# Triplet-Watershed for Hyperspectral Image Classification

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Abstract—Hyperspectral images (HSI) consist of rich spatial and spectral information, which can potentially be used for several applications. However, noise, band correlations and high dimensionality restrict the applicability of such data. This is recently addressed using creative deep learning network architectures such as ResNet, SSRN, and A2S2K. However, the last layer, i.e the classification layer, remains unchanged and is taken to be the softmax classifier. In this article, we propose to use a watershed classifier. Watershed classifier extends the watershed operator from Mathematical Morphology for classification. In its vanilla form, the watershed classifier does not have any trainable parameters. In this article, we propose a novel approach to train deep learning networks to obtain representations suitable for the watershed classifier. The watershed classifier exploits the connectivity patterns, a characteristic of HSI datasets, for better inference. We show that exploiting such characteristics allows the Triplet-Watershed to achieve state-of-art results in supervised and semi-supervised contexts. These results are validated on Indianpines (IP), University of Pavia (UP), Kennedy Space Center (KSC) and University of Houston (UH) datasets, relying on simple convnet architecture using a quarter of parameters compared to previous state-of-the-art networks.

The source code for reproducing the experiments and supplementary material (high resolution images) is available at https://github.com/ac20/TripletWatershed Code.

Index Terms—Hyperspectral Imaging, Watershed, Triplet Loss, Deep Learning, Classification

#### I. INTRODUCTION

TYPERSPECTRAL imaging has several applications ranging across different domains [1]. It has seen applications in earth observations [2], and land cover classification [3] etc. Hyperspectral datasets have rich information both spatially and spectrally. However, spectral and spatial correlations make a lot of such information redundant. One can obtain efficient representations using techniques such as band selection [4], [5] subspace learning [6], [7] multi-modal learning [8] low-rank representation [9].

Large number of bands, spatial and spectral feature correlations and curse of dimensionality make Hyperspectral image classification challenging. Conventional approaches use hand crafted features with techniques such as scale-invariant

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feature transform (SIFT) [10] sparse representation [11] principal component analysis [12] independent component analysis [13]. Classic approaches to classification such as support vector machines (SVM) [2], neural networks [14] and logistic regression [15] aimed at exploiting the spectral signatures alone. Using spatial features have been extremely useful to obtain better representations and higher classification accuracies [16]–[18], which the classic approaches ignore. Multiple kernel learning [19]–[21] use hand-designed kernels to exploit the spectral-spatial interactions. Deep learning approaches, especially CNNs, have been adapted to exploit the spectralspatial information. [22] proposes a 3D-CNN feature extractor to obtain combined spectral-spatial features. [23] adapts CNN to a two-branch architecture to extract joint spectral-spatial features. [24] used 3D volumes to extract spectral-spatial features, which may be improved using multi-scale approaches [25]. Spectral-spatial residual network (SSRN) proposed in [26] uses residual networks to remove the declining accuracy phenomenon. Residual Spectral-Spatial Attention Networks (RSSAN) [27] exploit the concept of attention to improve on SSRNs. [28] proposes Attention-Based Adaptive Spectral-Spatial Kernel Residual networks (A2S2K) by exploiting adaptive kernels. [29] uses graph convolution networks and [30] uses capsule networks. Most of these approaches tackle the problem of Hyperspectral image classification by considering novel architectures. Another prominent direction of research focusses on using unlabelled data for improving classification accuracies, referred to as semi-supervised learning. In [31], [32] the authors use hyperspectral data for improving inference on multispectral data. In [29] the authors propose a semisupervised approach to exploit multi-modal data for better inference. Graph Convolution Networks (GCN) have also been used to obtain state-of-art results on hyperspectral classification as evidenced by S2GCN [33] and DC-GCN (Dual Clustering GCN) [34]. Other approaches include local constraintbased sparse manifold hypergraph learning (LC-SMHL) [35], self-adaptive manifold discriminant analysis (SAMDA) [36], DLPNet [37] and adaptive residual convolutional neural network (ARCNN) [38].

In this article, we take a different route to propose a novel classifier based on the watershed operator. Watershed operator from Mathematical Morphology [39], [40] has been widely used for image segmentation, and, in particular, for Hyperspectral images [41], [42]. In [42], the authors combine (by majority voting) several watersheds computed on gradients of different bands. They observe that class-specific accuracies were improved by using the spatial information in the classifi-

TABLE I

OVERALL ACCURACY (OA) VS NUMBER OF PARAMETERS. OBSERVE THAT THE PROPOSED METHOD HAS VERY LESS NUMBER OF PARAMETERS BUT OUTPERFORMS THE CURRENT STATE-OF-THE-ART APPROACHES. IP INDICATES INDIAN PINES DATASET. UP DENOTES UNIVERSITY OF PAVIA DATASET AND KSC INDICATES THE KENNEDY SPACE CENTRE DATASET.

	# params	IP	UP	KSC
A2S2K [28]	370.7K	98.66	99.85	99.34
SSRN [26]	364.1K	98.38	99.77	99.29
ENL-FCN [50]	113.9K	96.15	99.76	99.25
ResNet34 [51]	21.9M	92.44	97.38	79.73
Triplet-Watershed	87.6K	99.57	99.98	99.72

cation for almost all the classes, a result that we are going to confirm in the present paper. To our knowledge, watersheds have not been used in conjunction with current state-of-art neural networks in the context of hyperspectral images. We propose a novel approach to achieve this in the current article.

In [43] the watershed operator is adapted to edge-weighted graphs. It is shown there that the watershed is closely related to the minimum spanning tree (MST) of the graph. Watersheds have also been highly successful as a post-processing tool for image segmentation [44]–[46]. In [47] the authors learn a representation suitable for MST-based classification. In [48] the authors learn a representation suitable to mutex-watershed, a different version of the watershed.

Departing from images, in our previous work [49] we have proposed to use the watershed operator as a generic classifier. We showed that it obtains a *maximum margin partition* similar to the support vector machine. We further showed that ensemble watersheds obtain comparable performance to other classifiers such as random forests. In this article we propose a novel approach, simple and efficient, called *Triplet-Watershed* to learn representations (also known as embeddings) suitable for the watershed classifier.

Why watershed classifier? Previous work on hyperspectral image classification, as discussed above, establish that one must use both spatial and spectral aspects to obtain good classifiers. They achieve this with creative approaches to design neural networks such as adaptive kernels, attention mechanism, etc. However, most of these still use conventional softmax classifier. The watershed classifier naturally uses spatial information for inference. Thus, it allows us to use simpler networks for representation. Table I shows the overall accuracy scores obtained by our approach and other state of art methods. It also shows the number of parameters used. Observe that Triplet-Watershed parameters are just 25% of those of the current state-of-art (A2S2K) approach.

The main contributions of this article are the following.

- (i) We propose a novel approach, namely *the Triplet-Watershed*, to learn a representation suitable to the watershed classifier. This representation is referred to as *watershed representations* in the rest of the article.
- (ii) The Triplet-Watershed achieves state-of-art results on the hyperspectral datasets with very simple networks, using much fewer parameters than the previous state-of-the-art approaches as described in table I.

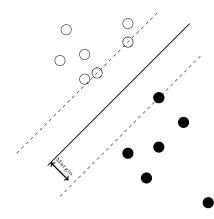


Fig. 1. Illustration of maximum margin for support vector machines (SVM) [49]. The key observation is - The margin is defined as the minimum distance between the training point labelled 0 and what would be labelled 1 after classification. And vice versa. The aim of the (linear) SVM classifier is to obtain a decision boundary that maximizes the margin. This can be extended to obtain a maximum-margin partition on an edge-weighted graph. Using (2), a solution of this is provided by the watershed classifier.

- (iii) The same Triplet-Watershed approach can be used for both supervised and semi-supervised tasks without any modification, still leading to state-of-the-art results compared to previous approaches.
- (iv) The framework used here to obtain representations is not restricted to watershed classifiers. It can be extended to use with other classifiers such as random forest or knearest neighbours as well, although watershed results outperform other classifiers on our datasets.
- (v) The main insight of our paper is that enforcing spatial connectivity (achieved thanks to the watershed classifier) during the training is a relevant constraint for hyperspectral classification.

Overview: Section II reviews the watershed classifier and the required terminology for the rest of the article. In section III we design the neural net (NN) and the training procedure to learn watershed representations. Section IV provides empirical analysis.

#### II. WATERSHED CLASSIFIER

The watershed classifier is defined on an edge-weighted graph. We follow the exposition as given in [49]. G=(V,E,W) denotes the edge-weighted graph. Here V denotes the set of vertices, E denotes the set of edges which is a subset of  $V\times V$  and  $W:E\to\mathbb{R}^+$  denotes the edge weight assigned to each edge. We assume that the edge weights are all positive in this article.

The (two-class) classification problem on the edge-weighted graph is stated as - Let  $X_0, X_1 \subset V$  denote the labelled subset of vertices labelled 0 and 1 respectively. Classification problem requires a partition of  $V = M_0 \cup M_1$  with  $M_0 \cap M_1 = \emptyset$ . With an additional constraint that  $X_0 \subset M_0$  and  $X_1 \subset M_1$ . Here  $M_0$  denotes all the vertices labelled 0 after classification and  $M_1$  denotes all the vertices labelled 1. We also assume there

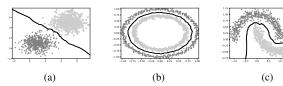
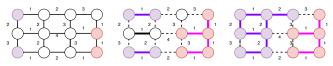


Fig. 2. Figure illustrating the watershed boundaries [49]. Observe that in all these cases the boundary is in-between the classes. Also, it is in the middle of the zero density (no points exist) regions. This maximizes the margin between the boundaries and the classes. This is consistent with the maximum margin principle of SVM.



(a) Original Graph

(b) Intermediate Step

(c) Watershed Labels

Fig. 3. Illustrating the watershed classifier. Let (a) denote the edge-weighted. The two distinct colours indicate two different classes. No colour indicates that the vertex is not yet labelled. (b) denotes the graph obtained by adding edges with weight 1. Each vertex is given a label accordingly. (c) denotes the graph obtained by adding the edges with weight 2 and Propagating the labels. Observe that all the points are now labelled and hence the algorithm terminates.

exists a dissimilarity measure  $\rho(x,y)$  between two vertices  $x,y\in V$ . This measure extends to subsets as

$$\rho(X,Y) = \min_{x \in X, y \in Y} \rho(x,y) \tag{1}$$

where X,Y are arbitrary subsets of V. Observe that there exist several partitions of  $V=M_0\cup M_1$  which satisfy the above conditions. Of these partitions, we only use the *Maximum Margin Partitions*, i.e the partitions which maximize

$$\min\{\rho(X_0, M_1), \rho(X_1, M_0)\}\tag{2}$$

This follows from the maximum margin principle of support vector machines (SVM). From figure 1, a key observation can be made - The margin for the SVM is the minimum distance between training point labelled 0 and what would be labelled 1 after classification. And vice versa. Linear SVM tries to obtain the boundary to maximize this margin. This can be extended to the edge-weighted graphs using (2).

The Watershed Classifier is defined by considering the dissimilarity measure to be

$$\rho(x,y) := \rho_{max}(x,y) = \min_{\pi \in \Pi(x,y)} \max_{e \in \pi} W(e)$$
 (3)

where  $\pi$  denotes a specific path between x,y.  $\Pi$  denotes the set of all possible paths.  $\rho_{max}$  is sometimes referred to as pass value.

If each edge-weight indicates the height of the corresponding edge, then  $\rho_{max}(x,y)$  indicates the minimum height one has to climb to reach y from x. When the points belong to a Euclidean space, the edge weight is given by Euclidean distance. That is, the edge weight indicates the distance between the points. Hence,  $\rho_{max}(x,y)$  would indicate the minimum "jump" one has to make to reach y from x. Thus, the boundaries (in cases where the classes are separable) would be along the low-density regions between classes. This is

illustrated in Figure 2. In all the cases, the boundary is between the classes such that we have the maximum margin. This is consistent with the maximum margin principle of SVM.

**Remark:** One can replace the pass value in (3) with several other measures, leading to different classifiers. Detailed analysis of replacing pass value with other measures is out of scope for the present article and may be considered for future work. For instance, using the Image Foresting Transform (IFT) [52] leads to a classifier similar to the one proposed in [53]. Few such techniques are discussed in [49].

Given the edge-weighted graph, the **Watershed algorithm** extends the Maximum Margin Partition principle to several classes and obtains the labels using the UNIONFIND data structure. This is described in algorithm 1.

#### **Algorithm 1** Watershed clustering algorithm [49]

**Input:** edge-weighted graph G = (V, E, W). A subset of labelled points  $V_l \subset V$ .

**Output:** Labels for each of the vertices L

- 1: Sort the edges E in increasing order w.r.t W.
- 2: Initialize the union-find data structure UF,
- 3: **for**  $e = (e_x, e_y)$  in sorted edge set E **do**
- 4: **if** both  $e_x$  and  $e_y$  are labelled **then**
- 5: do nothing
- 6: **else**
- 7: UF.union $(e_x, e_y)$
- 8: Assign same label for  $e_x$  and  $e_y$ .
- 9: end if
- 10: end for
- 11: Label each vertex of the connected component using labels  $V_l$ .
- 12: return Labels of the vertices.

Observe that step (10) is possible since each connected component would have exactly one unique label. One can see that watershed clustering is a semi-supervised algorithm, in the sense that it propagates the known labels to points with unknown label.

To illustrate the watershed classifier consider the simple edge-weighted graph in figure 3a. The two distinct colours indicate two classes. No colour indicates that the vertex is not yet labelled. In the first step, the least edge-weight is 1. Adding all these edges (thick edges in figure 3b) gives 4 distinct components. Each component is labelled according to the label present within the component. In case there exists no label, then label assignment is not yet carried out. We then add the edges with weight 2, and label the points accordingly. Observe that there are no more unlabelled points and hence the algorithm terminates.

In practice, it has been observed that ensemble techniques improve the robustness of watershed classifier. This is achieved using only a subset of labelled points and only a subset of features and taking the weighted average. Details can be found in [49]. We refer to these two approaches as *single watershed classifier* and *ensemble-watershed classifier*.

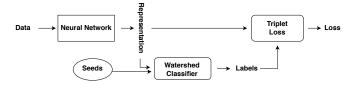


Fig. 4. Schematic of learning representations for the watershed classifier. Using a generic neural network we obtain the representation for the dataset. These representations are fed into the watershed classifier to obtain the labels using the seeds. Using the labels and the representation, we use triplet loss to compute the loss and also for obtaining the parameters for the neural network. Observe that the watershed classifier needs to be computed at every epoch.

## III. LEARNING REPRESENTATIONS FOR THE WATERSHED CLASSIFIER

The previous section described how one can obtain the labels using the watershed classifier. In [49], it was shown that this compares reasonably well to other classifiers such as SVM, random forests, etc. However, observe that this classifier has *no trainable parameters*. In this section, we develop an approach to train a neural network for learning representations suitable to the watershed classifier.

A key observation is - Watershed classifier reduces the distances within each component and increases the distance across components. This leads to the schematic in figure 4. First, we use a generic neural network to obtain the representations for the dataset. These representations, along with a subset of labelled points, are used with the watershed classifier to obtain the labels. Using these labels, we obtain a metric-learning loss to decide if two pixels are either in the same component (same label) of the watershed or in two different components (different label). More precisely, we use triplet loss [54], [55] to learn the watershed representation. For training, this cost is minimized using standard autograd packages such as pytorch.

Why schematic in figure 4 learns watershed representations? Triplet loss function uses {(anchor, postive, negative)} triplets for computation of the cost. It compares an anchor-input to a positive-input and a negative-input. The distance from the anchor-input to the positive-input is minimized, and the distance from the anchor-input to the negative-input is maximized using the cost

$$\min\{d(\text{anchor}, \text{positive}) - d(\text{anchor}, \text{negative}) + \alpha\}_{+}$$
 (4)

where  $\{*\}_+$  denotes the function  $\max\{0,*\}$ . By enforcing the order of distances, triplet loss models embed in the way that a pair of samples with the same label are smaller in distance than those with different labels. When watershed labels are used to obtain  $\{(\text{anchor}, \text{postive}, \text{negative})\}$  triplets, this leads to representations that are compatible with the watershed classifier.

Remark (Supervised vs Semi-Supervised): Recall that the watershed classifier uses a subset of training points (referred to as seeds) to obtain the labels of other training points. These labels are then used to the train the network with triplet loss. However, in the case of semi-supervised learning, unlabelled data is also available at train time. These points can be labelled and be used to train the network. In this article we

use the semi-supervised approach, randomly choosing some seeds for the watershed classifier that iteratively propagates their labels to their most resembling neighbours, obtaining the connected components. Hence, the combination of watershed clustering and triplet loss ensures that points with the most resembling representations are indeed clustered together, in the same connected component.

#### Training Dynamics

To summarize the entire training procedure of Triplet-Watershed, at each epoch

- 1) Obtain the representations for all the points using the neural network.
- We consider a randomly chosen subset of labelled points as seeds
- Propagate the labels to all points using the watershed classifier
- 4) Use the watershed labels to generate {(anchor, positive, negative)} triplets
- 5) Use the triplet loss to train the neural network.

Few obvious questions follow - (a) When would the training converge? (b) What is the steady-state obtained?

Note that the training would converge when there would be no further improvement in the triplet-loss. At this stage, the out-of-box score<sup>1</sup> of the watershed classifier would not improve as well. This implies that - all pairs of points with the same labels and within the same component have similar representation. Hence, we obtain 100% out-of-box accuracy<sup>2</sup> with watershed classifier.

Remark (Overfitting): Traditional machine learning advices against reaching 100% training accuracy as the models might be overfitting. However, recent deep learning trends point to the contrary. Several deep learning models can indeed fit random data with 100% accuracy [56]. It is still an open question to understand the generalization ability of these models. However, few observations point to the *inductive bias* [57] as the reason behind good generalization. In our case, the inductive bias is dictated by the graph constructed from the data.

Also, note that during training we use a single watershed classifier. While, at inference, we use an ensemble-watershed classifier. This ensures robustness during inference.

Remark (Complexity): Two main steps can be identified in the above procedure - (i) Obtaining a representation of the points and (ii) Propagating the labels using watershed. Time complexity for obtaining the representation is dictated by matrix multiplications with the neural network. This can easily be parallelized using GPU. Empirical study of the time taken for this is discussed in the following section. Table XI shows the actual time taken for both training and evaluation. Propagation of labels is done using binary partition trees and can be performed in quasi-linear time [58]. We use the routines available at [59] for implementation.

<sup>&</sup>lt;sup>1</sup>Accuracy on the training data excluding the seeds

<sup>&</sup>lt;sup>2</sup>Here we assume that there exists at least one seed per component

#### IV. EMPIRICAL ANALYSIS

In this section, we explore the application of the watershed classifier to the hyperspectral image classification task. We use the standard evaluation metrics for comparison:

- (i) Overall Accuracy (OA): it measures the overall accuracy across all samples, not considering the class imbalance.
- (ii) Average Accuracy (AA): it measures the average accuracy across the classes and
- (iii) Kappa Coefficient ( $\kappa$ ): it measures how well the estimates and groundtruth labels correspond, taking into account agreement by random chance.

Four datasets are used for comparison.

- Indian Pines (IP): Gathered by the Airborne Visible/Infrared Imaging Spectrometer (AVIRIS [60]) sensor over the test site in North-western Indiana. This data set contains 224 spectral bands within a wavelength range of 0.4 to 2.5 × 10<sup>-6</sup> meters. The 24 bands covering region of water absorption are removed. The image spatial dimension is 145 × 145, and there are 16 classes not all mutually exclusive.
- Kennedy Space Centre (KSC): The Kennedy Space Center (KSC) data set was gathered on March 23, 1996 by AVIRIS [60] with wavelengths ranging from 0.4 to 2.5 × 10<sup>-6</sup> meters. 176 spectral bands are used for analysis after removal of some low signal-to-noise ratio (SNR) bands and water absorption bands. 13 classes representing the various land cover types that occur in this environment are defined for the site.
- University of Pavia (UP): Acquired by the ROSIS [61] sensor during a flight campaign over Pavia, northern Italy. The number of spectral bands is 103 for Pavia University and is of size 610×610 pixels. The ground truth identifies 9 classes.
- University of Houston (UH): This dataset was acquired over the University of Houston campus and the neighbouring urban area. This dataset was captures with a spatial resolution of 2.5m and with 144 spectral bands in the 380 nm to 1050 nm region. This has 15 groundtruth classes. The dataset can be obtained from https://hyperspectral.ee.uh.edu/?page\_id=459<sup>3</sup>.

We preprocess the datasets using principal component analysis (PCA) [62] to obtain orthogonal components. We use 200 principal components for IP, 176 for KSC, 103 for UP and 144 for UH datasets. The train/test split is obtained randomly using 10% for training and 90% for testing.

*Graph Construction:* Note that the watershed classifier is defined on edge-weighted graphs. This is constructed as follows

- The set of vertices V is taken to be the set of all the pixels in the dataset ignoring the {labels = 0} class. Since, these points do not have any groundtruth labels.
- The edge set E is taken to be the union of 4-adjacency edges induced by the vertex set V (on the image) and edges obtained by EMST (Euclidean Minimum Spanning Tree [63]) for Indianpines (IP), University of Pavia (UP)

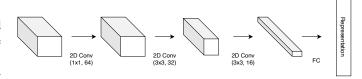


Fig. 5. Neural Network architecture used for obtaining the representations. The architecture is composed of 3 convolution layers followed by a fully connected layer to get the representation. Batch normalization is performed before each layer for efficient training. The number of parameters is 87K.

- and Kennedy Space Centre (KSC), and K-Neighbour edges with k=50 for University of Houston (UH) dataset. The EMST and K-Neighbour edges are obtained by considering the top 32 principal components.
- Given a representation obtained thanks to the neural network, the edge weights are computed using Euclidean distance. This representation (and hence the edge weights themselves) is updated at every epoch during training, while the edge set itself is never updated.

An illustration of the graph construction procedure is provided in appendix A.

In all the experiments we use the neural net architecture as shown in figure 5. We consider a patch  $(11 \times 11 \times \#Bands)$  around each pixel of the input hyperspectral image, suitably padded with 0s. We use 3 conv2d layers and a fully-connected layer to obtain the representation. These representations are then used for watershed classification and training. All models are trained using stochastic gradient descent (SGD) with cyclic learning rates [65]. We use 40% of the training data as seeds for the watershed classifier. The default weight initialization by pytorch [66] is used. We use 64 as the dimension for the representations. All accuracies are reported in the format mean  $\times 100\% \pm$  stdev to be consistent with [28]. The code is available at https://github.com/ac20/TripletWatershed Code.

Remark on evaluation: Different kind of evaluations of possible - Random train/test split or Patch-based evaluation as proposed in [67]. Here we use the former since - (i) Patch-based evaluation does not recommend using connectivity patterns, while watershed classifier is designed to exploit such patterns, (ii) Irrespective of the evaluation procedure, we remain consistent with baseline methods (A2S2K, SSRN). Hence, the observations in this article still remain valid.

### A. Supervised Classification

Firstly, we provide the results of Triplet-Watershed for supervised classification. We compare our approach with standard baselines (SVM [2] and Random Forest [64]), and also with the two recent state-of-art methods SSRN [26] and A2S2K [28]. Tables II, III, IV show the results (OA, AA,  $\kappa$ ) obtained. The train test splits per class are described in these tables. Note that Triplet-Watershed outperforms existing state-of-art A2S2KResNet [28] and other approaches in several aspects. This can be attributed to the fact that - Triplet Watershed exploits the connectivity patterns (edges within the pixels) in the dataset to propagate labels. Other approaches treat each pixel as a separate entity

<sup>&</sup>lt;sup>3</sup>Accessed on 30 April 2021.

TABLE II OVERALL ACCURACY (OA), AVERAGE ACCURACY (AA), AND KAPPA( $\kappa$ ) VALUES ON INDIANPINES (IP) DATASET USING 10% OF SAMPLES FOR TRAINING.

				Classic approa	nches		Deep-Learning approa	ches
Class	Train	Test	RF [64]	SVM [2]	Ensemble-Watershed [49]	SSRN [26]	A2S2K [28]	Triplet-Watershed
1	4	42	$28.46 \pm 0.061$	$51.22 \pm 0.190$	$41.43 \pm 0.2079$	$57.78 \pm 0.423$	$97.56 \pm 0.034$	$100.00 \pm 0.0000$
2	142	1286	$56.63 \pm 0.024$	$81.22 \pm 0.037$	$81.07 \pm 0.0202$	$98.37 \pm 0.012$	$98.62 \pm 0.010$	$98.62 \pm 0.0151$
3	83	747	$48.42 \pm 0.013$	$65.82 \pm 0.013$	$71.49 \pm 0.0250$	$97.47 \pm 0.010$	$98.58 \pm 0.006$	$100.00 \pm 0.0000$
4	23	214	$33.49 \pm 0.025$	$57.75 \pm 0.041$	$45.70 \pm 0.0327$	$99.12 \pm 0.010$	$98.29 \pm 0.014$	$100.00 \pm 0.0000$
5	48	435	$85.21 \pm 0.025$	$90.04 \pm 0.014$	$92.78 \pm 0.0286$	$97.79 \pm 0.013$	$99.02 \pm 0.003$	$97.98 \pm 0.0254$
6	73	657	$92.64 \pm 0.027$	$96.25 \pm 0.006$	$98.57 \pm 0.0033$	$98.50 \pm 0.010$	$98.71 \pm 0.010$	99.97 $\pm$ 0.0006
7	2	26	$2.67 \pm 0.038$	$73.33 \pm 0.019$	$99.17 \pm 0.0167$	$66.67 \pm 0.471$	$93.10 \pm 0.097$	$100.00 \pm 0.0000$
8	47	431	$97.67 \pm 0.015$	$97.98 \pm 0.006$	$98.14 \pm 0.0075$	$96.45 \pm 0.029$	$98.83 \pm 0.016$	$100.00 \pm 0.0000$
9	2	18	$9.26 \pm 0.094$	$50.00 \pm 0.045$	$37.50 \pm 0.1854$	$56.25 \pm 0.418$	$74.26 \pm 0.038$	$100.00 \pm 0.0000$
10	97	875	$60.91 \pm 0.047$	$73.87 \pm 0.018$	$85.81 \pm 0.0227$	$98.33 \pm 0.009$	$98.21 \pm 0.016$	$99.75 \pm 0.0040$
11	245	2210	$87.88 \pm 0.019$	$82.90 \pm 0.012$	$86.68 \pm 0.0105$	$99.08 \pm 0.005$	$99.09 \pm 0.001$	$99.61 \pm 0.0054$
12	59	534	$41.26 \pm 0.030$	$74.91 \pm 0.043$	$69.51 \pm 0.0182$	$98.46 \pm 0.009$	$98.37 \pm 0.013$	$99.89 \pm 0.0022$
13	20	185	$90.09 \pm 0.040$	$96.94 \pm 0.021$	$99.35 \pm 0.0079$	$100.0 \pm 0.000$	$99.80 \pm 0.002$	$100.00 \pm 0.0000$
14	126	1139	$95.46 \pm 0.014$	$93.82 \pm 0.010$	$92.59 \pm 0.0085$	$98.63 \pm 0.010$		$100.00 \pm 0.0000$
15	38	348	$41.11 \pm 0.029$	$60.42 \pm 0.044$	$54.48 \pm 0.0396$	$99.24 \pm 0.005$	$97.86 \pm 0.013$	$100.00 \pm 0.0000$
16	9	84	$79.37 \pm 0.030$	$91.27 \pm 0.054$	$79.29 \pm 0.1163$	$95.63 \pm 0.062$		$98.10\pm0.0267$
OA	1018	9231	$72.98 \pm 0.006$	$82.00 \pm 0.006$	$83.75 \pm 0.0076$	98.38 ± 0.004	$98.66 \pm 0.004$	99.57 ± 0.0026
AA			$59.41 \pm 0.005$	$77.36 \pm 0.019$	$77.10 \pm 0.0228$	$91.11 \pm 0.080$	$96.59 \pm 0.003$	$99.62 \pm 0.0029$
κ			$0.6862 \pm 0.007$	$0.7941 \pm 0.007$	$0.8143 \pm 0.0086$	$0.9815 \pm 0.003$	$0.9848 \pm 0.005$	$0.9951 \pm 0.0030$

TABLE III

OVERALL ACCURACY (OA), AVERAGE ACCURACY (AA), AND KAPPA( $\kappa$ ) VALUES ON UNIVERSITY OF PAVIA (UP) DATASET USING 10% OF SAMPLES FOR TRAINING.

				Classic approaches		De	Deep-Learning approaches		
Class	Train	Test	RF [64]	SVM [2]	Ensemble-Watershed [49]	SSRN [26]	A2S2K [28]	Triplet-Watershed	
1	663	5968	$91.11 \pm 0.007$	$94.30 \pm 0.008$	$94.34 \pm 0.0032$	$99.85 \pm 0.001$	$99.91 \pm 0.000$	$100.0 \pm 0.000$	
2	1864	16785	$98.11 \pm 0.003$	$97.65 \pm 0.002$	$95.24 \pm 0.0051$	$99.98 \pm 0.000$	$99.99 \pm 0.000$	$100.0\pm0.000$	
3	209	1890	$67.71 \pm 0.014$	$81.26 \pm 0.018$	$69.39 \pm 0.0151$	$99.68 \pm 0.003$	$99.88 \pm 0.001$	$99.8 \pm 0.004$	
4	306	2758	$88.20 \pm 0.006$	$94.63 \pm 0.004$	$78.69 \pm 0.0058$	$99.92 \pm 0.001$	$99.95 \pm 0.001$	$99.96 \pm 0.001$	
5	134	1211	$98.93 \pm 0.002$	$99.20 \pm 0.002$	$87.46 \pm 0.0110$	$99.94 \pm 0.000$	$100.0 \pm 0.000$	$100.0 \pm 0.000$	
6	502	4527	$72.14 \pm 0,022$	$90.58 \pm 0,008$	$61.37 \pm 0.0111$	$99.95 \pm 0.001$	$99.91 \pm 0,001$	$99.99 \pm 0.001$	
7	133	1197	$75.69 \pm 0.017$	$85.71 \pm 0.011$	$75.49 \pm 0.0295$	$100.0\pm0.000$	$100.0 \pm 0.000$	$100.0\pm0.000$	
8	368	3314	$89.64 \pm 0.013$	$88.20 \pm 0.003$	$74.65 \pm 0.0044$	$98.28 \pm 0.015$	$98.88 \pm 0.006$	99.97 $\pm$ 0.001	
9	94	853	$99.77 \pm 0.002$	$99.84 \pm 0.001$	$99.77 \pm 0.0015$	$99.39 \pm 0.003$	$99.78 \pm 0.003$	$\textbf{100.0}\pm\textbf{0.000}$	
OA	4273	38503	90.41 ± 0.001	$94.19 \pm 0.002$	86.13 ± 0.0023	$99.77 \pm 0.001$	99.85 ± 0.001	99.98 ± 0.001	
AA			$86.81 \pm 0.002$	$92.38 \pm 0.003$	$81.82 \pm 0.0039$	$99.66 \pm 0.001$	$99.81 \pm 0.001$	$99.97 \pm 0.001$	
$\kappa$			$0.8710 \pm 0.002$	$0.9229\pm0.002$	$0.8136 \pm 0.0030$	$0.9969 \pm 0.001$	$0.9981 \pm 0.001$	$\textbf{0.9998}\pm\textbf{0.001}$	

TABLE IV OVERALL ACCURACY (OA), AVERAGE ACCURACY (AA), AND KAPPA( $\kappa$ ) VALUES ON KENNEDY SPACE CENTRE (KSC) DATASET USING 10% OF SAMPLES FOR TRAINING.

				Classic approa	nches	D	eep-Learning approa	iches
Class	Train	Test	RF [64]	SVM [2]	Ensemble-Watershed [49]	SSRN [26]	A2S2K [28]	Triplet-Watershed
1	76	685	$94.79 \pm 0.012$	$95.43 \pm 0.023$	$96.23 \pm 0.0085$	$99.95 \pm 0.001$	$99.95 \pm 0.001$	100.0 ± 0.0000
2	24	219	$81.58 \pm 0.047$	$83.71 \pm 0.012$	$89.59 \pm 0.0247$	$100.0\pm0.000$	$98.68 \pm 0.019$	$100.0\pm0.0000$
3	25	231	$86.09 \pm 0.020$	$78.41 \pm 0.218$	$83.98 \pm 0.0341$	$99.66 \pm 0.005$	$98.72 \pm 0.012$	$100.0\pm0.0000$
4	25	227	$71.22 \pm 0.061$	$27.17 \pm 0.173$	$69.60 \pm 0.0406$	$91.22 \pm 0.047$	$94.27 \pm 0.042$	$96.56 \pm 0.0423$
5	16	145	$47.59 \pm 0.060$	$22.99 \pm 0.170$	$65.52 \pm 0.0474$	$100.0\pm0.000$	$94.46 \pm 0.050$	$99.86 \pm 0.0028$
6	22	207	$48.22 \pm 0.014$	$36.89 \pm 0.078$	$53.33 \pm 0.0526$	$98.45 \pm 0.022$	$99.82 \pm 0.003$	$99.52 \pm 0.0000$
7	10	95	$79.43 \pm 0.096$	$87.94 \pm 0.027$	$85.05 \pm 0.0234$	$95.42 \pm 0.050$	$99.61 \pm 0.005$	$100.0\pm0.0000$
8	43	388	$78.61 \pm 0.054$	$70.19 \pm 0.073$	$91.24 \pm 0.0297$	$99.80 \pm 0.003$	$100.0\pm0.000$	99.90 $\pm$ 0.0000
9	52	468	$89.46 \pm 0.011$	$85.33 \pm 0.021$	$93.08 \pm 0.0193$	$100.0\pm0.000$	$100.0\pm0.000$	$100.0\pm0.0000$
10	40	364	$88.43 \pm 0.034$	$78.88 \pm 0.069$	$92.64 \pm 0.0150$	$100.0\pm0.000$	$100.0\pm0.000$	$100.0\pm0.0000$
11	41	378	$95.58 \pm 0.014$	$93.81 \pm 0.008$	$94.44 \pm 0.0261$	$100.0\pm0.000$	$100.0\pm0.000$	$100.0\pm0.0000$
12	50	453	$82.63 \pm 0.032$	$86.98 \pm 0.009$	$86.98 \pm 0.0119$	$100.0\pm0.000$	$100.0\pm0.000$	$99.21 \pm 0.0159$
13	92	835	$99.60 \pm 0.002$	$\textbf{100.0}\pm\textbf{0.000}$	$99.69 \pm 0.0022$	100.0 $\pm$ 0.000	$\textbf{100.0}\pm\textbf{0.000}$	$100.0\pm0.0000$
OA	516	4695	$86.17 \pm 0.004$	$81.27 \pm 0.008$	89.54 ± 0.0038	99.29 ± 0.004	$99.34 \pm 0.0008$	99.72 ± 0.0023
AA			$80.25 \pm 0.004$	$72.90 \pm 0.021$	$84.72 \pm 0.0038$	$98.80 \pm 0.008$	$98.88 \pm 0.0018$	$99.62\pm0.0032$
$\kappa$			$0.8459 \pm 0.004$	$0.7909 \pm 0.009$	$0.8834 \pm 0.0042$	$0.9921 \pm 0.004$	$0.9927 \pm 0.001$	$0.9969 \pm 0.0026$

TABLE V OVERALL ACCURACY (OA), AVERAGE ACCURACY (AA), AND KAPPA( $\kappa$ ) VALUES ON UNIVERSITY OF HOUSTON (UH) DATASET USING 10% OF SAMPLES FOR TRAINING.

				Classic approa	iches	De	eep-Learning approa	iches
Class	Train	Test	RF [64]	SVM [2]	Ensemble-Watershed [49]	SSRN [26]	A2S2K [28]	Triplet-Watershed
1	125	1126	$82.52 \pm 0.0000$	$82.33 \pm 0.0000$	$93.68 \pm 0.0279$	$99.66 \pm 0.0012$	99.79 ± 0.0021	$98.99 \pm 0.0080$
2	125	1129	$83.30 \pm 0.0011$	$83.36 \pm 0.0000$	$81.97 \pm 0.0191$	$99.96 \pm 0.0004$	$100.0 \pm 0.0000$	$100.0 \pm 0.0000$
3	69	628	$97.62 \pm 0.0000$	$99.80 \pm 0.0000$	$99.90 \pm 0.0013$	$100.0 \pm 0.0000$	$100.0 \pm 0.0000$	$100.0 \pm 00000$
4	124	1120	$91.41 \pm 0.0027$	$98.95 \pm 0.0000$	$74.27 \pm 0.0240$	$99.66 \pm 0.0046$	$99.17 \pm 0.0095$	$100.0 \pm 0.0000$
5	124	1118	$96.49 \pm 0.0020$	$98.76 \pm 0.0000$	$82.15 \pm 0.0214$	$100.0 \pm 0.0000$	$100.0 \pm 0.0000$	$100.0 \pm 00000$
6	32	293	$99.30 \pm 0.0000$	$97.90 \pm 0.0000$	$92.22 \pm 0.0613$	$100.0 \pm 0.0000$	$100.0 \pm 0.0000$	$99.43 \pm 0.0080$
7	126	1142	$75.09 \pm 0.0020$	$77.42 \pm 0.0000$	$69.63 \pm 0.0272$	$99.10 \pm 0.0119$	$98.98 \pm 0.0088$	$99.65 \pm 0.0050$
8	124	1120	$33.04 \pm 0.0020$	$60.30 \pm 0.0000$	$78.25 \pm 0.0242$	$99.38 \pm 0.0016$	$99.72 \pm 0.0038$	$96.25 \pm 0.0338$
9	125	1127	$69.31 \pm 0.0042$	$76.77 \pm 0.0000$	$52.56 \pm 0.0159$	$99.30 \pm 0.0052$	$98.47 \pm 0.0101$	$97.96 \pm 0.0145$
10	122	1105	$44.11 \pm 0.0034$	$61.29 \pm 0.0000$	$63.66 \pm 0.0207$	$94.85 \pm 0.0152$	$94.90 \pm 0.0178$	$100.0 \pm 0.0000$
11	123	1112	$70.20 \pm 0.0020$	$80.55 \pm 0.0000$	$56.83 \pm 0.0379$	$99.23 \pm 0.0075$	$99.42 \pm 0.0040$	$99.07 \pm 0.0131$
12	123	1110	$54.81 \pm 0.0036$	$79.92 \pm 0.0000$	$54.77 \pm 0.0319$	$98.76 \pm 0.0028$	$99.46 \pm 0.0033$	$99.64 \pm 0.0000$
13	46	423	$60.23 \pm 0.0129$	$70.87 \pm 0.0000$	$06.52 \pm 0.0130$	$99.90 \pm 0.0013$	$99.01 \pm 0.0101$	$98.74 \pm 0.0089$
14	42	386	$99.32 \pm 0.0019$	$100.0 \pm 0.0000$	$94.15 \pm 0.0089$	$98.63 \pm 0.0193$	$100.0 \pm 0.0000$	$100.0 \pm 0.0000$
15	66	594	$97.25 \pm 0.0017$	$96.40 \pm 0.0000$	$98.55 \pm 0.0051$	$100.0\pm0.0000$	100.0 $\pm$ 0.0000	$\textbf{100.0}\pm\textbf{00000}$
OA			$73.02 \pm 0.0004$	$81.86 \pm 0.0000$	$72.50 \pm 0.0030$	$99.10 \pm 0.0013$	$99.12 \pm 0.0030$	99.25 ± 0.0039
AA			$76.93 \pm 0.0004$	$84.31 \pm 0.0000$	$73.27 \pm 0.0046$	$99.23 \pm 0.0016$	$99.26 \pm 0.0020$	$99.32 \pm 0.0031$
Kappa			$71.01 \pm 0.0003$	$80.42 \pm 0.0000$	$70.22 \pm 0.0033$	$99.03 \pm 0.0015$	$99.05 \pm 0.0033$	$99.19 \pm 0.0042$

TABLE VI OVERALL ACCURACY (OA), AVERAGE ACCURACY (AA), AND KAPPA( $\kappa$ ) VALUES ON INDIANPINES (IP) DATASET USING SEMI-SUPERVISED APPROACHES.

Class	Train	Test	S2GCN [33]	SSRN [26]	DC-GCN [34]	Triplet-Watershed
1	30	16	$100.0 \pm 0.0000$	$93.24 \pm 0.0263$	$100.00 \pm 0.0000$	$100.00 \pm 0.0000$
2	30	1398	$84.43 \pm 0.0250$	$76.63 \pm 0.0596$	$91.28 \pm 0.0360$	$91.69 \pm 0.0194$
3	30	800	$82.87 \pm 0.0553$	$68.78 \pm 0.0753$	$92.88 \pm 0.0396$	$95.25 \pm 0.0610$
4	30	207	$93.08 \pm 0.0195$	$87.64 \pm 0.0249$	$98.11 \pm 0.0151$	$100.00\pm0.0000$
5	30	453	$97.13 \pm 0.0134$	$86.72 \pm 0.0154$	$95.54 \pm 0.0339$	$98.63 \pm 0.0171$
6	30	700	$97.29 \pm 0.0127$	$92.05 \pm 0.0182$	$98.67 \pm 0.0104$	$100.00\pm0.0000$
7	15	13	$92.31 \pm 0.0000$	$95.66 \pm 0.0051$	$100.00\pm0.0000$	$100.00\pm0.0000$
8	30	448	$99.03 \pm 0.0093$	$95.90 \pm 0.0297$	$100.00\pm0.0000$	$100.00\pm0.0000$
9	15	5	$100.00\pm0.0000$	$100.00\pm0.0000$	$100.00\pm0.0000$	$100.00\pm0.0000$
10	30	942	$93.77 \pm 0.0373$	$82.42 \pm 0.0324$	$91.91 \pm 0.0378$	$98.22 \pm 0.0232$
11	30	2425	$84.98 \pm 0.0282$	$82.23 \pm 0.0288$	$91.79 \pm 0.0379$	$94.43 \pm 0.0229$
12	30	563	$80.05 \pm 0.0517$	$69.09 \pm 0.0436$	$90.17 \pm 0.0554$	$99.08 \pm 0.0185$
13	30	175	$99.43 \pm 0.0000$	$95.78 \pm 0.0075$	$99.65 \pm 0.0027$	$100.00\pm0.0000$
14	30	1235	$96.73 \pm 0.0092$	$86.52 \pm 0.0243$	$99.73 \pm 0.0066$	$99.87 \pm 0.0026$
15	30	356	$86.80 \pm 0.0342$	$73.12 \pm 0.0528$	$99.94 \pm 0.0016$	$100.00\pm0.0000$
16	30	63	$100.00\pm0.0000$	$86.21 \pm 0.0130$	$100.00\pm0.0000$	$99.37 \pm 0.0078$
OA		-	$89.4 \pm 0.0108$	$88.34 \pm 0.0173$	$94.65 \pm 0.1210$	96.74 ± 0.0194
AA			$92.9 \pm 0.0104$	$85.75 \pm 0.0069$	$96.85 \pm 0.0040$	$98.53 \pm 0.0098$
$\kappa$			$0.880 \pm 0.012$	$0.866 \pm 0.019$	$0.944 \pm 0.014$	$0.9627\pm0.0221$

TABLE VII OVERALL ACCURACY (OA), AVERAGE ACCURACY (AA), AND KAPPA( $\kappa$ ) VALUES ON UNIVERSITY OF PAVIA (UP) DATASET USING SEMI-SUPERVISED APPROACHES.

Class	Train	Test	S2GCN [33]	SSRN [26]	DC-GCN [34]	Triplet-Watershed
1	30	6601	$92.78 \pm 0.0379$	$98.80 \pm 0.0110$	$92.85 \pm 0.0351$	99.56 ± 0.0088
2	30	18619	$87.06 \pm 0.0447$	$98.45 \pm 0.0054$	$97.53 \pm 0.0140$	$100.00\pm0.0000$
3	30	2069	$87.97 \pm 0.0477$	$77.05 \pm 0.1024$	$97.94 \pm 0.0118$	$99.85\pm0.0084$
4	30	3034	$90.85 \pm 0.0094$	$83.02 \pm 0.0907$	$94.57 \pm 0.0109$	$99.99 \pm 0.0003$
5	30	1315	$100.00\pm0.0000$	$99.96 \pm 0.0009$	$99.49 \pm 0.0068$	$100.00\pm0.0000$
6	30	4999	$88.69 \pm 0.0264$	$87.03 \pm 0.0626$	$98.57 \pm 0.0278$	$99.99\pm0.0001$
7	30	1300	$98.88 \pm 0.0108$	$83.92 \pm 0.0897$	$100.00\pm0.0000$	$100.00\pm0.0000$
8	30	3652	$89.97 \pm 0.0328$	$88.41 \pm 0.0463$	$96.00\pm0.0277$	$92.15 \pm 0.1560$
9	30	917	$98.89 \pm 0.0053$	$99.97\pm0.0004$	$97.51\pm0.0140$	$100.00\pm0.0000$
OA			89.74 ± 0.0170	$92.81 \pm 0.0190$	96.87 ± 0.0111	99.20 ± 0.0129
AA			$92.80 \pm 0.0047$	$90.73 \pm 0.0226$	$97.16 \pm 0.0076$	$98.95 \pm 0.0165$
$\kappa$			$0.8665 \pm 0.020$	$0.9059 \pm 0.024$	$0.9677 \pm 0.012$	$\textbf{0.9894}\pm\textbf{0.0170}$

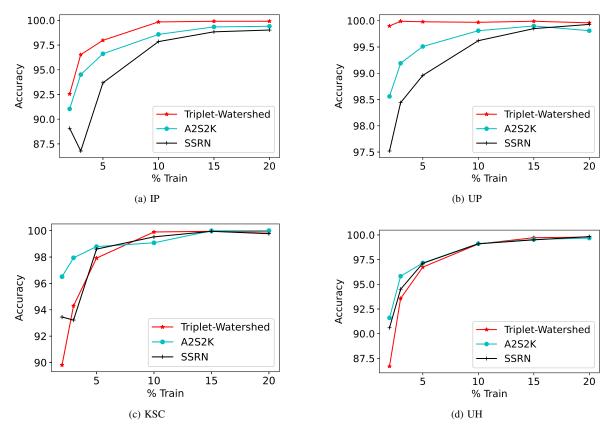


Fig. 6. Overall Accuracy (OA) vs % training data. We observe that Triplet-Watershed outperforms other approaches even at small sizes of training data for Indianpines and University of Pavia Dataset. IP denotes Indianpines dataset, UP denotes University of Pavia dataset, KSC denotes Kennedy Space Centre dataset and UH denotes University of Houston dataset.

TABLE VIII

COMPARISON OF TRIPLET-WATERSHED WITH TRIPLET-RANDOM-FOREST AND TRIPLET-K-NEAREST-NEIGHBORS. REPLACE WATERSHED CLASSIFIER WITH RANDOM FOREST AND KNN CLASSIFIER TO UNDERSTAND THE IMPORTANCE OF WATERSHED CLASSIFIER.

	Triplet-Watershed	Triplet-RF	Triplet-KNN
IN	$99.57\pm0.0026$	$91.46 \pm 0.011$	$90.86 \pm 0.013$
UP	$99.98 \pm 0.001$	$98.06 \pm 0.007$	$99.62 \pm 0.000$
KSC	$99.72\pm0.0023$	$87.80 \pm 0.039$	$82.38 \pm 0.031$
UH	$99.25 \pm 0.004$	$89.02 \pm 0.018$	$96.15 \pm 0.0086$

TABLE IX
MEAN AVERAGE PRECISION (MAP) SCORES FOR THE REPRESENTATIONS.
OBSERVE THAT TRIPLET-WATERSHED OBTAINS BETTER
REPRESENTATIONS THAN COMPETING APPROACHES ON ALL DATASETS.

	Triplet-Watershed	A2S2K [28]	SSRN [26]
IN	0.9819	0.9713	0.9135
UP	0.9970	0.9821	0.9703
KSC	0.9822	0.9837	0.9846
UH	0.9821	0.9799	0.9692

which would not exploit this observation. Other approaches treat each pixel as a separate entity which would not exploit this observation. Simple Ensemble-Watershed results are shown in the tables as well. Classification maps for Triplet-Watershed along with competing approaches are shown in

TABLE X TRIPLET-WATERSHED: ACCURACY VS EMBED DIMENSION. NOTE THAT DIFFERENCES ACROSS VARIOUS EMBEDDING DIMENSIONS ARE NOT SIGNIFICANT.

Dimension	KSC	IN	UP	UH
16	$99.53 \pm 0.0031$	$99.45 \pm 0.0025$	$99.95 \pm 0.0002$	$98.74 \pm 0.0034$
32	$99.70 \pm 0.0029$	$99.72 \pm 0.0010$	$99.97 \pm 0.0003$	$98.73 \pm 0.0018$
64	$99.54 \pm 0.0017$	$99.67 \pm 0.0011$	$99.98 \pm 0.0001$	$99.25 \pm 0.0039$
128	$99.72 \pm 0.0004$	$99.84 \pm 0.0009$	$99.97 \pm 0.0001$	$98.87 \pm 0.0025$

TABLE XI
RUN-TIMES (IN SECONDS) OF TRIPLET-WATERSHED AND OTHER
APPROACHES. OBSERVE THAT THE RUNNING TIME OF
TRIPLET-WATERSHED IS COMPARABLE TO OTHER APPROACHES.

	Time(s)	Triplet-Watershed	A2S2K [28]	SSRN [26]
IN	Train	520.56	829.23	779.33
	Test	3.77	10.55	11.44
UP	Train	791.22	2582.31	1964.66
	Test	46.23	47.33	33.02
KSC	Train	978.25	757.46	535.20
	Test	1.58	8.37	5.84
UH	Train	1460.15	947.73	1145.38
	Test	8.74	11.55	7.85

figures 8,9,10,11. High resolution stand-alone images can also be found in https://github.com/ac20/TripletWatershed\_Code/tree/main/img/classification\_maps.

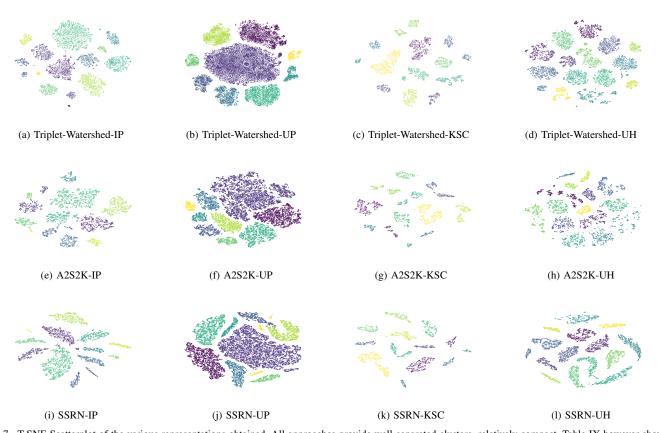


Fig. 7. T-SNE Scatterplot of the various representations obtained. All approaches provide well-separated clusters, relatively compact. Table IX however shows that triplet-watershed achieves a better precision (MAP score). IP denotes Indianpines dataset, UP denotes University of Pavia dataset, KSC denotes Kennedy Space Centre dataset and UH denotes University of Houston dataset.

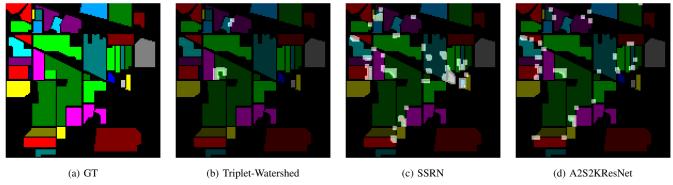


Fig. 8. Classification maps for Indianpines (IP) dataset. The main differences with respect to groundtruth have been highlighted. As one can observe, the number of errors of Triplet-Watershed is small compared to SSRN and A2S2K.

#### B. Semi-Supervised Classification

We compare the Triplet-Watershed with three recent state-of-art semi-supervised approaches - S2GCN [33], SSRN [26] and DC-GCN (Dual Clustering GCN) [34]. We consider 30 samples for training if the class size is greater than 30 and 15 if the class size is less than 30. Tables VI, VII show the results obtained. Observe that, once again, Triplet-Watershed obtains the state-of-art in several aspects.

#### C. Evaluation of Representation

Recall that accuracies in tables II-VII for Triplet-Watershed use ensemble watershed classifier. However, ensemble watershed exploits the connectivity patterns in the data. We now try to understand how well watershed representations compare with representations obtained by other approaches. Qualitatively, we use the TSNE [68] plots as in Figure 7. Note that there does not exist any major differences except that within a class, A2S2K and SSRN have "clumps" points while Triplet-Watershed has a uniform density. Quantitatively

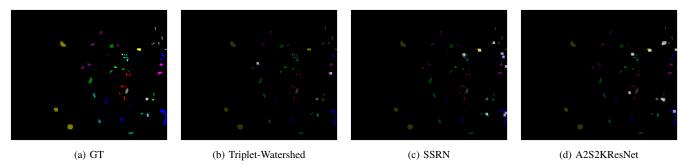


Fig. 9. Classification maps for Kennedy Space Centre (KSC) dataset. The main differences with respect to groundtruth have been highlighted. As one can observe, the number of errors of Triplet-Watershed is small compared to SSRN and A2S2K.

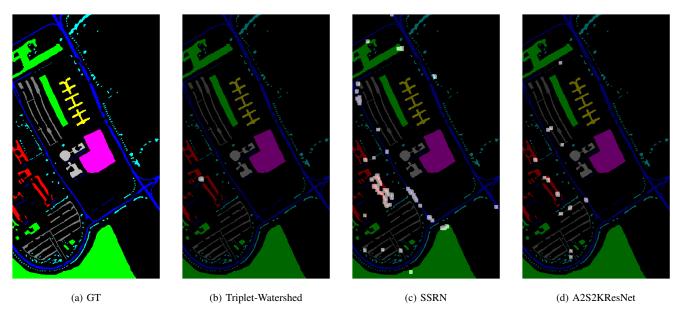


Fig. 10. Classification maps for University of Pavia (UP) dataset. The main differences with respect to groundtruth have been highlighted. As one can observe, the number of errors of Triplet-Watershed is small compared to SSRN and A2S2K.

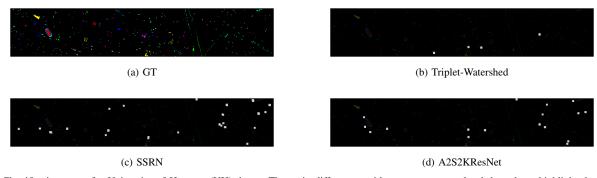


Fig. 11. Classification maps for University of Houston (UH) dataset. The main differences with respect to groundtruth have been highlighted. As one can observe, the number of errors of Triplet-Watershed is small compared to SSRN and A2S2K.

TABLE XII
TRIPLET-WATERSHED: ACCURACY VS PATCH SIZE.

		7	9	11	13
IN	OA	99.72 ± 0.0021	99.56 ± 0.0021	99.63 ± 0.0017	99.63 ± 0.0022
	AA	$99.72 \pm 0.0024$	$98.57 \pm 0.0180$	$99.82 \pm 0.0012$	$99.75 \pm 0.0009$
	κ	$0.9968 \pm 0.0024$	$0.9949 \pm 0.0023$	$0.9958 \pm 0.0020$	$0.9957 \pm 0.0025$
UP	OA	$99.96 \pm 0.0008$	$99.98 \pm 0.0002$	$99.99 \pm 0.0002$	$99.98 \pm 0.0002$
	AA	$99.93 \pm 0.0012$	$99.96 \pm 0.0005$	$99.98 \pm 0.0004$	$99.96 \pm 0.0005$
	$\kappa$	$0.9994 \pm 0.0010$	$0.9997 \pm 0.0003$	$0.9999 \pm 0.0002$	$0.9997 \pm 0.0003$
KSC	OA	$99.77 \pm 0.0023$	$99.96 \pm 0.0010$	$99.96 \pm 0.0010$	$99.96 \pm 0.0010$
	AA	$99.55 \pm 0.0050$	$99.95 \pm 0.0010$	$99.95 \pm 0.0010$	$99.95 \pm 0.0010$
	$\kappa$	$0.9975 \pm 0.0025$	$0.9995 \pm 0.0010$	$0.9995 \pm 0.0010$	$0.9995 \pm 0.0010$
UH	OA	$98.22 \pm 0.0024$	$98.78 \pm 0.0014$	$99.25 \pm 0.0011$	$99.23 \pm 0.0031$
	AA	$98.28 \pm 0.0038$	$98.89 \pm 0.0015$	$99.32 \pm 0.0013$	$99.26 \pm 0.0031$
	$\kappa$	$0.9807 \pm 0.0026$	$0.9868 \pm 0.0015$	$0.9919 \pm 0.0012$	$0.9915 \pm 0.0034$

we use the mean average precision (MAP) over all points. The computation procedure is as follows:

- 1) Given a data point  $x_k$ , we order all other data points  $\{y_i\}_i$  using an inverse function of distance,  $\exp(-\text{distance})$ .
- 2) Labels are assigned based on whether the points  $\{y_i\}_i$  belong to the same class as  $x_k$  or not with class label 1 and 0 respectively.
- 3) Average precision (AP) computes the area under the precision-recall curve.
- 4) The AP scores are averages over all points  $\{x_k\}_k$  to obtain the MAP score.

This procedure is as suggested in [69] to evaluate representations. The results are shown in Table IX. Observe that the watershed outperforms the current state-of-art techniques.

#### D. Ablation Study

We now study the importance of various aspects of Triplet-Watershed for the accuracies.

- 1) Accuracy vs % training data: Figure 6 shows the plots of overall accuracy (OA) vs % training data. For IP and UP datasets, it can be seen that Triplet-Watershed outperforms other approaches even at small sizes of training data. This can be attributed to the fact that the watershed classifier propagates the information to unlabelled nodes, which is in turn used for training. (See Figure 4). For optimal performance, the watershed classifier requires at least one labelled node per component. In cases of very small training data and many components, Triplet-Watershed does not perform well. This is the case for the KSC dataset at 2% and 3% training data, as shown in Figure 6. Detailed analysis of the underperformance of Triplet-Watershed at low train sizes for Kennedy Space Center (KSC) and University of Houston (UH) dataset can be found in appendix B.
- 2) Replacing Watershed With Other Classifiers: To illustrate the importance of the watershed classifier in the training pipeline (Figure 4), we replace it with Random Forest (RF) classifier and K-Nearest Neighbors (KNN) classifier with k=5, referring to these as Triplet-Random Forest and Triplet-K-Nearest-Neighbors. The results are shown in Table VIII. Firstly observe the dramatic improvement of accuracies with respect to vanilla classifiers (Tables II, III, IV). Also, observe that Triplet-Watershed outperforms the other techniques. This, once again, is attributed to the fact that watershed exploits the observation that classes in the groundtruth consist of connected components.

Remark: Both Random Forest (RF) and K-Nearest Neighbors (KNN) are considered for this experiment since the labels generated by these are not differentiable with respect to the input representations. This property is shared with the watershed classifier. However, Multi-layered perceptron (MLP) and Support vector machines (SVM) obtain labels using specific costs and are indeed differentiable with respect to their input representations. Hence, the latter approaches are not considered for comparison.

- 3) Accuracy vs embed dimension: Table X shows the effect of embedding dimension on accuracy. Observe that there does not exist any significant trend with respect to the embedding dimension. We use 64 as the default embedding dimension.
- 4) Accuracy Vs Patch Size: Recall that one of the hyperparameter of the approach is patch size The size of the window around the pixel. Table XII shows the results obtained by varying the patch sizes across different datasets. Observe that larger window size implies more information for inference and hence scope for better inference. Thus, as a rule of thumb, larger window size obtain better results. But, it also implies higher computational requirement. However in several cases increasing the window size beyond a threshold would not lead to significant improvements. For example, in table XII IN and UP datasets do not show much improvement with larger window sizes. UH dataset improves with larger window size, but no significant improvement is obtained by increasing the window size from 11 to 13.

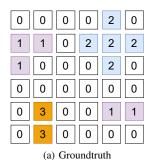
#### V. CONCLUSION

In this article, we proposed a novel approach to train for the watershed classifier. We refer to this as Triplet-Watershed. We show that the watershed classifier exploits the connectivity patterns in the datasets. This leads to huge performance gains compared to other approaches which use simple softmax classifier. We prove this empirically by comparing Triplet-Watershed with existing state-of-art deep learning approaches such as A2S2K [28], SSRN [26] and also classic approaches - RF [64] and SVM [2]. We also compare the current technique with semi-supervised approaches such as S2GCN [33] and DC-GCN [34]. In each case, we achieve better accuracy while using a quarter of the parameters of the previous state-of-the-art approaches.

## APPENDIX A CONSTRUCTING THE GRAPH ON HSI

Here, we illustrate the process of constructing the graph on HSI dataset. Figure 12a considers a simple hypothetical image with the groundtruth classes as shown. Figure 12b shows the graph obtained using the following steps:

- (i) Firstly, only points with groundtruth available, i.e  $\{labels \neq 0\}$  are considered. This can be trivially extended to other points depending on requirement. These points constitute the vertex set.
- (ii) The edge set is obtained by taking the union of (a) 4 adjacency edges denoted by colour black and (b) "other" edges which span across components. These "other" edges are constructed using Euclidean Minimum



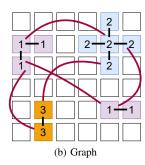


Fig. 12. Constructing the graph on HSI data. (a) shows a simple toy HSI data with groundtruth classes. Note that class 0 implies that groundtruth is not available. (b) illustrates the graph constructed by considering only points with {labels  $\neq 0$ } as vertices. 4-adjacency edges (black) along with other edges (red) spanning across components are considered. These "other" edges are constructed using techniques such as Euclidean Minimum Spanning Tree (EMST) or K-Neighbors graph.

Spanning Tree (EMST) for IP, UP, and KSC datasets. For UH dataset these edges are constructed using K-Neighbors graph with k=50.

The two main principles for selecting the graph are - (i) We require each label-induced subgraph<sup>4</sup> such that the number of connected components are as few as possible and (ii) We also require the number of edges to be as few as possible. Both these act against each other and the right combination is obtained through trial and error.

## APPENDIX B TRIPLET-WATERSHED AT SMALL TRAIN SIZES

Note that from figure 6, at low train sizes (2% and 3%, Triplet-Watershed performs better than A2S2KResNet and SSRN on IP, UP datasets. While, Triplet-Watershed is slightly inferior to A2S2KResNet and SSRN on KSC, UH datasets. In this section we analyze and explain this in detail.

There are two main reasons for the different behaviours of Triplet-Watershed at high (10%) and low (2%, 3%) train sizes - (i) At low train sizes, not all components within the data are covered and (ii) There aren't enough points near the boundary to allow for better separation. To understand this better, we perform a post-hoc analysis on UH and IP datasets.

For each label, (both groundtruth and prediction) we consider the subgraph induced by the vertices<sup>5</sup> of the given label. In this subgraph, we count the size of each connected component. Table XIII shows these values for UH/IP datasets, for groundtruth labels, and labels predicted for 10% and 2%. Both the above phenomenon can be observed in table XIII.

(i) Observe that for several classes in UH dataset, there exists small components for UH (example: class 1 with 178 points) which are not represented when only 2% of the data is considered for training. While, this happens for IP dataset (class 5, 147 points), it is relatively low in magnitude. This partly explains why we achieve better

- results at 10% train size. And also why IP performs better at 2% train size comparatively.
- (ii) The other main reason is Boundaries are not sufficiently represented at 2% train size. As an example of this, consider class 13 for UH dataset which has a single component 469 points. At 2% train size, this component splits into small components. However, at 10% train size, the component is preserved. This is due to insufficient boundary information at 2% train size. Moreover, as can be intuitively expected, this happens when there is a relatively high standard deviation within the class.

The above observations explain the behaviour of Triplet-Watershed at low train sizes.

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#### REFERENCES

- [1] P. Ghamisi, N. Yokoya, J. Li, W. Liao, S. Liu, J. Plaza, B. Rasti, and A. Plaza, "Advances in hyperspectral image and signal processing: A comprehensive overview of the state of the art," *IEEE Geoscience and Remote Sensing Magazine*, vol. 5, no. 4, pp. 37–78, 2017.
- [2] F. Melgani and L. Bruzzone, "Classification of hyperspectral remote sensing images with support vector machines," *IEEE Trans. Geosci. Remote. Sens.*, vol. 42, no. 8, pp. 1778–1790, 2004. [Online]. Available: https://doi.org/10.1109/TGRS.2004.831865
- [3] P. O. Gislason, J. A. Benediktsson, and J. R. Sveinsson, "Random forests for land cover classification," *Pattern Recognit. Lett.*, vol. 27, no. 4, pp. 294–300, 2006. [Online]. Available: https://doi.org/10.1016/ j.patrec.2005.08.011
- [4] Y. Cai, X. Liu, and Z. Cai, "Bs-nets: An end-to-end framework for band selection of hyperspectral image," *IEEE Trans. Geosci. Remote. Sens.*, vol. 58, no. 3, pp. 1969–1984, 2020. [Online]. Available: https://doi.org/10.1109/TGRS.2019.2951433
- [5] S. K. Roy, S. Das, T. Song, and B. Chanda, "Darecnet-bs: Unsupervised dual-attention reconstruction network for hyperspectral band selection," *IEEE Geoscience and Remote Sensing Letters*, pp. 1–5, 2020.
- [6] D. Hong, N. Yokoya, J. Chanussot, J. Xu, and X. X. Zhu, "Joint and progressive subspace analysis (jpsa) with spatial-spectral manifold alignment for semi-supervised hyperspectral dimensionality reduction," 2020.
- [7] D. Hong, N. Yokoya, J. Xu, and X. Zhu, "Joint and progressive learning from high-dimensional data for multi-label classification," in *Computer Vision – ECCV 2018*, V. Ferrari, M. Hebert, C. Sminchisescu, and Y. Weiss, Eds. Cham: Springer International Publishing, 2018, pp. 478–493.
- [8] D. Hong, L. Gao, N. Yokoya, J. Yao, J. Chanussot, Q. Du, and B. Zhang, "More diverse means better: Multimodal deep learning meets remotesensing imagery classification," *IEEE Transactions on Geoscience and Remote Sensing*, pp. 1–15, 2020.

<sup>&</sup>lt;sup>4</sup>Given a graph G=(V,E,W), the subgraph induces by a subset of vertices  $V'\subset V$  is given by G'=(V',E',W). Here  $E'=\{(e_x,e_y)\in E \text{ such that } e_x,e_y\in V'\}$ 

<sup>&</sup>lt;sup>5</sup>See footnote 4.

#### TABLE XIII

SIZES OF COMPONENTS OF LABEL-INDUCED SUBGRAPHS FOR DATASETS UH AND IP. THREE KINDS OF LABELS ARE CONSIDERED - GROUNDTRUTH, LABELS PREDICTED AT 2% AND 10%. ALSO, SHOWN ARE THE RELATIVE (TO MAXIMUM) STANDARD DEVIATIONS OF THE GROUNDTRUTH COMPONENTS.

		UH					IP		
Label	Groundtruth	Rel. Stdev.	10%	2%	Label	Groundtruth	Rel. Stdev.	10%	2%
1	[178, 1073]	[0.41, 0.68]	[154, 1073]	[1073]	1	[46]	[0.79]	[46]	[46]
2	[1096, 158]	[0.68, 0.4]	[1096, 158]	[312, 1130]	2	[1428]	[0.76]	[1416, 1, 1]	[1440]
3	[697]	[0.47]	[697]	[697]	3	[830]	[0.73]	[830]	[830]
4	[1174, 70]	[0.61, 0.39]	[1268]	[1268]	4	[237]	[0.83]	[237]	[237]
5	[1242]	[0.72]	[1272]	[1315]	5	[318,147,18]	[0.65, 0.74, 0.72]	[318, 147, 18]	[318, 88]
6	[40, 6, 279]	[0.51, 0.64, 0.65]	[40, 6, 279]	[6, 279]	6	[730]	[0.68]	[730]	[730]
7	[1268]	[0.56]	[1268]	[1225]	7	[28]	[0.68]	[28]	[28]
8	[1011, 170, 34, 20, 9]	[0.93, 0.42, 0.66, 0.56, 0.63]	[998, 170, 34, 20]	[290, 224, 476]	8	[478]	[0.85]	[478]	[478]
9	[1243, 9]	[0.66, 0.64]	[1257, 9]	[1302, 9, 1]	9	[20]	[0.67]	[20]	[20]
10	[901, 326]	[0.65, 0.42]	[905, 326]	[908, 368]	10	[912, 60]	[0.69, 0.84]	[912, 60]	[912, 60]
11	[1235]	[0.56]	[1204]	[1176, 118]	11	[2455]	[0.74]	[2465]	[2520]
12	[1233]	[0.65]	[1237]	[1726]	12	[593]	[0.82]	[633]	[624]
13	[469]	[1.0]	[461]	[3, 13, 1, 1]	13	[205]	[0.68]	[205]	[205]
14	[428]	[0.95]	[428]	[428]	14	[1265]	[0.74]	[1265]	[1265]
15	[660]	[0.67]	[669]	[680]	15	[386]	[0.76]	[386]	[386]
					16	[93]	[1.0]	[53]	[62]

- [9] L. Gao, D. Yao, Q. Li, L. Zhuang, B. Zhang, and J. M. Bioucas-Dias, "A new low-rank representation based hyperspectral image denoising method for mineral mapping," *Remote Sensing*, vol. 9, no. 11, 2017. [Online]. Available: https://www.mdpi.com/2072-4292/9/11/1145
- [10] Y. Li, Q. Li, Y. Liu, and W. Xie, "A spatial-spectral sift for hyperspectral image matching and classification," *Pattern Recognition Letters*, vol. 127, pp. 18–26, 2019, advances in Visual Correspondence: Models, Algorithms and Applications (AVC-MAA). [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0167865518305117
- [11] Y. Shao, N. Sang, C. Gao, and L. Ma, "Spatial and class structure regularized sparse representation graph for semi-supervised hyperspectral image classification," *Pattern Recognition*, vol. 81, pp. 81–94, 2018. [Online]. Available: https://www.sciencedirect.com/ science/article/pii/S0031320318301171
- [12] G. Licciardi, P. R. Marpu, J. Chanussot, and J. A. Benediktsson, "Linear versus nonlinear pca for the classification of hyperspectral data based on the extended morphological profiles," *IEEE Geoscience and Remote Sensing Letters*, vol. 9, no. 3, pp. 447–451, 2012.
- [13] A. Villa, J. A. Benediktsson, J. Chanussot, and C. Jutten, "Hyperspectral image classification with independent component discriminant analysis," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 49, no. 12, pp. 4865–4876, 2011.
- [14] Y. Zhong and L. Zhang, "An adaptive artificial immune network for supervised classification of multi-/hyperspectral remote sensing imagery," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 50, no. 3, pp. 894–909, 2012.
- [15] J. Li, J. M. Bioucas-Dias, and A. Plaza, "Semisupervised hyperspectral image segmentation using multinomial logistic regression with active learning," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 48, no. 11, pp. 4085–4098, 2010.
- [16] P. Ghamisi, E. Maggiori, S. Li, R. Souza, Y. Tarablaka, G. Moser, A. De Giorgi, L. Fang, Y. Chen, M. Chi, S. B. Serpico, and J. A. Benediktsson, "New frontiers in spectral-spatial hyperspectral image classification: The latest advances based on mathematical morphology, markov random fields, segmentation, sparse representation, and deep learning," *IEEE Geoscience and Remote Sensing Magazine*, vol. 6, no. 3, pp. 10–43, 2018.
- [17] L. He, J. Li, C. Liu, and S. Li, "Recent advances on spectral-spatial hyperspectral image classification: An overview and new guidelines," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 56, no. 3, pp. 1579–1597, 2018.
- [18] S. Li, W. Song, L. Fang, Y. Chen, P. Ghamisi, and J. A. Benediktsson, "Deep learning for hyperspectral image classification: An overview," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 57, no. 9, pp. 6690–6709, 2019.
- [19] G. Camps-Valls, L. Gomez-Chova, J. Munoz-Mari, J. Vila-Frances, and J. Calpe-Maravilla, "Composite kernels for hyperspectral image classification," *IEEE Geoscience and Remote Sensing Letters*, vol. 3, no. 1, pp. 93–97, 2006.
- [20] M. Fauvel, J. Chanussot, and J. Benediktsson, "A spatial–spectral kernel-based approach for the classification of remote-sensing images," *Pattern Recognition*, vol. 45, no. 1, pp. 381–392, 2012. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0031320311002019
- [21] L. Fang, S. Li, W. Duan, J. Ren, and J. A. Benediktsson, "Classification of hyperspectral images by exploiting spectral-spatial information of

- superpixel via multiple kernels," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 53, no. 12, pp. 6663–6674, 2015.
- [22] Y. Chen, H. Jiang, C. Li, X. Jia, and P. Ghamisi, "Deep feature extraction and classification of hyperspectral images based on convolutional neural networks," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 54, no. 10, pp. 6232–6251, 2016.
- [23] J. Yang, Y. Zhao, and J. C. Chan, "Learning and transferring deep joint spectral–spatial features for hyperspectral classification," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 55, no. 8, pp. 4729–4742, 2017.
- [24] A. Ben Hamida, A. Benoit, P. Lambert, and C. Ben Amar, "3-d deep learning approach for remote sensing image classification," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 56, no. 8, pp. 4420–4434, 2018.
- [25] M. He, B. Li, and H. Chen, "Multi-scale 3d deep convolutional neural network for hyperspectral image classification," in 2017 IEEE International Conference on Image Processing (ICIP), 2017, pp. 3904–3908.
- [26] Z. Zhong, J. Li, Z. Luo, and M. Chapman, "Spectral-spatial residual network for hyperspectral image classification: A 3-d deep learning framework," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 56, no. 2, pp. 847–858, 2018.
- [27] M. Zhu, L. Jiao, F. Liu, S. Yang, and J. Wang, "Residual spectral–spatial attention network for hyperspectral image classification," *IEEE Trans*actions on Geoscience and Remote Sensing, vol. 59, no. 1, pp. 449–462, 2021
- [28] S. K. Roy, S. Manna, T. Song, and L. Bruzzone, "Attention-based adaptive spectral-spatial kernel resnet for hyperspectral image classification," *IEEE Transactions on Geoscience and Remote Sensing*, pp. 1–13, 2020.
- [29] D. Hong, L. Gao, J. Yao, B. Zhang, A. Plaza, and J. Chanussot, "Graph convolutional networks for hyperspectral image classification," *IEEE Transactions on Geoscience and Remote Sensing*, pp. 1–13, 2020.
- [30] M. E. Paoletti, J. M. Haut, R. Fernandez-Beltran, J. Plaza, A. Plaza, J. Li, and F. Pla, "Capsule networks for hyperspectral image classification," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 57, no. 4, pp. 2145–2160, 2019.
- [31] D. Hong, N. Yokoya, N. Ge, J. Chanussot, and X. X. Zhu, "Learnable manifold alignment (lema): A semi-supervised cross-modality learning framework for land cover and land use classification," *ISPRS Journal* of Photogrammetry and Remote Sensing, vol. 147, pp. 193–205, 2019. [Online]. Available: https://www.sciencedirect.com/science/article/pii/ S0924271618302843
- [32] D. Hong, N. Yokoya, J. Chanussot, and X. X. Zhu, "Cospace: Common subspace learning from hyperspectral-multispectral correspondences," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 57, no. 7, pp. 4349–4359, 2019.
- [33] A. Qin, Z. Shang, J. Tian, Y. Wang, T. Zhang, and Y. Y. Tang, "Spectral-spatial graph convolutional networks for semisupervised hyperspectral image classification," *IEEE Geoscience and Remote Sensing Letters*, vol. 16, no. 2, pp. 241–245, 2019.
- [34] H. Zeng, Q. Liu, M. Zhang, X. Han, and Y. Wang, "Semi-supervised hyperspectral image classification with graph clustering convolutional networks," 2020.
- [35] Y. Duan, H. Huang, and Y. Tang, "Local constraint-based sparse manifold hypergraph learning for dimensionality reduction of hyperspectral

- image," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 59, no. 1, pp. 613–628, 2021.
- [36] H. Huang, Z. Li, H. He, Y. Duan, and S. Yang, "Self-adaptive manifold discriminant analysis for feature extraction from hyperspectral imagery," *Pattern Recognition*, vol. 107, p. 107487, 2020. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0031320320302909
- [37] Z. Li, H. Huang, Y. Duan, and G. Shi, "Dlpnet: A deep manifold network for feature extraction of hyperspectral imagery," *Neural Networks*, vol. 129, pp. 7–18, 2020. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0893608020301921
- [38] H. Huang, C. Pu, Y. Li, and Y. Duan, "Adaptive residual convolutional neural network for hyperspectral image classification," *IEEE Journal* of Selected Topics in Applied Earth Observations and Remote Sensing, vol. 13, pp. 2520–2531, 2020.
- [39] L. Vincent and P. Soille, "Watersheds in digital spaces: An efficient algorithm based on immersion simulations," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 13, no. 6, pp. 583–598, 1991. [Online]. Available: https://doi.org/10.1109/34.87344
- [40] S. Beucher and F. Meyer, The Morphological Approach to Segmentation: The Watershed Transformation. CRC Press., 01 1993, vol. Vol. 34, p. 433–481.
- [41] G. Noyel, J. Angulo, and D. Jeulin, "Morphological segmentation of hyperspectral images," *Image Analysis & Stereology*, vol. 26, no. 3, pp. 101–109, 2007.
- [42] Y. Tarabalka, J. Chanussot, and J. A. Benediktsson, "Segmentation and classification of hyperspectral images using watershed transformation," *Pattern Recognition*, vol. 43, no. 7, pp. 2367–2379, 2010.
- [43] J. Cousty, G. Bertrand, L. Najman, and M. Couprie, "Watershed cuts: Minimum spanning forests and the drop of water principle," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 31, no. 8, pp. 1362–1374, 2009. [Online]. Available: https://doi.org/10.1109/TPAMI.2008.173
- [44] S. C. Turaga, K. L. Briggman, M. Helmstaedter, W. Denk, and H. S. Seung, "Maximin affinity learning of image segmentation," in Advances in Neural Information Processing Systems 22: 23rd Annual Conference on Neural Information Processing Systems 2009. Proceedings of a meeting held 7-10 December 2009, Vancouver, British Columbia, Canada, Y. Bengio, D. Schuurmans, J. D. Lafferty, C. K. I. Williams, and A. Culotta, Eds. Curran Associates, Inc., 2009, pp. 1865–1873. [Online]. Available: https://proceedings.neurips.cc/paper/ 2009/hash/68d30a9594728bc39aa24be94b319d21-Abstract.html
- [45] K. Maninis, J. Pont-Tuset, P. Arbeláez, and L. Van Gool, "Convolutional oriented boundaries: From image segmentation to high-level tasks," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 40, no. 4, pp. 819–833, 2018.
- [46] S. Wolf, L. Schott, U. Köthe, and F. A. Hamprecht, "Learned watershed: End-to-end learning of seeded segmentation," in *IEEE International Conference on Computer Vision, ICCV 2017, Venice, Italy, October 22-29, 2017.* IEEE Computer Society, 2017, pp. 2030–2038. [Online]. Available: https://doi.org/10.1109/ICCV.2017.222
- [47] J. Funke, F. Tschopp, W. Grisaitis, A. Sheridan, C. Singh, S. Saalfeld, and S. C. Turaga, "Large scale image segmentation with structured loss based deep learning for connectome reconstruction," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 41, no. 7, pp. 1669–1680, 2019.
- [48] S. Wolf, A. Bailoni, C. Pape, N. Rahaman, A. Kreshuk, U. Köthe, and F. A. Hamprecht, "The mutex watershed and its objective: Efficient, parameter-free graph partitioning," *IEEE Transactions on Pattern Anal*ysis and Machine Intelligence, pp. 1–1, 2020.
- [49] A. Challa, S. Danda, B. S. D. Sagar, and L. Najman, "Watersheds for semi-supervised classification," *IEEE Signal Process. Lett.*, vol. 26, no. 5, pp. 720–724, 2019. [Online]. Available: https://doi.org/10.1109/LSP.2019.2905155
- [50] Y. Shen, S. Zhu, C. Chen, Q. Du, L. Xiao, J. Chen, and D. Pan, "Efficient deep learning of nonlocal features for hyperspectral image classification," *IEEE Transactions on Geoscience and Remote Sensing*, pp. 1–15, 2020.
- [51] K. He, X. Zhang, S. Ren, and J. Sun, "Deep residual learning for image recognition," in 2016 IEEE Conference on Computer Vision and Pattern Recognition, CVPR 2016, Las Vegas, NV, USA, June 27-30, 2016. IEEE Computer Society, 2016, pp. 770–778. [Online]. Available: https://doi.org/10.1109/CVPR.2016.90
- [52] A. X. Falcão, J. Stolfi, and R. de Alencar Lotufo, "The image foresting transform: Theory, algorithms, and applications," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 26, no. 1, pp. 19–29, 2004. [Online]. Available: http://doi.ieeecomputersociety.org/10.1109/TPAMI.2004.10012
- [53] W. P. Amorim, A. X. Falcão, and M. H. de Carvalho, "Semisupervised pattern classification using optimum-path forest," in

- 27th SIBGRAPI Conference on Graphics, Patterns and Images, SIBGRAPI 2014, Rio de Janeiro, Brazil, August 27-30, 2014. IEEE Computer Society, 2014, pp. 111–118. [Online]. Available: https://doi.org/10.1109/SIBGRAPI.2014.45
- [54] E. Hoffer and N. Ailon, "Deep metric learning using triplet network," in 3rd International Conference on Learning Representations, ICLR 2015, San Diego, CA, USA, May 7-9, 2015, Workshop Track Proceedings, Y. Bengio and Y. LeCun, Eds., 2015. [Online]. Available: http://arxiv.org/abs/1412.6622
- [55] M. Schultz and T. Joachims, "Learning a distance metric from relative comparisons," in Advances in Neural Information Processing Systems 16 [Neural Information Processing Systems, NIPS 2003, December 8-13, 2003, Vancouver and Whistler, British Columbia, Canada], S. Thrun, L. K. Saul, and B. Schölkopf, Eds. MIT Press, 2003, pp. 41–48. [Online]. Available: https://proceedings.neurips.cc/paper/2003/hash/d3b1fb02964aa64e257f9f26a31f72cf-Abstract.html
- [56] C. Zhang, S. Bengio, M. Hardt, B. Recht, and O. Vinyals, "Understanding deep learning requires rethinking generalization," in 5th International Conference on Learning Representations, ICLR 2017, Toulon, France, April 24-26, 2017, Conference Track Proceedings. OpenReview.net, 2017. [Online]. Available: https://openreview.net/ forum?id=Sy8gdB9xx
- [57] P. W. Battaglia, J. B. Hamrick, V. Bapst, A. Sanchez-Gonzalez, V. F. Zambaldi, M. Malinowski, A. Tacchetti, D. Raposo, A. Santoro, R. Faulkner, Ç. Gülçehre, H. F. Song, A. J. Ballard, J. Gilmer, G. E. Dahl, A. Vaswani, K. R. Allen, C. Nash, V. Langston, C. Dyer, N. Heess, D. Wierstra, P. Kohli, M. Botvinick, O. Vinyals, Y. Li, and R. Pascanu, "Relational inductive biases, deep learning, and graph networks," CoRR, vol. abs/1806.01261, 2018. [Online]. Available: http://arxiv.org/abs/1806.01261
- [58] L. Najman, J. Cousty, and B. Perret, "Playing with kruskal: Algorithms for morphological trees in edge-weighted graphs," in Mathematical Morphology and Its Applications to Signal and Image Processing, 11th International Symposium, ISMM 2013, Uppsala, Sweden, May 27-29, 2013. Proceedings, ser. Lecture Notes in Computer Science, C. L. L. Hendriks, G. Borgefors, and R. Strand, Eds., vol. 7883. Springer, 2013, pp. 135–146. [Online]. Available: https://doi.org/10.1007/978-3-642-38294-9 12
- [59] B. Perret, G. Chierchia, J. Cousty, S. J. F. Guimarães, Y. Kenmochi, and L. Najman, "Higra: Hierarchical graph analysis," *SoftwareX*, vol. 10, p. 100335, 2019. [Online]. Available: https://doi.org/10.1016/j.softx.2019. 100335
- [60] R. O. Green, M. L. Eastwood, C. M. Sarture, T. G. Chrien, M. Aronsson, B. J. Chippendale, J. A. Faust, B. E. Pavri, C. J. Chovit, M. Solis, M. R. Olah, and O. Williams, "Imaging spectroscopy and the airborne visible/infrared imaging spectrometer (aviris)," *Remote Sensing* of Environment, vol. 65, no. 3, pp. 227–248, 1998. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0034425798000649
- [61] B. Kunkel, F. Blechinger, R. Lutz, R. Doerffer, H. van der Piepen, and M. Schroder, "Rosis (reflective optics system imaging spectrometer) - a candidate instrument for polar platform missions," in *Optoelectronic Technologies for Remote Sensing from Space*, C. S. Bowyer and J. S. Seeley, Eds. SPIE, Apr 1988. [Online]. Available: http://dx.doi.org/10.1117/12.943611
- [62] K. P. F.R.S., "On lines and planes of closest fit to systems of points in space," *The London, Edinburgh, and Dublin Philosophical Magazine* and Journal of Science, vol. 2, no. 11, pp. 559–572, 1901. [Online]. Available: https://doi.org/10.1080/14786440109462720
- [63] W. B. March, P. Ram, and A. G. Gray, "Fast euclidean minimum spanning tree: algorithm, analysis, and applications," in *Proceedings of the 16th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, Washington, DC, USA, July 25-28, 2010*, B. Rao, B. Krishnapuram, A. Tomkins, and Q. Yang, Eds. ACM, 2010, pp. 603–612. [Online]. Available: https://doi.org/10.1145/1835804.1835882
- [64] J. Ham, Yangchi Chen, M. M. Crawford, and J. Ghosh, "Investigation of the random forest framework for classification of hyperspectral data," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 43, no. 3, pp. 492–501, 2005.
- [65] L. N. Smith, "Cyclical learning rates for training neural networks," in 2017 IEEE Winter Conference on Applications of Computer Vision, WACV 2017, Santa Rosa, CA, USA, March 24-31, 2017. IEEE Computer Society, 2017, pp. 464–472. [Online]. Available: https://doi.org/10.1109/WACV.2017.58
- [66] A. Paszke, S. Gross, F. Massa, A. Lerer, J. Bradbury, G. Chanan, T. Killeen, Z. Lin, N. Gimelshein, L. Antiga, A. Desmaison, A. Kopf, E. Yang, Z. DeVito, M. Raison, A. Tejani,

S. Chilamkurthy, B. Steiner, L. Fang, J. Bai, and S. Chintala, "Pytorch: An imperative style, high-performance deep learning library," in *Advances in Neural Information Processing Systems* 32, H. Wallach, H. Larochelle, A. Beygelzimer, F. dÁlché-Buc, E. Fox, and R. Garnett, Eds. Curran Associates, Inc., 2019, pp. 8024–8035. [Online]. Available: http://papers.neurips.cc/paper/9015-pytorch-an-imperative-style-high-performance-deep-learning-library pdf

[67] J. Nalepa, M. Myller, and M. Kawulok, "Validating hyperspectral image segmentation," *IEEE Geoscience and Remote Sensing Letters*, vol. 16, no. 8, pp. 1264–1268, 2019.

[68] L. Van der Maaten and G. Hinton, "Visualizing data using t-sne." *Journal of machine learning research*, vol. 9, no. 11, 2008.

[69] K. Musgrave, S. Belongie, and S.-N. Lim, "A metric learning reality check," in *European Conference on Computer Vision*. Springer, 2020, pp. 681–699.



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