Kawamura of the Osaka University and Takahiro Hatano of the Tokyo University and their collaborators made extensive simulation studies on a similar numerical version of the Burridge–Knopoff model, with more realistic friction forces, etc. It may be mentioned here that none of these model studies could reproduce the Omori law for aftershocks. A review of those studies and of other statistical physics models was published along with us in an extensive review on the statistical physics of earthquake dynamics in 2012 in *Reviews of Modern Physics*.⁵⁰

In the summer of 2012, Soumyajyoti, Purusattam and I were attending a fracture meeting in the SINTEF Petroleum Research, Trondheim, organized by Srutarshi.

One evening, during a discussion in the guest house there, Soumyajyoti and Purusattam came to a novel computationally simpler Burridge–Knopoff type model, where the block motions are discretized (by the lattice structure of the underlying table) and more importantly, the difficult-to-handle nonlinear friction force is replaced by random (threshold type) pinning forces. Though, no analytic calculation could be done, Soumyajyoti, on return to Kolkata, made extensive numerical studies and the results showed extremely encouraging features in the avalanche statistics: both the Gutenberg–Richter law as well as the Omori law were reproduced⁵¹ (see also Ref. 52).

4.2. *Self-organized criticality: Bak and others*

The power laws in the distribution functions observed in nature, like the above-mentioned Omori or Gutenberg–Richter laws and the universal values of those powers, clearly indicate the presence of some kind of self-similarities or scale-independent features in such complex dynamics. Such self-similarities keep the power invariant, like the fractal dimensionality of the effective space of dynamics. The distribution function may, if we wish, be viewed as an effective “volume” in such a self-similar (fractal) space (geometry) and it varies with the event size (viewed as some effective inverse “length”). In any geometry, the power law relation between the length and volume follows naturally. Changes in the length scale would result in the change in the volume (in that embedding geometry) by a corresponding power law, with the power given by the (fractal) dimension of the space or geometry. These observations therefore clearly indicate the role of critical phenomena in earthquake statistics. However, in the cases of liquid–gas or ferromagnet–paramagnet phase transitions, where criticality occurs at specific points, the systems need to be brought to the critical point by tuning externally the (thermo-) dynamic parameters like temperature, etc. Here, in the example of the earthquake we are considering, the system seems to be self-tuned to criticality!

Per Bak (from Copenhagen) and collaborators proposed in 1987 a toy model, called the sand pile model, which dynamically evolves towards such a self-organized critical state and continues its dynamics there without any tuning.⁵³ Imagine a horizontally placed square lattice of finite but large size (having boundaries), where on any randomly chosen site one throws unit height (sand grain). The process of throwing heights on randomly chosen sites of the lattice (adding sand to the pile goes on at a constant but slow rate, much slower than the dynamics for local failure or toppling discussed next). The dynamics of (local) failure is such that if the height at any site becomes four at any time, the site topples (height becomes zero at that site) and four of its neighbors receives one unit of height each. If that causes the height of any of the neighbors to become equal to four, that site topples in the next time unit and four of its neighbors (including the neighbor whose toppling caused its own) receive a unit each. This happens everywhere, except for sites on boundary, where the share of the height for the neighbor(s) beyond the boundary leaves the system

(the total mass or height at any time leaving the system contributes to the size of the avalanches). Needless to say, although the (input) addition of sand grains or heights to the system occurs at a constant rate, the (output) rate of mass or height ejections from the system occurs in bursts or through avalanches. Numerical studies show that after some initial “charging” period, when the average height at any site reaches about 2.1 (for square lattice and with 4 as the threshold height at any site, as in the example above), the dynamics stabilize to a self-organized critical state where the avalanche frequencies decrease with its mass or size following an universal power law with an exponent value around 1.3 (independent of the lattice or threshold details).⁵³ Several extensions of the model were proposed immediately afterwards to make the model more realistic. However, they all led to the same universality class for the critical behavior of their statistics.

Subhrangshu studied a novel stochastic version of the toppling dynamics in a computationally efficient version of the model, where the threshold height becomes 2 and after the site topples, two neighbors are chosen randomly of the four neighbors, and they get one unit of height. If any of these two chosen neighboring sites had one unit of height earlier, that site also topples in the next instant, and so on. I was visiting Forschung Zentrum Julich in the summer of 1990, where Subhrangshu explained the model and its results to me. Because of the stochastic nature of failed load (height) sharing and the stable values of load or height at any point having binary values (0 or 1), computationally the model had been much more efficient and the numerical results seemed to suggest a new universality class.⁵⁴ The model, now known as Manna model, has since been extensively studied and a new (Manna) universality class for such dynamical critical phenomena is more or less established. A more realistic version of the model for earthquake was proposed by Olami *et al.*,⁵⁵ where each time a toppling occurs at any site, the entire load (force) is not shared by the neighbors, but a fraction is assumed to be lost and dissipated locally. As mentioned earlier (see Sec. 3.5) early in 2012, Soumyajyoti discovered a brilliant version of a two-dimensional fiber bundle model with local load sharing, which was shown to possess interesting self-organized critical behavior. In the model, a horizontally held two-dimensional network of hook springs, having random breaking thresholds, is pulled downwards from a central site at a constant rate using a motor. As with time more and more fibers break, they immediately join the pulling string leaving the springs beyond the periphery of the central defect patch unaffected. All the springs on the growing periphery of the central broken patch share equally the constantly growing central pulling force. This dynamic equilibrium has interesting critical statistics of failure⁵⁶ (see also Ref. 52), particularly because there was no externally imposed dissipation scale, but dissipation came from the increase of the effective system size.

Such models therefore provide natural and generic ways of explaining the Gutenberg–Richter type universal behavior of the released energy bursts or avalanches in earthquakes. One may note, as such, these models cannot distinguish between the main shock and aftershocks and therefore they do not capture at all any Omori type behavior of the aftershocks.

4.3. Two-fractal overlap model

As discussed in Sec. 3.1, the fracture surfaces have well-established self-similar geometries and resultant scaling properties. As earthquakes occur due to the slips of the rough crust surface over the moving tectonic plate surface, one can model the earthquake time series by counting the changes in the measure of the overlaps of two fractal surfaces, as one of them moves with a fixed velocity over the other. One can assume that the elastic energy stored during sticking period in the interfacial contacts between the crust and creeping tectonic plate (measured by the overlaps) gets released as slip occurs. The time series of these energy bursts are then given by the time series of the overlaps between two fractals, as one moves with constant velocity over the other. This maps the entire earthquake dynamics into a geometric model of finding the two-fractal (mass) overlap time series. Analytical results for the simplest two Cantor set overlap series showed not only the Gutenberg–Richter type law, but also a built-in Omori law⁵⁷ (see also Refs. 50 and 52 for details). Indeed, Srutarshi started his research career with study of “two-fractal overlap model”, as part of his Post MSc project and later published a detailed numerical study on it with Purusattam and others.⁵⁸

5. Comparison of Activities with Those in Other Contemporary Topics of Condensed Matter Physics

In this section, we wish to compare how the activities in the (statistical) physics of fracture and earthquake compare with other branches of condensed matter physics that are generally considered mature (with being awarded Nobel prizes). One objective way to compare is to look at the number of papers mentioning the subject unambiguously in the topics of a published paper (ISI Web of Science data). We compare the data from websites such as Google Scholar and ISI Web of Science (data compiled in January 2017; see Figs. 1–4).

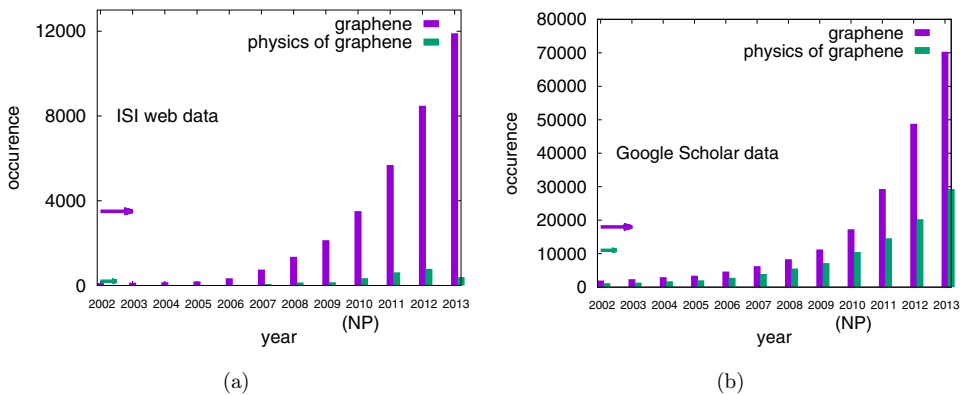
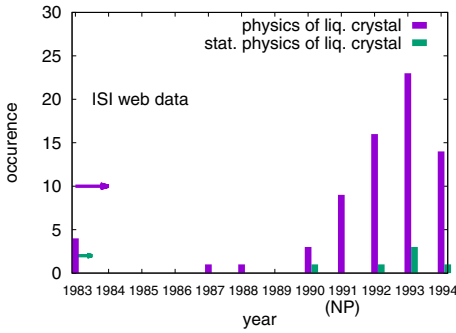
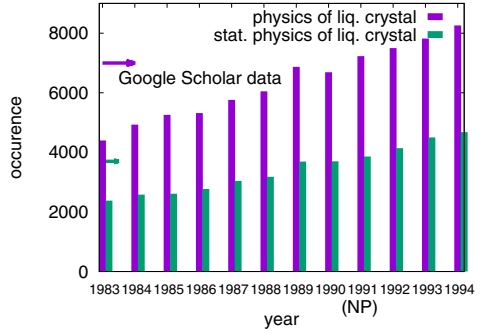


Fig. 1. (a) The number of papers on graphene listed in ISI Web of Science around the year the Nobel prize was awarded in graphene (2010); (b) The same is shown for the Google Scholar data.

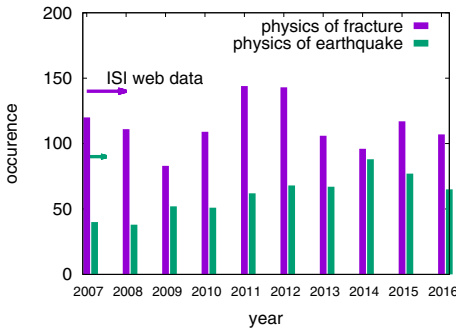


(a)

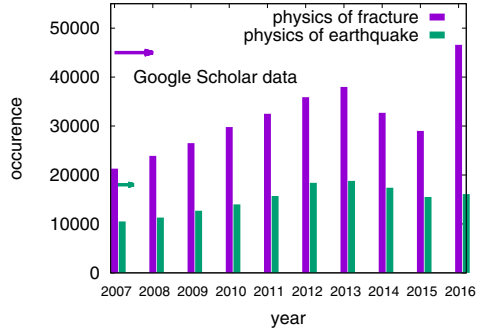


(b)

Fig. 2. (a) The number of papers on liquid crystal listed in ISI Web of Science around the year the Nobel prize was awarded in liquid crystal (1991); (b) The same is shown for the Google Scholar data.

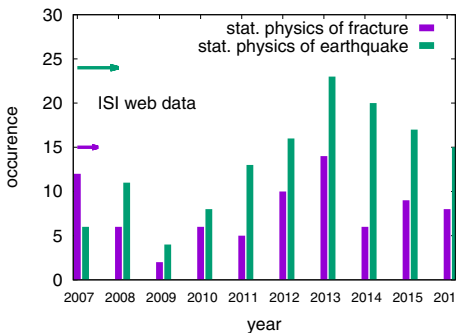


(a)

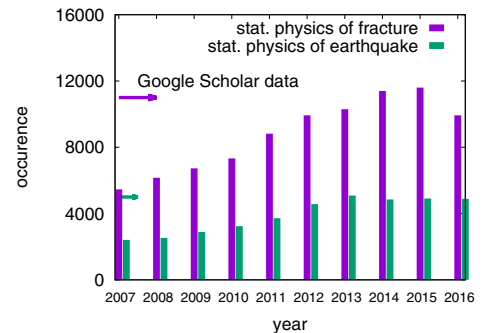


(b)

Fig. 3. (a) The number of papers on physics of fracture and earthquakes listed in ISI Web of Science; (b) The same is shown for the Google Scholar data.



(a)



(b)

Fig. 4. (a) The number of papers on statistical physics of fracture and earthquakes listed in ISI Web of Science; (b) The same is shown for the Google Scholar data.

Figures 1 and 2 show the number of papers published each year on the topics graphene and liquid crystals, respectively, around the years in which the Nobel prizes were awarded. Both ISI Web of science data (published papers; topics search) and Google Scholar data (term anywhere in published/unpublished documents in the Internet). Graphene data are much higher than those for physics of graphene data. Statistical physics of graphene is not an appropriate topic for search (unlike the rest of the datasets shown in Figs 2–4). Still, the data are shown here, just to indicate the scale of research activities in the field at the time of recognition for such a popular and contemporary condensed matter physics topic. The physics and statistical physics of liquid crystals are of course much more appropriate for comparison with the corresponding data (from both ISI Web of Science and Google Scholar) for fracture as well as earthquake, and are shown in Figs. 3 and 4.

As might be noted, the contemporary rates of publications (research activities) in both physics and statistical physics of fracture and earthquake are quite comparable and even higher than the respective rates for liquid crystals research activities around the year of its recognition. The data for contemporary research activities in physics of fracture and of earthquake are also comparable to those for physics of graphene around the year of its recognition.

6. Perspectives and Concluding Remarks

The data shown in Figs. 1–4 clearly indicate that the progress in the studies on the statistical physics of fracture and earthquake and their impact in the contemporary literature has already been extremely significant. It is therefore unfortunate that the standard condensed matter physics graduate courses and researches do not include even minimal discussions on physics of fracture (introduce, say, the elegant and versatile Fiber Bundle Model, used and explored extensively by engineers and physicists) and of earthquake (introduce, say, the Burridge–Knopoff model). It may be mentioned that four recently published books, namely *Earthquakes: Models, Statistics, Testable Forecasts*,⁵⁹ *Desiccation Cracks & their Patterns*,⁶⁰ *Fiber Bundle Model: Modeling Failure in Materials*³⁸ and *Statistical Physics of Fracture, Breakdown & Earthquakes*,⁵² in the series *Statistical Physics of Fracture and Breakdown* by Wiley-VCH and edited by Purusattam and me tried to capture all these developments in details. A suitably picked and chosen set of topics from these set of books can be utilized to generate an appropriate graduate level course. We do believe such a course would be very timely and has been long overdue.

Acknowledgments

This is an expanded version of the presentation in the 5th event (January, 2017) in the series of the International Conferences on Statistical Physics of Fracture (Fracmeet) held in the Institute of Mathematical Sciences, Chennai, organized by Purusattam Ray and his colleagues. Several encouraging suggestions and comments from the participants helped me to complete this story. I am thankful to Soumyajyoti

Biswas of the Max Planck Institute for Dynamics and Self-Organization, Goettingen and Editorial board member of this journal to encourage me to finish the account and publish. I am also grateful to the Editor, Ophir Flomenbom, for a careful reading of the entire manuscript, thoughtful suggestions, appreciation and ready support. I am grateful to the J. C. Bose National Fellowship (DST, Govt. India) grant for supporting the publication of this paper.

References

1. A. Skal and B. Shklovskii, *Sov. Phys.-Semicond.* **8** (1975) 1029.
2. P-G. de Gennes, *J. Physique Lett.* **52** (1976) L1.
3. J. R. Lund and J. P. Bryne, *Civil. Eng. and Env. Syst.* **18** (2000) 243.
4. <https://ebooks.adelaide.edu.au/g/galileo/dialogues/complete.html>.
5. A. A. Griffith, *Phil. Trans. Royal Soc. London A* **221** (1921) 163.
6. F. T. Pierce, *J. Textile Indust.* **17** (1926) 355.
7. H. E. Daniels, *Proc. Royal Soc. London A* **183** (1945) 404.
8. N. F. Mott, *Engineering* **165** (1948) 16.
9. E. Bouchaud, *J. Phys.: Condensed Matter* **9** (1997) 4319.
10. J. Schmittbuhl and K. J. Måløy, *Phys. Rev. Lett.* **78** (1997) 3888.
11. S. Santucci, K. J. Måløy, A. Delaplace, J. Mathiesen, A. Hansen, J. Øistein, H. Bakke, J. Schmittbuhl, L. Vanel and P. Ray, *Phys. Rev. E* **75** (2007) 016104.
12. I. M. Lifshitz, *Sov. Phys.-Usp.* **7** (1965) 549.
13. W. Weibull, *J. Appl. Mech.* **18** (1951) 293.
14. E. J. Gumbel, *Statistical Theory of Extreme Values & Some Practical Applications* (Columbia University Press, New York, 1954).
15. P. Ray and B. K. Chakrabarti, *Solid State Commun.* **53** (1985) 477.
16. B. K. Chakrabarti and L. G. Benguigui, *Statistical Physics of Fracture & Breakdown in Disordered Systems* (Oxford University Press, Oxford, 1997).
17. P. M. Duxbury, P. D. Beale and P. L. Leath, *Phys. Rev. Lett.* **57** (1986) 1052.
18. B. B. Mandelbrot, D. E. Passoja and A. J. Paullay, *Nature* **308** (1984) 721.
19. D. Stauffer and A. Aharony, *Introduction to Percolation Theory* (Taylor & Francis, London, 1994).
20. P. Ray and B. K. Chakrabarti, *J. Phys. C* **18**, L185 (1985).
21. B. K. Chakrabarti, D. Chowdhury and D. Stauffer, *Zeitschrift fur Physik B* **62** (1986) 343.
22. L. de Arcangelis, S. Redner and H. J. Herrmann, *J. Phys. Lett.* **46** (1985) 585.
23. S. S. Manna and B. K. Chakrabarti, *Phys. Rev. B* **36** (1987) 4078.
24. L. Benguigui, *Phys. Rev. B* **38** (1988) 7211.
25. L. Benguigui and P. Ron, in *Nonlinearity & Breakdown in Soft Condensed Matter*, eds. K. K. Bardhan, B. K. Chakrabarti and A. Hansen (Springer, Berlin, 1994).
26. P. Ray and G. Date, *Physica A* **229** (1996) 26.
27. A. Shekhawat, S. Zapperi and J. P. Sethna, *Phys. Rev. Lett.* **110** (2013) 185505.
28. A. A. Moreira, C. L. N. Oliveira, A. Hansen, N. A. M. Araujo, H. J. Herrmann and J. S. Andrade Jr., *Phys. Rev. Lett.* **109** (2012) 255701.
29. B. K. Chakrabarti, *Rev. Solid State Sci. (World Scientific)* **2** (1988) 559.
30. M. Sahimi, *Heterogeneous Materials II: Nonlinear & Breakdown Properties & Atomistic Modelling* (Springer, NY, 2003).
31. D. Sornette, *J. Phys. A* **22** (1989) L243.
32. S. Zapperi, P. Ray, H. E. Stanley and A. Vespignani, *Phys. Rev. Lett.* **78** (1997) 1408.
33. S. Pradhan and B. K. Chakrabarti, *Phys. Rev. E* **65** (2001) 016113.

34. P. Bhattacharyya, S. Pradhan and B. K. Chakrabarti, *Phys. Rev. E* **67** (2003) 046122.
35. S. Pradhan, A. Hansen and B. K. Chakrabarti, *Rev. Mod. Phys.* **82** (2010) 499.
36. C. Roy, S. Kundu and S. S. Manna, *Phys. Rev. E* **87** (2013) 062137.
37. P. C. Hemmer and A. Hansen, *J. Appl. Mech.* **59** (1992) 909.
38. A. Hansen, P. C. Hemmer and S. Pradhan, *The Fiber Bundle Model: Modeling Failure in Materials* (Wiley-VCH, 2015).
39. U. Divakaran and A. Dutta, *Phys. Rev. E* **78** (2008) 021118.
40. S. Biswas and P. Sen, *Phys. Rev. Lett.* **115** (2015) 155501.
41. S. Biswas and B. K. Chakrabarti, *Eur. Phys. J. B* **86** (2013) 160.
42. D. G. Harlow and S. L. Phoenix, *J. Composite Mater.* **12** (1978) 195.
43. S. Roy, S. Biswas and P. Ray, arxiv:1610.06942.
44. F. Omori, *J. Coll. Sci. Imp. Univ. Tokyo* **7** (1895) 111.
45. T. Utsu, *Geophys. Mag.* **30** (1961) 521.
46. B. Gutenberg and C. F. Richter, *Bull. Seismol. Soc. Am.* **46** (1956) 105.
47. R. Burridge and L. Knopoff, *Bull. Seismol. Soc. Am.* **57** (1967) 341.
48. R. Burridge, in *Modelling Critical & Catastrophic Phenomena in Geoscience*, eds. P. Bhattacharyya and B. K. Chakrabarti (Springer, Heidelberg, 2006), pp. 113–280.
49. J. M. Carlson, J. S. Langer and B. E. Shaw, *Rev. Mod. Phys.* **66** (1994) 657.
50. H. Kawamura, T. Hatano, N. Kato, S. Biswas and B. K. Chakrabarti, *Rev. Mod. Phys.* **84** (2012) 839.
51. S. Biswas, P. Ray and B. K. Chakrabarti, *Euro. Phys. J B.* **86** (2013) 160.
52. S. Biswas, P. Ray and B. K. Chakrabarti, *Statistical Physics of Fracture, Breakdown & Earthquake* (Wiley-VCH, 2015).
53. P. Bak, C. Tang and K. Wiesenfeld, *Phys. Rev. Lett.* **59** (1987) 381.
54. S. S. Manna, *J. Phys. A: Math. Gen.* **24** (1991) L363.
55. Z. Olami, H. J. S. Feder and K. Christensen, *Phys. Rev. Lett.* **68** (1992) 1244.
56. S. Biswas and B. K. Chakrabarti, *Phys. Rev. E* **88** (2013) 042112.
57. B. K. Chakrabarti and R. B. Stinchcombe, *Physica A* **270** (1999) 27.
58. S. Pradhan, B. K. Chakrabarti, P. Ray and M. K. Dey, *Physica Scripta* **T106** (2003) 77.
59. Y. Kagan, *Earthquakes: Models, Statistics, Testable Forecasts* (Wiley-VCH, 2014).
60. L. Goehring, A. Nakahara, T. Dutta, S. Kitsunezaki and S. Tarafdar, *Desiccation Cracks and their Patterns* (Wiley-VCH, 2015).