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Letter to Editors

Phase Space Maps of a Simulated Sand Dune: A Scope

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This note is complemented to the earlier work of the author, in which the equations to model the symmetrical fold with two and three limbs under the influence of horizontally acting forces, and a sand dune that propagates towards peek points are given. Present note gives, a scope on how to incorporate remote sensing based information in the process to validate these equations.

In earlier works [1, 2], certain equations have been derived, which are based on the first order nonlinear difference equation to estimate the attracting interlimb (fold case) and the attracting interslipface (sand dune case) angles that are of interest to simulate the morphological evolution of the mentioned phenomena of interest to geodynamicists. In earlier studies, symmetrical fold cases with 2 and 3 limbs under horizontal compression with fixed limb lengths [1], and a pyramidal sand dune case with two slipfaces with a fixed base length [2] have been modelled qualitatively through bifurcation theory [3]. The definitions of symmetrical folds, and a pyramidal sand dune may be seen in the earlier works [1, 2]. To model these two distinct phenomena of interest to geodynamicists, the first order nonlinear difference equation [3] that has the physical viability to model the geodynamic phenomena has been considered as a base (for more details, the reader may kindly refer to [1, 2]).

MATHEMATICAL EQUATIONS

Sand Dune and Fold Cases with 2 Slipfaces and 3 Limbs Respectively

For the sand dune with 2 slipfaces and the fold with 3 limbs, the θ_{t+1} is a function of θ_t

$$\theta_{t+1} = f(\theta_t) \tag{1}$$

The function is defined in the earlier works [1, 2] as

$$\theta_{t+1} = 2\,\mathrm{Sin}^{-1}\left[\frac{10\exp \mathrm{Log}\,N/\{\lambda\{\mathrm{Log}\,N/[\mathrm{Log}\,[2\mathrm{Sin}\,\theta_t/2]] - D_T\}\{1-\{\mathrm{Log}\,N/[\mathrm{Log}[2\mathrm{Sin}\,\theta_t/2]] - D_T\}\} + D_T}{2}\right] \tag{2}$$

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θ_t and θ_{t+1} = Interlimb or Interslipface angles at discrete times
λ = Strength of regulatory force, $0 < \lambda < 4$
D_T = Topological dimension
$90° < \theta < 180°$.

Symmetrical Fold Case with 3 Limbs

$$\theta_{t+1} = \mathrm{Cos}^{-1}\left[\frac{5 - 10\exp 2\mathrm{Log}\, N/\{\lambda\{2\mathrm{Log}\, N/[\mathrm{Log}[5-4\mathrm{Cos}\,\theta]] - D_T\}\{1 - \{2\mathrm{Log}\, N/[\mathrm{Log}[5-4\mathrm{Cos}\,\theta]] - D_T\}\} + D_T}{4}\right] \quad (3)$$

where $60° < \theta < 180°$.

The interlimb and interslipface angles can be computed by iterating the functions defined in Eqs. (2) and (3) to represent them in θ-space to visualize them in the form of symmetrical geological fold and sand dune phase maps, further to characterize them by following fractal techniques to quantify the degree of predicting predictability of their dynamical behaviour. The whole issue of this note is that how these equations can be used in understanding the spatiotemporal organization of geodynamical phenomena of interest using the multitemporal multiscale digital elevation models (DEMs) derived directly from remotely sensed data. In the field of geodynamics, the modelling of the morphological dynamics of a symmetrical fold may not be possible as the morphological changes in short time intervals cannot be observed. Hence, this type of exercise can be carried out forthe interest of computer simulations. However, it is rather clear to extend to test the validity of Eq. (2) by incorporating the interslipface angles, of a pyramidal sand dune, the retriveal of which is possible with the advent of the availability of high resolution interferometrically generated DEMs.

PHASE SPACE AND RETURN MAPS IN θ SPACE

The fold and sand dune maps, by plotting values θ_t *vs.* θ_{t+1}, and return maps, by plotting values $\theta_{t+1} - \theta_t$ *vs.* $\theta_{t+2} - \theta_{t+1}$, can be constructed for the regulatory forces that are represented in Eqs. (2) and (3) as λ that ranges between 0 and 4. The values follow the sequence of threshold regulatory forces further to compute the universality in the dynamical behaviour of a specific geodynamical phenomenon. To have a visual appeal, a phase space map and a return map are constructed for a pyramidal sand dune case by considering the interslipface angles arrived at by iterating Eq.

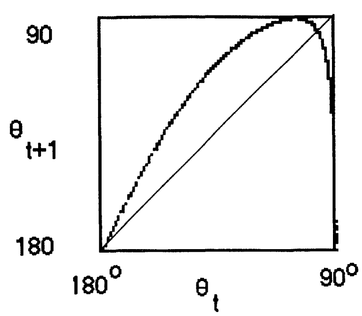

FIGURE 1a Attractor map plotted between θ_t *vs.* θ_{t+1} for sand dune case; $\lambda = 4$.

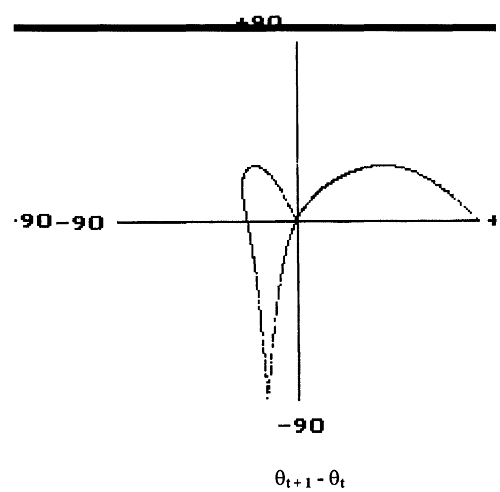

FIGURE 1b Return map plotted between $\theta_{t+1} - \theta_t$ *vs.* $\theta_{t+2} - \theta_{t+1}$ for sand dune case; $\lambda = 4$.

(2). The λ is considered as 4 to construct these maps (Figs. 1a and 1b).

Certain geodynamic problems such as the morphological evolutionary behaviour of a sand dune can be better modelled by using the multidate DEMs derived from high resolution remotely sensed data. The sand dune evolution that is addressed in the present note is a good example to test the exactness in testing the successfulness of the model. With the advent of availability of multitemporal remotely sensed data the DEMs can be precisely computed. Such continuous and long time series of DEMs facilitate the morphological changes in the sand dune. The changing sand dune profiles can be retrieved from such multitemporal DEMs. Future works may lie in developing a framework to exactly model the sand dune morphological evolution by considering the high resolution interferometrically generated multidate DEMs that facilitate changing interslipace angles of sand dunes.

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