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Short Communication

Quantitative Spatial Analysis of Randomly Situated Surface Water Bodies Through f-α Spectra

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A notable similarity is observed between the probability distributions obtained from a data set that contains a large number of randomly situated surface water bodies and the probability distributions estimated by binomial multiplicative process. From these well conformed probability distributions, the generalised information dimensions have been computed through f-α spectra to characterise and quantify the spatial organisation of the surface water bodies. It is noticed from the investigated case study that the results tend to vary by changing the direction of bisecting process. The experimental results on spatial distribution of surface water bodies of the study area qualify that the computed generalised information dimensions for the vertical bisecting is rather uniform than that of the horizontal bisecting process.

Keywords: Water bodies; Binomial multiplicative process; Iterated bisecting process; Spatio-temporal organisation; f-α spectra

INTRODUCTION

Understanding the spatio-temporal organisation of the randomly situated and fluctuating geometry of surface water bodies is an important physical phenomenon. The recent extensions to estimate the local and generalised information dimensions enable to have a better understanding of spatio-temporal organisation of surface water bodies. Many works have been appeared in relation to spatio-temporal changes in the surface water bodies. However, these models are of exceedingly descriptive nature. As the information dimension has the capability to capture the characteristic alterations by means of an analytical value, one can adopt this multifractal technique to better quantify and characterise the spatio temporal organisation of water bodies.

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Self Similar Size Distribution of Water Bodies by Iterated Bisecting

A landscape section contains a large number of surface water bodies which are quite apparent on any remotely sensed data. The total area occupied by the water bodies in a section of landscape is A. This section of landscape is bisected in both vertically and horizontally. The defined regions are according to the vertical and horizontal bisecting. One piece of the landscape contains βA water bodies and the other $(1-\beta)$A. This bisected landscape is bisected further; and four equal parts of the landscape with equal area contains β^2A, $\beta(1-\beta)$A, $(1-\beta)\beta$A and $(1-\beta)^2$A of the water bodies respectively. It is observed that in the every bisecting the water body occupied area is divided in the ratio $\beta:1-\beta$. The data set containing large number of water bodies in 528 km^2 region is taken for this study. This sample consists of a number of surface water bodies larger than (32.5 mt^2). The limit 36.25 mt^2 represents the smallest water body that could be traced accurately from IRS 1A (LISS II) data in geocoded format. It lies in between the geographical coordinates of 18° 15′ and 18° 30′N and 83° 30′ and 83° 45′E belonging to the 65N/11 Survey of India (SOI) topographic map. This data that contains water body and no-water body regions has been cut into two equal area pieces and it has been found that 51% of water body area is covered in the left half (while bisecting vertically) which is considered as "β" and then $1-\beta$ is 49%. Whereas in horizontal bisecting, β is computed as 63% of water body occupied area in upper half and $1-\beta$ is 37%. This bisecting process is carried out iteratively. Figures 1a and b show the self similar distribution by iterated bisecting process of water body data after respective number of vertical and horizontal bisections. Table I shows the probability distributions computed from the percentage of water body area occupied at different regions at respective generation of bisecting. It is interesting to see the considerable similarity between the values depicted for the areal extents of water bodies at the respective level of bisecting and that of the values predicted through binomial multiplicative process. The comparison is depicted in the Table I. From the probability distributions it has been observed the remarkable similarity between the observed and computed values.

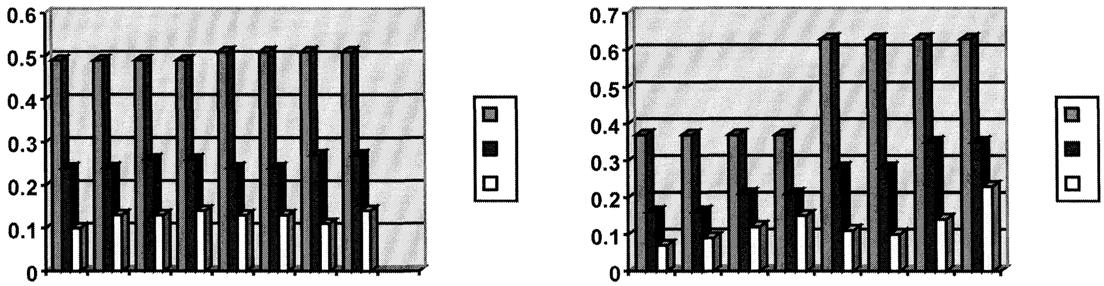
As the observed and estimated probability distributions are considerably tallied Eq. (1) which is due to Halsey *et al.* (1986) has been used to compute the information dimension (D_1) by taking the values $\beta=0.51$ and $1-\beta=0.49$ for vertical bisecting, and $\beta=0.63$ and $1-\beta=0.37$ for horizontal bisecting.

$$D = -\{\beta \log_2 \beta + (1-\beta)\log_2(1-\beta)\} \qquad (1)$$

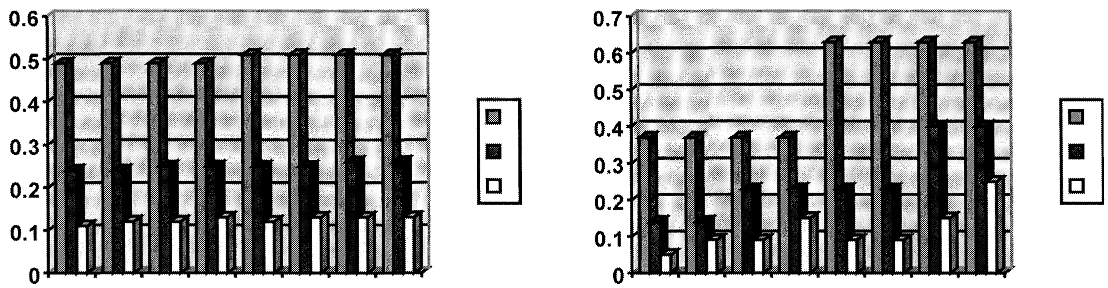
The β denotes the division rate derived from the section of surface water bodies after vertical and horizontal bisectings. The probability distribution of this section compared with that of the probability distributions estimated by binomial multiplicative process, and found tallied significantly well. The information dimensions computed from Eq. (1) for the section of the landscape containing surface water bodies after vertical and horizontal bisecting processes are respectively 0.99 and 0.95. The computed information dimension indicates that the spatial organisation of the surface water bodies is highly uniform for the vertical bisecting process, whereas the spatial organisation is rather non-uniform for the horizontal bisection. This disparity needs to be paid attention while trying to understand the spatio-temporal organisation of the surface water bodies.

The $f(\alpha)$ is also calculated for this spatially distributed surface water body data, which is a fractal. This is done to construct the f-α spectra. The localised fractal dimension (α_q) which is akin to the Lipshitz Holder exponent, and $f(\alpha_q)$, the global fractal dimension have been computed by following Eqs. (3) and (4) which are also due to Halsey *et al.* [1].

$$D_q = -\frac{1}{q-1}\log_2\{\beta^q + (1-\beta)^q\} \qquad (2)$$



(a) Selfsimilar size distribution of surface water bodies after iterated vertical and horizontal bisection process of the data consists of randomly situated surface water bodies



(b) Selfsimilar size distribution for the data set estimated through binomial multiplicative process for vertical and horizontal bisectings

FIGURE 1 Comparative self similar distributions of surface water bodies (a) observed from the data set and (b) estimated through binomial multiplicative process. Gray (long bars), black (medium bars), and white (short bars) indicate the probability distributions at first, second, and third levels of bisectings respectively. The four right side long bars (half unit) in the first bisecting indicate the β, and the left side four long bars indicate $1-\beta$. In the second bisecting, two medium bars are considered as a quarter unit. In the third bisection, each bar is considered as 1/8 of a unit.

TABLE I Self similar distribution of a section of landscape containing surface water bodies

Vertical bisecting						Horizontal bisecting					
I		II		III		I		II		III	
Probability distributions in terms of percentage area occupied by the water body area at respective iterated bisecting											
0.51	0.49	0.24	0.26	0.11	0.14	0.63	0.37	0.35	0.21	0.23	0.15
				0.14	0.13					0.14	0.12
		0.27	0.24	0.13	0.14			0.28	0.16	0.10	0.09
				0.13	0.10					0.11	0.07
Probability distribution estimated from binomial multiplicative process											
0.51	0.49	0.26	0.25	0.13	0.13	0.63	0.37	0.40	0.23	0.25	0.15
				0.13	0.12					0.15	0.09
		0.25	0.24	0.13	0.12			0.23	0.14	0.09	0.09
				0.12	0.11					0.09	0.05

The generalised information dimensions for the considered section of randomly situated surface water bodies after vertical and horizontal iterated bisecting process have been estimated from the f-α spectra (Fig. 2). The parameters to construct this multifractal spectra have been computed from

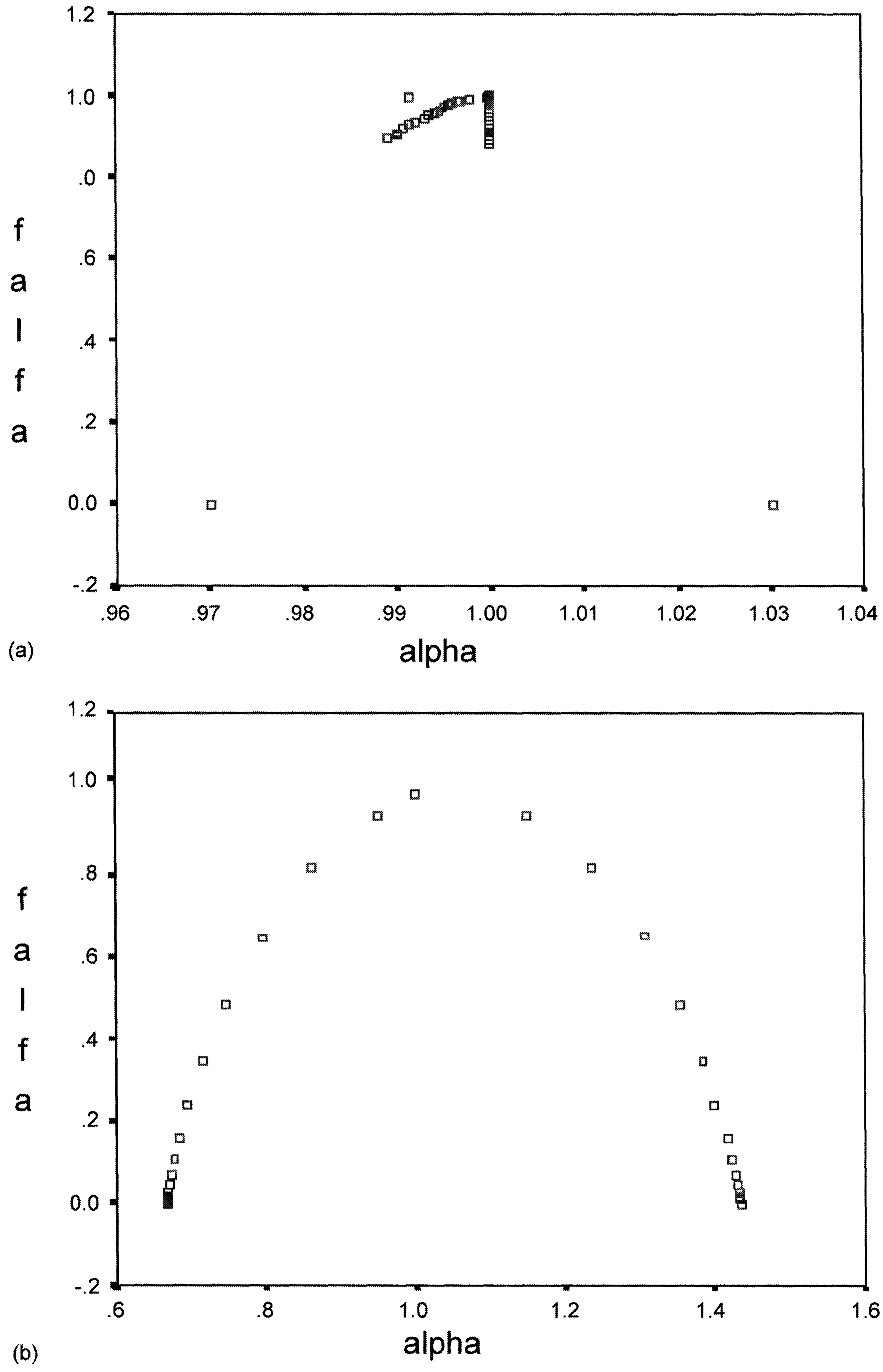


FIGURE 2 f-α spectra for a section of landscape containing a large number of randomly situated surface water bodies. These spectra are constructed for the binomial multiplicative process with (a) $\beta = 0.51$ (vertical bisecting) and (b) $\beta = 0.63$ (horizontal bisecting).

TABLE II Generalised information dimensions of randomly situated surface water bodies

	D_0	D_1	D_2	D_3
Vertical bisection	1	0.9997114	0.999423	0.9991349
Horizontal bisection	1	0.950672	0.9056287	0.8668016

Eqs. (3) and (4).

$$\alpha_q = -\frac{\beta^q \log_2\beta + (1-\beta)^q \log_2(1-\beta)}{\beta^q + (1-\beta)^q} \tag{3}$$

$$f(\alpha_q) = q\alpha_q + \log_2(\beta^q + (1-\beta)^q) \tag{4}$$

The q ranges between any integer values.

From the spectra, it can be observed that the maximum of $f(\alpha)$ is equal to capacity dimension (D_0). The D_1, the information dimension, can be obtained as the slope of the tangent drawn to the curve of $f(\alpha)$ from the origin in the f-α spectram. This is exactly tallied with that of the information dimension computed from Eq. (1). The capacity dimension is computed from Eq. (1), and the generalised information dimensions for the data set that contains large number of surface water bodies can be depicted from the f-α spectra constructed for the two types of bisecting processes. From these multifractal spectra, the fractal dimensions D_0, D_1, and D_2 that are depicted from the f-α spectra have been shown in the Table II. It is deduced from the results that the spatial distribution of the surface water bodies in the sample section of the landscape is rather uniform while quantifying by vertical bisections, whereas, it is nonuniform while bisecting the sample horizontally. From this study, it is deduced that the distribution of water body occurrence can be well quantified. It can be shown whether the spatial distribution of surface water bodies is in uniform or in non-uniform. As the dimension is close to 1, *i.e.*, 0.99 computed for the vertical bisecting process, it is deduced that the distribution of water bodies in the considered section of landscape is more or less uniform. In contrast, the distribution pattern of surface water bodies from the same landscape is non-uniform while following the horizontal bisecting process. Hence from this study, it is also deduced that the degree of uniformity in the spatial distribution of surface water bodies depends not only on the physiographic set up of the landscape but also on the bisecting process that is adopted. This statement is justified from the fact that the generalised information dimensions derived from the vertical bisecting process is quite different from that of the dimensions derived from horizontal bisecting process. This information can be considered as a tool to better characterise the spatio temporal organisation of the surface water bodies. The multifractal modelling is a powerful tool to study the spatio temporal organisation of the randomly situated lakes that can be extracted from the multidate remotely sensed data. Our future work is aimed at studying the spatio temporal organisation of the lakes derived from the multidate remotely sensed data. As it is intuitively valid that the spatio temporal pattern may vary with the sizes of the lakes the future direction will be taken in this line by considering the multiscale multi temporal data that consists of lakes of various sizes and shapes.

Reference

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