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Technical note

Is the spatial organization of larger water bodies heterogeneous?

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Abstract. The intuitive argument is that the spatial organization of larger water bodies is more heterogeneous than that of smaller water bodies. To validate this argument, a section containing a large number of water bodies was distributed according to size by following a multiscale *opening* transformation. A self-similar iterated bisecting process was followed to observe the probability distribution (division rate) pattern of all the size distributed water bodies. These observed probability distributions conformed well to that of the values computed from a binomial multiplicative process. Hence, f-α multifractal spectra were constructed for all the distributed sections containing water bodies of different sizes, and it was found that the degree of heterogeneity in the spatial organization increased with the increasing size category of the water bodies, as the estimated generalized information dimensions from spectra are in an increasing order.

1. Introduction

Analysis of spatial heterogeneity, in quantitative terms, of water bodies according to their sizes is one of the interesting problems of a limnologist. One of the best examples of a natural fractal that occurs in any landscape is a section of surface water bodies of various sizes and shapes. Several mathematical tools are available now to quantify such fractals to understand the spatial heterogeneity. Recent studies on surface water bodies applying mathematical techniques include automatic computation of dimensional parameters (Sagar *et al.* 1995 a), distribution of surface water bodies according to their shapes and sizes (Sagar *et al.* 1995 b), ranking of lakes according to the dynamical behaviour (Sagar and Rao 1995 a, b, c), fractal-based dimensional analysis (Sagar 1994, Sagar and Rao 1995 d), morphological dynamical

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behaviour of lakes (Sagar *et al.* 1998), fractal and morphometric relationships of topological networks of water bodies (Sagar *et al.* 1999), estimation of number-area-frequency dimensions of surface water bodies (Sagar and Srinivas 1999), and also the fractal relation of medial axis length to the water body area (Sagar 2000). In all these studies, the application of mathematical morphology, fractals, and nonlinear concepts are shown on the surface water bodies extracted from remotely sensed data. However, in this technical note, the degrees of heterogeneity in the spatial organization of water bodies of different size categories are quantified by estimating multifractal measures. It is intuitively justifiable that there may be a variation in the generalized information dimensions that can be used as a quantitative measure to understand the heterogeneity in spatial distribution of water bodies. This incited the distribution of the surface water bodies by following a mathematical morphological transformation to further study the changes in the spatial distribution pattern with size of the spatially distributed surface water bodies.

2. Materials and methods

The discrete binary image that contains water bodies, and the no-water body region extracted from IRS 1A satellite data defined as a finite subset of IR^2 are considered. The geometrical properties of these binary data, which contain water bodies (set) and non-water bodies (set complement), were subjected to morphological functionals (Serra 1982) by means of a defined sub-image (or) kernel that is termed here as a structuring template. With the help of a multiscale *opening* transformation (Maragos and Schafer 1986) the water body data were distributed into four size categories. More details on distribution of surface water bodies by applying a multiscale opening transformation can be found in Sagar *et al.* (1995 b) and Sagar and Srinivas (1999).

The multiscale opening transformation was performed to distribute surface water bodies according to their sizes in order to further estimate the degree of heterogeneity in the spatial distribution of surface water bodies of various sizes. These were extracted from IRS-1A remotely sensed data of a region situated between the geographical co-ordinates 18° 15′ and 18° 30′ N and 83° 30′ and 83° 45′ E belonging to the 65 N/11 Survey of India topographic map that covers a part of Vizianagaram district of Andhra Pradesh, India. Since the resolution of IRS-1A (LISS II) data is 36.25 m by 36.25 m, the minimum limit considered was 36.25 m^2 to trace the water bodies for this analysis. The images of IRS-1A (LISS II) of the study region, a section of traced surface water bodies, and a section of size-distributed surface water bodies may be seen in Sagar *et al* (1995 b). In the present note, the published data from earlier work were used to compute the multifractal spectra according to the water body size category. This distribution process was done automatically to compute the number of water bodies according to their sizes. During the multiscale opening transformation, the water bodies that were smaller than the size of the structuring template were vanished by leaving the bigger water bodies. The number of retained water bodies after opening transformation was considered to be an estimate of the number of surface water bodies within a specified size range. This size range was specified by the diameter of the structuring element.

To determine the generalized information dimensions, we counted the number of water bodies between the specified diameters. The considered water body data (figure 1) were distributed into four categories by following a multiscale opening transformation. The categories included the water bodies that were larger than the

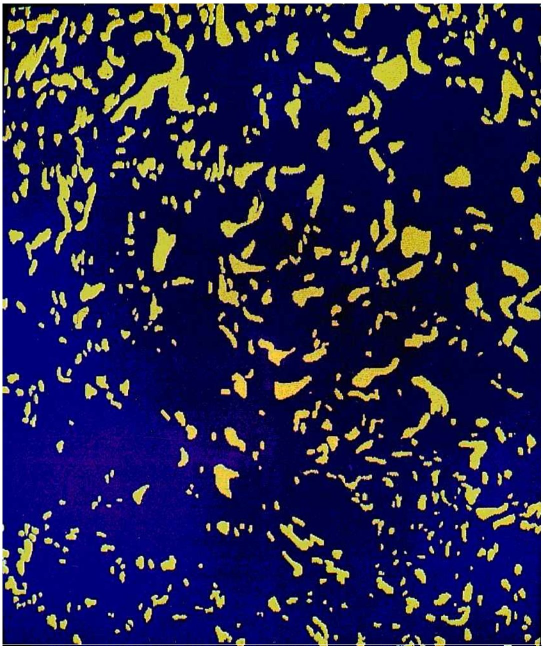

Figure 1. Image shows discretized water bodies that were traced manually from Geocoded IRS-1A (LISS II) data acquired on 3 August 1993. This section of water bodies is considered to be the source data to segregate them according to their sizes by performing multiscale opening transformation (IRS-1A image and section showing water bodies of different size categories can be seen in Sagar *et al.* 1995b).

structuring element with a specified diameter. An iterated horizontal bisecting process of individual categories of water bodies represented in 2-dimensional space was carried out. Each water body was counted as full if the area of the water body was more than its half in the respective level of bisecting. This was done to avoid confusion while counting the number of water bodies at different levels of bisecting. After the first bisecting, for instance, one piece of the landscape contains β (denotes division rate) water bodies in a normalized scale, and the other $(1-\beta)$. This bisected landscape is bisected further; and four equal parts of the landscape with equal area contain β^2, $\beta(1-\beta)$, $(1-\beta)\beta$ and $(1-\beta)^2$ division rates of the water bodies respectively. It was observed that in every bisecting the number of water bodies included was divided in the ratio β:$(1-\beta)$. The four categories of the size-distributed water body sections containing water bodies included those greater than 15 pixel diameter, between 11 and 15 pixel diameter, 7 and 11 pixel diameter and less than 7 pixel diameter. Table 1 shows the number of water bodies within the range specified in terms of the structuring element diameter. Table 1 also shows the number of water bodies counted at their respective level of bisecting, and the division rates observed at respective bisecting levels. Since the division rates estimated through the binomial multiplicative process tallied well with the observed values, they are not shown in table 1. In the present investigation the bisecting was performed up to the second level. Further bisecting is only possible with a high spatial resolution image that generally shows more water body size categories. Up to the second level of bisecting, the observed probability distribution values tallied well with those of the values

Table 1. Self-similar distribution of size distributed water bodies, division rates, and estimated generalized dimensions.

Size distribution of surface water bodies by a multiscale opening transformation		Number of water bodies after iterated bisecting with division rates in parentheses		Information (D_1) and correlation (D_2) dimensions derived from f-α spectra	
Diameter of the structuring template in pixels	Number of water bodies at zeroth level of bisecting	I	II	D_1	D_2
>15	20	16 ($\beta=0.8$)	12 ($\beta^2=0.60$)	0.721	0.322
			4 ($\beta(1-\beta)=0.20$)		
		4 ($1-\beta=0.2$)	3 ($(1-\beta)\beta=0.15$)		
			1 ($(1-\beta)^2=0.05$)		
11–15	36	23 ($\beta=0.64$)	15 ($\beta^2=0.42$)	0.942	0.795
			8 ($\beta(1-\beta)=0.22$)		
		13 ($1-\beta=0.36$)	8 ($(1-\beta)\beta=0.22$)		
			5 ($(1-\beta)^2=0.14$)		
7–11	40	24 ($\beta=0.6$)	14 ($\beta^2=0.35$)	0.97	0.89
			10 ($\beta(1-\beta)=0.25$)		
		16 ($1-\beta=0.4$)	10 ($(1-\beta)\beta=0.25$)		
			6 ($(1-\beta)^2=0.15$)		
<7	143	81 ($\beta=0.57$)	46 ($\beta^2=0.32$)	0.987	0.95
			35 ($\beta(1-\beta)=0.25$)		
		62 ($1-\beta=0.43$)	35 ($(1-\beta)\beta=0.25$)		
			27 ($(1-\beta)^2=0.18$)		

computed through the binomial multiplicative process. This well tallied results further motivated to construct the $f(\alpha)$ spectra. The detailed mathematical theory involved in the construction of the $f-\alpha$ spectrum may be seen in Halsey *et al.* (1986), Chhabra *et al.* (1989) and Takayasu (1990). The $f(\alpha)$ spectra were constructed for all the size-distributed water body categories. To construct the $f-\alpha$ spectra by considering the water body data, the localized fractal dimension, (α_q), which is akin to the Lipshitz Holder exponent, and $f(\alpha_q)$, the global fractal dimension, were computed by equations (1) and (2), which are due to Halsey *et al.* (1986).

$$\alpha_q = -\frac{\beta^q \log_2 \beta + (1-\beta)^q \log_2 (1-\beta)}{\beta^q + (1-\beta)^q} \tag{1}$$

$$f(\alpha_q) = q\alpha_q + \log_2[\beta^q + (1-\beta)^q] \tag{2}$$

The q ranges between any integer values.

3. Results and conclusions

The higher the dimensions (D_1 or D_2), the higher the degree of spatial homogeneity. One of these two dimensions can be considered to quantify the degree of heterogeneity in terms of an analytical value that can be derived from $f(\alpha)$ spectra. The information (D_1) and correlation (D_2) dimensions for these four sections that possess the spatially distributed water bodies of different size categories were estimated from the $f(\alpha)$ spectra (figure 2(*a–d*)). From these spectra, it can be observed that the maximum of $f(\alpha)$ is equal to the capacity dimension; that is 1 for all four size categories. Hence, this is not shown in table 1. The D_1 can be obtained as the slope of the tangent drawn to the curve of $f(\alpha)$ from the origin in the $f(\alpha)$ spectra. From these multifractal spectra, the information and correlation dimensions (D_1 and D_2) were computed and are shown in table 1. From table 1 it is understood that the degree of spatial heterogeneity is higher in the larger water body category, as both the information dimension (0.721) and the correlation dimension (0.322) were lower than for the other water body size categories. As the information dimension (0.987) and the correlation dimension (0.95) were higher for the smaller than for the larger water body categories, it was deduced that the degree of spatial homogeneity is higher in the smaller water body category. In summary, it was deduced that the degree of heterogeneity in the spatial organization of the surface water bodies increased with the increasing size category of the water bodies. This information can be considered as a tool to better characterize the spatiotemporal organization of the surface water bodies. Multifractal modelling is a powerful tool to study the spatiotemporal organization of the randomly situated lakes, which can be extracted from the multidate, multiscale remotely sensed data. Our future work is aimed at studying the spatiotemporal organization of the lakes derived from the multidate remotely sensed data that consists of lakes of various sizes and shapes. It is expected that this study, which demonstrates the spatial organization of randomly situated objects of various sizes and shapes, will be useful as more attention is focused on the usage of multifractal models in the context of limnological research. In any case, the study to model the degree of heterogeneity in the spatial distribution of water bodies according to their sizes across time intervals can be performed on the data extracted from multiscale, multitemporal, remotely sensed digital data. It is intuitively justified that larger features or highly concentrated features are fewer in number compared with the number of available smaller features. Because of the smaller number, it may be

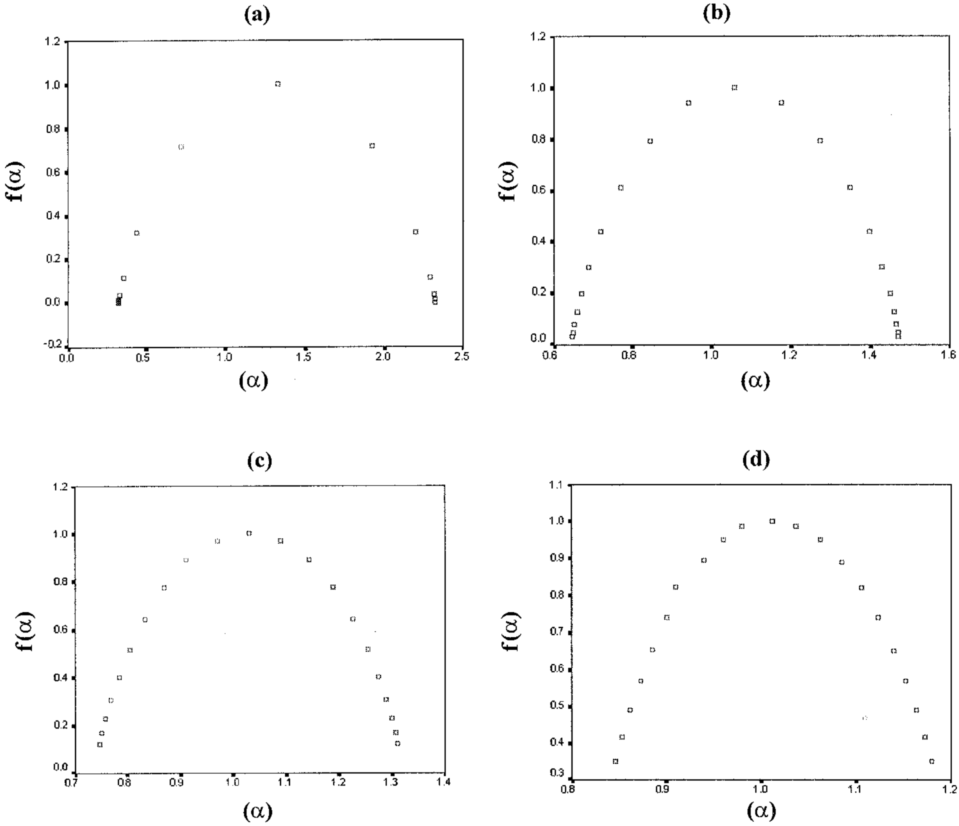

Figure 2. Multifractal spectra for four different size categories of water body sections. (*a*) The larger water bodies between 15 and 17 pixel diameter, (*b*) the water bodies between 11 and 15 pixel diameter, (*c*) the water bodies between 7 and 11 pixel diameter, and (*d*) the water bodies less than 7 pixel diameter.

heuristically true that the spatial organization of larger water bodies is more heterogeneous than that of the smaller features which are generally larger in number. If a dataset consisting of a very large number of surface water bodies with a larger number of size categories over a large landmass is available, this postulate can be further strengthened by modelling the degree of heterogeneity in the spatiotemporal organization in a time sequential mode through multifractal measures that may give a relationship between heterogeneity in the spatial organization of water bodies and their stability, in the sense, the fluctuation in the areal or volume extents.

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