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Estimation of 'drainable' storage – A geomorphological approach

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ABSTRACT

Storage of water within a river basin is often estimated by analyzing recession flow curves as it cannot be 'instantly' estimated with the aid of available technologies. In this study we explicitly deal with the issue of estimation of 'drainable' storage, which is equal to the area under the 'complete' recession flow curve (i.e. a discharge vs. time curve where discharge continuously decreases till it approaches zero). But a major challenge in this regard is that recession curves are rarely 'complete' due to short inter-storm time intervals. Therefore, it is essential to analyze and model recession flows meaningfully. We adopt the well-known Brutsaert and Nieber analytical method that expresses time derivative of discharge (dQ/dt) as a power law function of $Q : -dQ/dt = kQ^{\alpha}$. However, the problem with dQ/dt–Q analysis is that it is not suitable for late recession flows. Traditional studies often compute α considering early recession flows and assume that its value is constant for the whole recession event. But this approach gives unrealistic results when $\alpha \geqslant 2$, a common case. We address this issue here by using the recently proposed geomorphological recession flow model (GRFM) that exploits the dynamics of active drainage networks. According to the model, α is close to 2 for early recession flows and 0 for late recession flows. We then derive a simple expression for drainable storage in terms the power law coefficient k, obtained by considering early recession flows only, and basin area. Using 121 complete recession curves from 27 USGS basins we show that predicted drainable storage matches well with observed drainable storage, indicating that the model can also reliably estimate drainable storage for 'incomplete' recession events to address many challenges related to water resources.

1. Introduction

Terrestrial water is a key entity that takes part in many important roles like regulating regional climate, shaping natural landscapes and maintaining fresh water ecosystems (e.g., [1,13,21,22,25,28]). Its spatio-temporal distribution displays great variability, which is often studied in the context of river basins that are commonly viewed as hydrologically independent units. River basins receive water in the form of precipitation and release it through various mechanisms like evapotranspiration and surface outflow. Thanks to their ability to hold water, they sustain flow in channels even during prolonged drought periods, ensuring continuous supply of water for various human needs. The flow characteristics of a basin during drought periods will, of course, depend on physio-climatological features of the basin like basin-scale hydraulic conductivity, climate and drainage area [6,9–11,36]. Thus, to manage the limited freshwater resources more efficiently it is necessary to accurately model storage and discharge for river basins using the measurable physio-climatological features (e.g., [6,7,11,17,29,36]).

While discharge being routinely measured by government agencies, measurement of storage is not a straightforward job. The main problem being that it is not possible to measure storage in a basin 'instantly' because of our inability to access subsurface systems with the use of available technologies. For example, GRACE (Gravity Recovery and Climate Experiment) satellites can measure storage fluctuation, but cannot measure absolute storage (e.g., [21]). In fact, it is quite hard to define storage objectively. Up to what depth does a basin's storage extend? To our knowledge, there is no objective answer for this question given in the hydrologic literature. Generally, mass balance equation is used to define storage for a basin in terms of its inflow and outflow components (see Fig. 1), which can be written as:

$$\frac{dS}{dt} = P - ET - Q - LS \quad (1)$$

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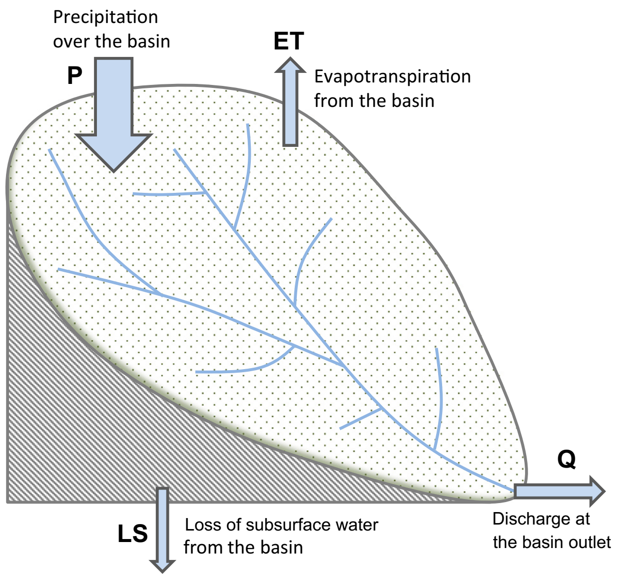

Fig. 1. A schematic diagram indicating mass balance for a hypothetical basin. While precipitation is the only input term in the system, storage loss can occur due to evapotranspiration, discharge through the channel network and water flow via deep subsurface flow pathways.

S in Eq. (1) is the volume of water stored in the basin at time *t*. *P* is the precipitation input rate, the only input term in the equation. *ET* is the rate of water loss from the basin through evapotranspiration. *Q* is discharge from the basin at the root of the channel network or the surface water outlet point of the basin. *LS* is the rate of loss of water from the basin via deep subsurface flow pathways. Note that *Q* will also be composed of water drained by subsurface storage units (e.g., [6]) into stream channels, but the component *LS* will never pass through the surface water outlet of the basin. If precipitation input stops completely, water stored in a basin will keep on depleting and, eventually, each of the outflow components (*ET*, *Q* and *LS*) will approach zero.

In this study, we exclusively focus on the issue of estimating the volume of hydrologically active storage or water stored in a basin that is drained by the channel network, which is also called 'drainable' storage (*SD*) here. This is not because the other two outflow components should not be of concern, but because it is beyond the scope of the present study to consider them for the analysis. When precipitation is absent, the mass balance equation for *SD* becomes quite simple:

$$\frac{d(SD)}{dt} = -Q \tag{2}$$

However, estimation of drainable storage can still be very challenging. In this study, we fist discuss the problems one is supposed to face while using Eq. (2). We then use the recently proposed geomorphological recession flow model (GRFM) and follow suitable analytical methods to obtain a simple expression for drainable storage. Finally, using observed daily streamflow data from 27 USGS basins we evaluate the predictability of the proposed model.

2. Recession flow analysis and the problem of a single storage–discharge relationship

Discharge during drought periods decreases continuously over time till it approaches zero, which is also known as recession flow. If a drought event is sufficiently long, discharge will eventually approach zero or the basin will stop flowing (see Fig. 2(a)). We call such a recession event as 'complete' recession event. Similarly, an

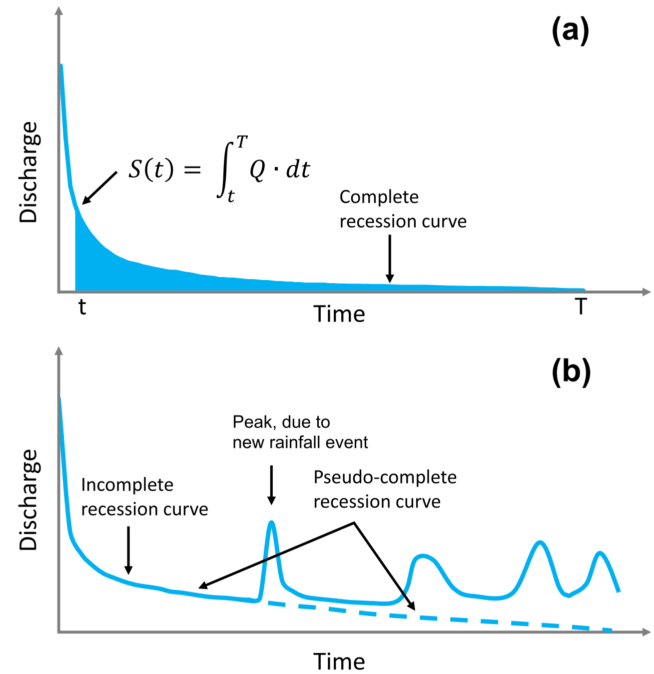

Fig. 2. (a) The recession event is 'complete', i.e. discharge during the recession event decreases till it approaches zero. Drainable storage for the complete recession curve can be estimated at any point of time *t* by computing area under the curve from *t* to *T*. (b) The recession event is 'incomplete', because the inter-storm time interval is not long enough to allow discharge decrease till it approaches zero. Drainable storage can not be computed for this event. However, a 'pseudo-complete' recession curve (dotted line) can be obtained by using a recession flow model and assuming that no rainfall occurred over the basin till discharge approached zero. Once we have the pseudo-complete recession curve, drainable storage at any point can be obtained by following the same integration method.

'incomplete' recession event is defined as a recession event which is not long enough to allow discharge approach zero (see Fig. 2(b)). For a complete recession event, drainable storage at any point of time can be computed by integrating Eq. (2) as (see Fig. 2(a)):

$$SD(t) = \int_t^T Q \cdot dt \tag{3}$$

where T is the timescale of the recession curve or the time period for which the recession event lasts, i.e. $Q(T) = 0$. A major problem in regard to estimation of drainable storage, however, is that most of the times we do not observe a complete recession curve, because inter-storm time gaps are usually shorter than recession timescales, particularly in case of large basins (Fig. 2(b)). (Perennial basins by definition never dry up, i.e. they never witness a complete recession event.) In such a case, our only option is to estimate drainable storage using the incomplete recession curve. This can be achieved by obtaining a 'pseudo-complete' recession curve by assuming that the inter-storm time gap is sufficiently long and extending the incomplete recession curve till discharge equal to zero with the help of a recession flow model (Fig. 2(b)). Drainable storage for the incomplete recession event can then be estimated by applying Eq. (3) for the pseudo-complete recession curve. Of course, the accuracy of drainable storage estimation by this method will depend on how well recession flow curves are modeled.

A radical change in recession analysis was introduced by Brutsaert and Nieber [11] who expressed dQ/dt as a function of Q, thus eliminating the necessity of defining a reference time. $-dQ/dt$ vs. Q curves generally tend to follow a power law relationship (e.g., [3,6,11,12,32–36]):

$$-\frac{dQ}{dt} = kQ^{\alpha} \tag{4}$$

Now combining Eqs. (2) and (4) one can obtain the relationship between drainable storage and discharge in integral form for a recession duration with constant α and k as:

$$\int_{Q}^{Q(t^*)} Q^{1-\alpha} dQ = k \int_{SD}^{SD(t^*)} d(SD) \tag{5}$$

where $Q(t^*)$ and $SD(t^*)$ are discharge and drainable storage, respectively, at a reference time t^*. We distinguish two main scenarios for Eq. (5) as: $\alpha = 2$ and $\alpha \neq 2$. When $\alpha = 2$, Eq. (5) gives:

$$SD(t) = SD(t^*) + \frac{1}{k}(\ln Q - \ln Q(t^*)) \tag{6}$$

or discharge is an exponential function of drainable storage: $Q(t) = Q(t^*) \cdot e^{k(SD - SD(t^*))}$. When $\alpha \neq 2$,

$$SD(t) = SD(t^*) + \frac{1}{k(2-\alpha)}\left(Q^{2-\alpha} - Q(t^*)^{2-\alpha}\right) \tag{7}$$

Eq. (6) or (7) can be applied to a recession period for which α and k are constant, and at any point of time drainable storage can be estimated once we have the values of $Q(t^*)$ and $SD(t^*)$. Here arises the second major problem: practically, it is almost impossible to compute α and k for late recession periods, because $-dQ/dt$ is very sensitive to observational errors during these periods when discharge is typically very low. For example if two consecutive discharge values are equal in a discharge time series, $-dQ/dt$ will be zero, making it unsuitable for a power-law analysis (Eq. (4)). That means, dQ/dt–Q analysis is not possible for a complete recession curve. In practice $-dQ/dt$ vs. Q curves are analyzed only for early recession periods when discharge is relatively high, and then dQ/dt–Q relationships for complete (or pseudo-complete) recession curves are constructed using a recession flow model. Often it is assumed that α is constant throughout a recession event, and its value is either assumed or computed by fitting a regression line to the $(Q, -dQ/dt)$ data cloud obtained by using the available early recession flow data from the basin (e.g., [2,15,19]). However, Biswal and Marani [3] found that while α remains fairly constant for a basin, $-dQ/dt$ vs. Q curves from individual recession events maintain significant distance from one another, i.e. the value of k varies significantly across recession events. Therefore, recession curves should be analyzed individually as fitting a regression line to the $(Q, -dQ/dt)$ data cloud of a basin will result in significant underestimation of α [3,6,7,18,20,32,33].

A logical puzzle here is that if α is assumed to be constant throughout a recession event, Eqs. (6) and (7) will suggest that drainable storage SD is infinite for any finite discharge Q when $\alpha \geqslant 2$, because both drainable storage and discharge at the end of a recession event (reference values) will be zero. This means that, for the case of $\alpha \geqslant 2$, flow characteristics during late recession periods must be different from those during early recession periods, such that storage becomes finite. Using Dupuit–Boussinesq aquifer model Brutsaert and Nieber [11] suggested that the value of α is 3 for a short period of time in the beginning of a recession event and then it becomes 1.5 or 1 depending on the aquifer geometry. However, they did not suggest an objective method to find out when the transition of α from 3 to 1.5 or 1 occurs. Furthermore, individual $-dQ/dt$ vs. Q curves from real basins (representing early recession flows) do not show α from 3 to 1.5 or 1 type transition [3,6]. Other major limitations of Dupuit–Boussinesq are discussed in Biswal and Nagesh Kumar [6]. Individual $-dQ/dt$ vs. Q curve analysis generally reveal that the value of α for early recession flows is nearly equal to 2 [3,4,6,20,32,33]. Therefore, we use GRFM in this study to construct pseudo-complete recession curves for incomplete recession events with $\alpha = 2$ to compute drainable storage.

3. GRFM and drainable storage estimation

Many details of the hydrological processes occurring in a basin can be found to be encoded in the morphology of the drainage network. Over the past few decades much research has been carried out to identify the signatures of the channel network morphology in the hydrological response generated by it, particularly in regard to surface flows (e.g., [23,24,26,27]). The recently proposed GRFM suggests that the gradual shrinkage of the active drainage network (ADN), i.e. the part of the drainage network actively draining water at a particular time (e.g., [14,37]), controls recession flows in the basin at that time [3,5–7,20]. GRFM connects recession flow properties with the channel network morphology by assuming that the flow generation per unit ADN length, q, remains constant during a recession event. Q can thus be expressed as: $Q = q \cdot G(t)$, where $G(t)$ is the total length of the ADN at time t [3]. Furthermore, it is assumed that the speed at which the ADN heads move in downstream direction, c ($c = dl/dt$ or $l = c \cdot t$, where l is the distance of a ADN head from its farthest source or channel head at time t), remains constant in space and time. That means, discharge can also be expressed as a function of l:

$$Q = q \cdot G(l) \tag{8}$$

with $G(l)$ being the geomorphic recession curve for the basin. Fig. 3(a) shows $G(l)$ vs. l curve for Arroyo basin (106.71 sq km, California) obtained by using 30 m resolution USGS digital elevation model and imposing a flow accumulation threshold of 100 pixels (for details regarding the computation of $G(l)$, see [3]). The expression for $-dQ/dt$ can then be obtained as:

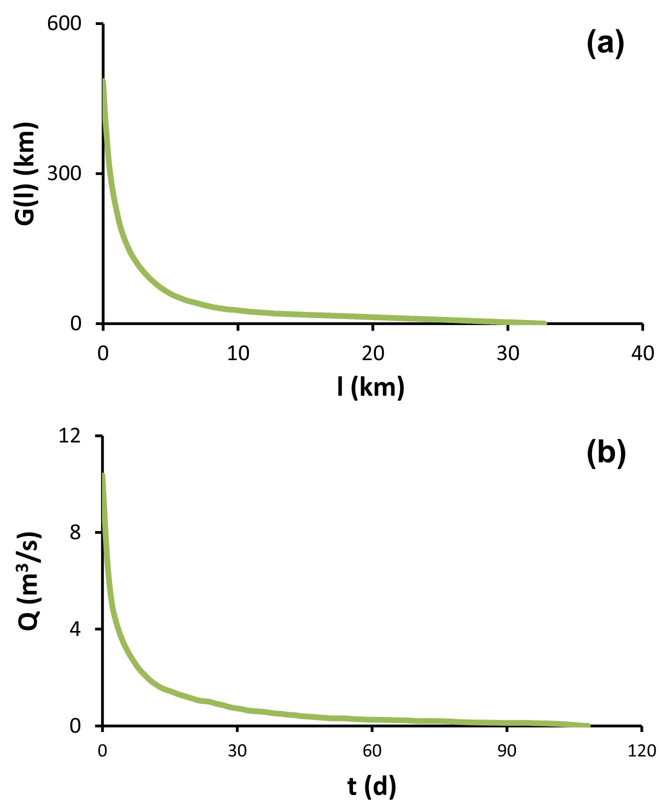

Fig. 3. (a) The $G(l)$ vs. l curve (or the geomorphic recession curve) for Arroyo basin (106.71 sq km). The channel network for the basin was obtained by imposing a flow accumulation threshold equal to 100 pixels. (b) A sample observed recession curve (Q vs. t curve, from 3/20/1973 to 7/8/1973) from the basin, which looks similar to the geomorphic recession curve of the basin.

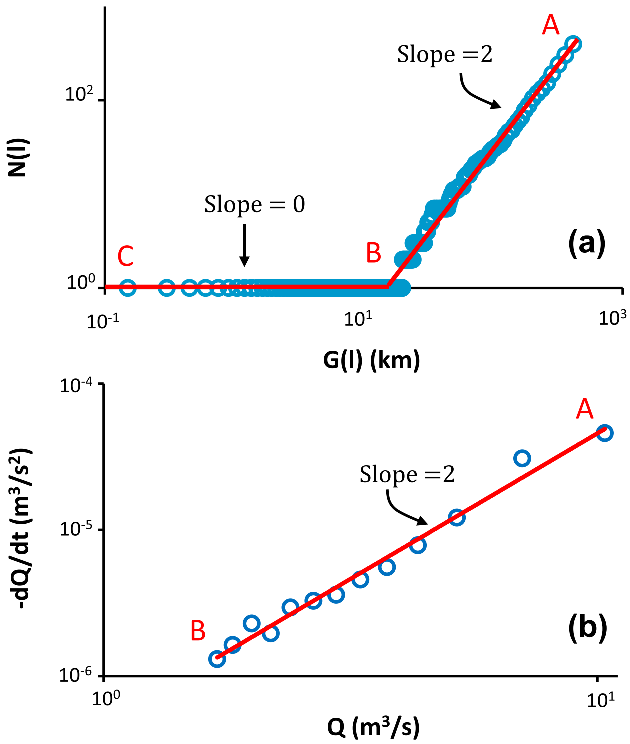

Fig. 4. (a) $N(l)$ vs. $G(l)$ curve for Arroyo basin (106.71 sq km), which displays two distinct scaling regimes: AB that corresponds to early recession flows ($\alpha = 2$) and BC that corresponds to late recession flows ($\alpha = 0$). The channel network for the basin was obtained by imposing a flow accumulation threshold equal to 100 pixels. (b) AB part of an observed recession curve (lasting from 3/20/1973 to 7/8/1973) from the basin displaying $-dQ/dt$ vs. Q power law relationship with power law exponent nearly equal to 2. Note that BC parts of observed recession curves are generally dominated by significant errors (Red lines indicate slopes in the log–log planes.). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$$-\frac{dQ}{dt} = -q \cdot \frac{dl}{dt} \cdot \frac{dG(l)}{dl} = q \cdot c \cdot N(l) \tag{9}$$

where $N(l)$ is the number of ADN heads at distance l or time t. Using Eqs. (8) and (9), the expression for the geomorphic counterpart of $-dQ/dt$ vs. Q curve (Eq. (4)) can be obtained as [3]:

$$N(l) = \rho \cdot G(l)^{\alpha} \tag{10}$$

where $\rho = kq^{\alpha-1}/c$.

The $N(l)$ vs. $G(l)$ curve of a basin typically displays two scaling regimes, AB and BC, easily distinguishable from one another ([3], also see Fig. 4(a)). The regime AB corresponds to early recession flows, and for most basins the geomorphic α for this phase is also nearly equal to 2, i.e. both geomorphic α and observed α are nearly equal to 2 for the regime AB [3,6], suggesting that the model is able to capture key details of a recession flow curve. Defining the geomorphic storage $\Lambda(l)$ as $-d(\Lambda(l))/dl = G(l)$, the expression for the geomorphic storage–discharge relationship in integral form for $\alpha = 2$ can be obtained by using Eq. (10):

$$\int_{G(l)}^{G(l^*)} \frac{dG(l)}{G(l)} = \rho \int_{\Lambda(l)}^{\Lambda(l^*)} \cdot d\Lambda(l) \tag{11}$$

where $l^* = c \cdot t^*$, the reference distance. Now Eq. (11) produces the geomorphic equivalent of drainable storage as:

$$\Lambda(l) = \Lambda(l^*) + \frac{1}{\rho}(\ln G(l) - \ln G(l^*)) \tag{12}$$

Eq. (12) suggests that the geomorphic storage–discharge relationship for part AB is exponential: $G(l) = G(l^*) \cdot e^{\rho(\Lambda(l)-\Lambda(l^*))}$. Fig. 5(a) shows $\Lambda(l)$ vs. $G(l)$ curve for Arroyo basin displaying exponential relationship for its AB portion, with the transition point B being very noticeable. Note that Eq. (6) can be easily retrieved from Eq. (12) using the relationships $Q = q \cdot G(l)$ and $SD = -\int Qdt = -q \cdot \int G(l) \cdot dt/dl \cdot dl = q/c \cdot \Lambda(l)$. $N(l)$ is always equal to 1 for the phase BC as only the mainstream of the channel network contributes, which also means that $\alpha = 0$ for this phase (see Fig. 4(a)). $G(l)$ for this phase is thus $L - l$, where L is the length of the mainstream of the channel network, and $\Lambda(l) = 1/2 \cdot (L - l)^2$. That means,

$$\Lambda(l) = \frac{1}{2} G(l)^2 \tag{13}$$

for the BC part of the recession curve. Fig. 6(a) separately shows BC portion of the $\Lambda(l)$ vs. $G(l)$ curve for Arroyo basin. Using the expressions for SD ($SD = q/c \cdot \Lambda(l)$, obtained from Eq. (2)) and Q ($Q = q \cdot G(l)$) it is found that drainable storage–discharge relationships for BC portions or late recession flows according to GRFM should follow a power law relationship with exponent equal to 2:

$$SD \propto Q^2 \tag{14}$$

If the power law scaling transition (i.e. α changes from 2 to 0) takes place at length l^*, Eq. (10) suggests that $N(l^*) = 1 = \rho G(l^*)^2$ (i.e., BC phase starts at l^* where $N(l^*)$ is 1). We therefore find that $\ln G(l) - \ln G(l^*)$ is $\ln(G(l)/G(l^*)) = 1/2 \cdot \ln N(l)$ and $\Lambda(l^*)$ is $1/2 \cdot (L - l^*)^2 = 1/2 \cdot G(l^*)^2 = 1/(2\rho)$. Thus, Eq. (12) can be expressed as:

$$\Lambda(l) = \frac{1}{2\rho}(1 + \ln N(l)) \tag{15}$$

According to Biswal and Marani [7], $N(l) = \Upsilon(l) \cdot A$ for topologically random networks [31] when c is a constant. $\Upsilon(l)$ is the constant of proportionality and A is the basin area. We assume that the basins selected here follow this criterion. Geomorphic storage $\Lambda(l)$ can thus be written as:

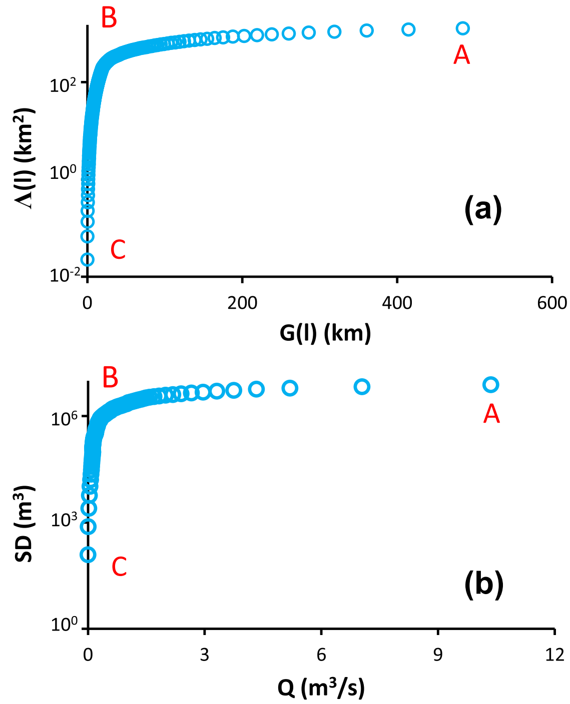

Fig. 5. (a) The $\Lambda(l)$ vs. $G(l)$ curve or the geomorphic storage–discharge curve of Arroyo basin (106.71 sq km) and (b) a selected observed recession curve from the basin (from 3/20/1973 to 7/8/1973) displaying two distinct scaling relationships in semi logarithmic planes: exponential relationship for the regime AB and power law relationship for the regime BC (not clearly visible here).

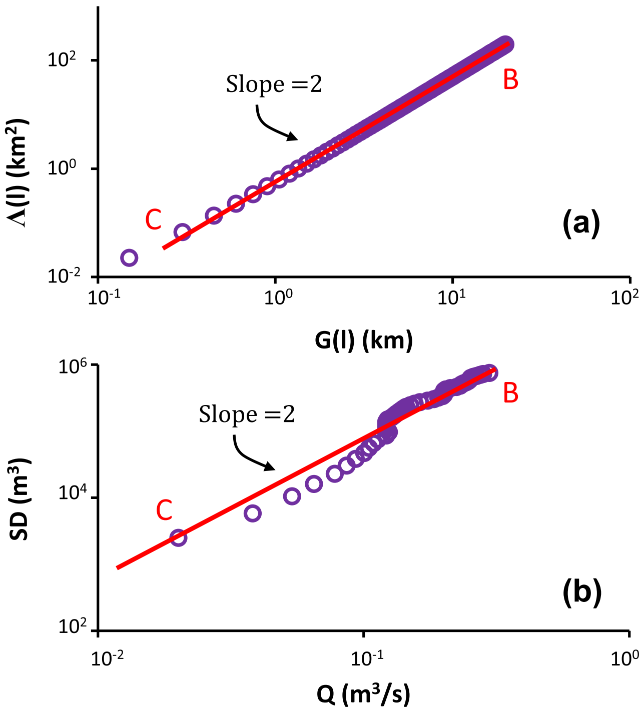

Fig. 6. (a) The BC portion of the $\Lambda(l)$ vs. $G(l)$ curve of Arroyo basin (106.71 sq km) in log–log plot displaying a power law relationship with exponent equal to 2 (indicated by red line). (b) The BC portion of a selected observed recession curve (from 3/20/1973 to 7/8/1973) also displaying a power law relationship with exponent close to 2. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$$\Lambda(l) = \frac{1}{2\rho}(1 + \ln A + \psi(l)) \tag{16}$$

where $\psi(l) = \ln \Upsilon(l)$. Recalling that storage $SD(l) = q/c \cdot \Lambda(l)$ and $k = c\rho/q$ for $\alpha = 2$, the expression for drainable storage at any point of time t can be obtained as

$$SD(t)^m = SD(l)^m = \frac{1}{2k}(1 + \ln A + \psi(l)) \tag{17}$$

The superscript m explicitly denotes that Eq. (17) gives modeled drainable storage. Since $\psi(l)$ is supposed to be constant across basins, the variability of drainable storage $SD(t)$ within a basin is represented by the variability of k only (17). It is interesting to note here that Eq. (17) supports the earlier speculation that the dynamic parameter k is mainly controlled by the characteristic storage in the basin [4,6,7,33].

In short, GRFM defines the shape of a recession curve. Thus, it can be used to obtain a pseudo-complete recession curve for an incomplete recession event. As computation of k requires only a few early recession flow data points [3,6,32], Eq. (17) can be used to estimate drainable storage for incomplete recession events once we have the value of the universal constant $\psi(l)$. However, the predictability of Eq. (17) can be verified only when we have a complete recession curve, for which drainable storage can be directly estimated using Eq. (3). In the next section we analyze complete recession curves from a number of basins compare observed drainable storage (Eq. (17)) with modeled drainable storage (Eq. (3)). Note that even for a complete recession curve we use only early recession flow data points (belonging to the AB part) to compute k.

4. Analysis of observed recession flow curves

We use daily average streamflow data and identify complete recession curves for 27 USGS basins that are relatively unaffected by human activities (see Table S1 of the online supporting material). Theoretically, both Q and $-dQ/dt$ should continuously decrease over time during a recession period. However, almost always, this criterion is not satisfied due to errors (numerical errors, measurement errors, etc.), particularly associated with late recession flows (i.e. with the BC parts of recession curves). Here we visually select relatively smooth looking recession curves starting from their respective peaks and lasting till discharge approaching zero (Fig. 3(b)). For convenience, we denote discharge in discrete form as Q_z, the average discharge in the zth day during a recession event after the recession peak. For time $t = z + 1/2$ days, Q and $-dQ/dt$ are computed by following Brutsaert and Nieber [11] as: $Q = (Q_z + Q_{z+1})/2$ and $-dQ/dt = (Q_z - Q_{z+1})/\Delta t$ (here Δt is 1 day). $-dQ/dt$ often increases from $t = 1/2$ day ($z = 0$) to $z = 3/2$ days ($z = 1$), possibly because discharge during this period is likely to be significantly controlled by storm flows, and then it keeps on decreasing [3,6]. We thus discard the recession peak and compute drainable storage for $z = 1$. Note that $z = 0$ does not necessarily mean the beginning of a recession event. In fact, it is widely acknowledged that there is no objective definition for the origin of a recession event (e.g., [11]). A recession curve is then considered if at least 3 data points starting from $z = 1$ (i.e. they belong to the AB part, see Table S1) show a robust dQ/dt–Q power law relationship ($R^2 > 0.7$) with $\alpha = 2 \pm 0.25$ (see, for e.g., Fig. 4(b)). Note that BC phases of observational $-dQ/dt$ vs. Q curves cannot be produced, even for the complete recession curves, as $-dQ/dt$ is very sensitive to errors during late recession periods. In total, we select 121 complete recession curves from the 27 basins for our analysis (see Table S1).

SD_z, drainable storage in a basin in the zth day after the beginning of a complete recession event can be computed for a complete recession curve by discretizing Eq. (3) as:

$$SD_z^o = \Delta t \cdot \sum_{i=z}^{Z} Q_i \tag{18}$$

where Z is the number of days for which the recession event lasts or the timescale of the recession curve (i.e. $T = Z$ days). The superscript o suggests that Eq. (18) deals with observed drainable storage. The observed recession curves display drainable storage–discharge patterns very similar to those of the geomorphic recession curves. The AB parts of observed recession curves display exponential SD–Q relationship as predicted by Eq. (12) (see Fig. 5(b)). The BC parts exhibit power law SD–Q relationship with exponent nearly equal to 2 (see Fig. 6(b)), though not in all cases because of the dominance of errors during this phase. It should be noted here that the discontinuation of exponential SD–Q relationship was also reported in some past studies (e.g., [2,16]). However, none of those studies had investigated when and how the discontinuation occurs and, more importantly, its physical significance.

Now we focus our attention on evaluating the applicability of Eq. (17). Particularly, for each recession curve we obtain observed SD for $z = 1$ (SD_1^o) following Eq. (18). Then we follow least square linear regression method and compute k for each recession curve by fitting a line with slope $\alpha = 2$ to the selected early recession period ($Q, -dQ/dt$) data points (see Table S1) in the double logarithmic plane [6,7]. Note that for modeling drainable storage for $z = 1$ (SD_1^m) using Eq. (17) the value of ψ needs to be determined from recession flow data as we do not have information on the values of $N(l)$ and $G(l)$ at $z = 1$. Therefore, we compute the value of ψ_1 (ψ for $z = 1$) for each recession curve by putting SD_1^o (computed by using Eq. (18)) in Eq. (17). The mean value ψ_1 obtained by considering all the recession curves is found to be nearly equal to 1 (1.02), which gives the expression for SD_1 as:

$$SD_1^m = \frac{1}{k}(1 + 0.5 \ln A) \quad (19)$$

We thereafter use Eq. (19) to compute SD_1^m for all the recession curves (i.e. by considering that $\psi_1 = 1$ for all the recession curves from all the basins). Fig. 7 shows the plot between SD_1^m and SD_1^o for the selected recession curves with correlation $R^2 = 0.96$ and the slope of SD_1^m vs. SD_1^o line is 1.06, which is quite close to 1, indicating that $SD_1^m \approx SD_1^o$ in general. Another convincing evidence is that the condition $\psi_1 = \ln(N_1/A) = 1$ (N_1 being N_1 at $z = 1$), which implies that $N_1 = e \cdot A$, is physically very plausible. According to Shreve [31], for random networks $A \approx 1/D_1^2 \cdot (2N_1 - 1)$, where D_1 is the active drainage density [7] at $z = 1$. For real basins N_1 is large enough to consider that $2N_1 - 1 \approx 2N_1$. Thus we obtain $D_1 = \sqrt{2 \cdot e} \approx 2.33\ \text{km}^{-1}$, which is quite realistic (see, for e.g., [8]). These observations strongly suggest that the proposed model is quite robust to predict drainable storage. Most importantly, Eq. (19) can be used to estimate drainable storage for incomplete recession events as it requires only the value of k, which can be computed by considering a few days of early recession flow discharge data.

It should be noted that various errors and uncertainties might affect estimation of drainable storage with the use of Eq. (19). Errors associated with discharge measurements might affect the computation k. Spatial rainfall variation might also introduce errors [4]. The value of ψ_1 might vary across events and basins. Though Eq. (19) is valid for $\alpha = 2$, our analysis suggests computation k by allowing some deviation for α, which might introduce some uncertainty. Thus, future studies need to estimate drainable storage for scenarios when α shows large deviation from 2 as Eq. (19) is meant for scenarios when α is nearly equal to 2. Furthermore, Eq. (19) is based on GRFM whose assumption that both q and c remain constant for a recession event might not be very accurate always. However, despite of all these limitations, the results obtained by using Eq. (19) seem to be promising. Potential implications of the observations in this study, therefore, include more accurate prediction in ungauged basins for better management of fresh water resources and ecosystems. The strong R^2 correlation between SD_1^m and SD_1^o may also be suggesting that the value of ψ_1 is universal, meaning that for a basin not considered in this study SD_1 can be estimated for any of its recession event by computing k from the early recession flow data and considering that $\psi_1 = 1$. This aspect needs to be rigorously tested as our dataset includes basins only from some selected parts of the US. Also note that the present study uses data from relatively small and homogeneous basins (drainage area ranging from 2.85 km^2 to 595.70 km^2) as it is not possible for us to obtain complete recession curves for large basins (say the Mississippi river basin). Thus, further investigation is required to analyze drainable storage–discharge relationships of large river basins that can even witness spatial variation in climate and geology (e.g., [30]). These objectives can possibly be achieved by introducing meaningful modifications to GRFM (e.g., [5,20]) and then obtaining a more robust expression for drainable storage.

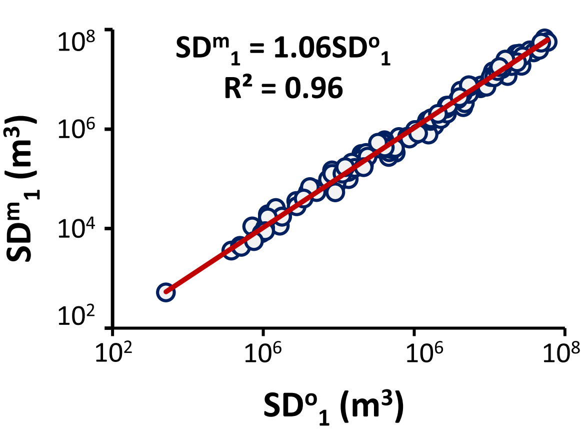

Fig. 7. Modeled drainable storage one day after the recession peak (SD_1^m) vs. observed drainable storage one day after the recession peak (SD_1^o) for the 121 complete recession curves selected in this study. Good correlation ($R^2 = 0.96$) indicates that the predictions are reliable.

5. Summary

The state of the art technologies do not enable us to instantly estimate water stored within a drainage basin. Therefore, hydrologists generally estimate storage by expressing it as a function of basin inflow and outflow components that are measurable. Our aim in this study was to estimate only the part of basin storage that transforms later into streamflows or 'drainable' storage. While it is straightforward to estimate drainable storage when recession events are 'complete', the same cannot be done when recession events are 'incomplete' due to short inter-storm time intervals. One possible solution is to use a hydrological model to construct 'pseudo-complete' recession curves from incomplete recession curves. Thus, it is essential to carefully analyze and model recession flow curves. Brutsaert and Nieber [11] suggested to expresses time rate of change of discharge (dQ/dt) as a function of Q, which commonly takes the form: $-dQ/dt = kQ^\alpha$. However, the problem is that dQ/dt–Q analysis is not practically possible for late recession periods when discharge is relatively low. Studies dealing with recession flows often consider early discharge observations for a recession event to compute α and assume that its value is constant for the whole recession period. Interestingly, α for early recession flows from natural basins is generally close to 2, in which case the constant α assumption suggests that storage is infinite for any finite discharge. That means α for late recession flows must be different from that of early recession flows. We addressed this issue in this study using geomorphological recession flow model (GRFM).

GRFM suggests that a $-dQ/dt$ vs. Q curve exhibits two distinct scaling regimes: AB, which corresponds to early recession flows, and BC, which corresponds to late recession flows. While the regime AB gives $\alpha \approx 2, \alpha$ for the regime BC is 0 according to the model. That means drainable storage–discharge relationship according to GRFM is exponential for AB regimes and power law with exponent equal to 2 for BC regimes. The transition thus makes drainable storage finite. Using data from 27 basins we found that the observed recession curves, like the modeled (geomorphic) recession curves, indeed display exponential discharge-storage relationship for AB parts and power law relationships for BC parts. We then followed suitable analytical methods and obtained a simplified expression for drainable storage one day after a recession peak (SD_1) as a function of k and basin area: $SD_1 = 1/k(1 + 0.5 \ln A)$. We observed that SD_1^m matches well with SD_1^o ($R^2 = 0.96$ and the slope of the linear regression line being 1.06). Despite many critical assumptions in the model formulation, the observations in this study are quite encouraging. Another interesting observation is that according to our model the active drainage density one day after a recession peak (D_1) should be approximately equal to 2.33 km^{-1}, which is quite realistic. Results in this study, therefore, are suggestive of the possibility that the proposed model can be used to estimate drainable storage for various important purposes related to water resources management, particularly when the recession events are incomplete.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.advwatres.2014.12.009.

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