Multicriterion *Q*-Analysis and Compromise Programming for Irrigation Planning

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Selection of the best compromise alternative plan in irrigation development strategies is examined in the multiobjective context with the aid of cluster analysis for a case study of Sri Ram Sagar Project, Andhra Pradesh. The study deals with three conflicting objections, namely, net benefits, crop production and labour employment. Present analysis combines multiobjective optimization, cluster analysis and multicriterion decision making (MCDM) methods. Two MCDM methods compromise programming (CP) and multicriterion Q-analysis (MCQA) are employed to select the best compromise alternative. It is observed that net benefits, crop production and labour employment per hectare on average for the culturable command area (CCA) are Rs 8 907.9, 4.16 t and 226 man-days, respectively with irrigation intensity of 126.4%. Sensitivity analysis studies indicated that the ranking pattern is quite robust to the parameter changes.

Keywords : Irrigation planning; Multicriterion Q-analysis; Compromise programming; Cluster analysis; Linear programming

NOTATION

i	:	crop index, 1, ..., ..., 16
f	:	fertilizer index, f=1,2,3
s	:	summer
srf	:	summer rainfed
t	:	month index t=1, ..., 12
ts	:	two seasons
w	:	winter
A_i	:	area of crop i, ha
B_i	:	unit gross return from i th crop, Rs
BEM	:	net benefits, Rs
CCA	:	culturable command area, ha
e_k	:	error value for each cluster group k
E_K, E_{K+1}	:	total square error value for cluster group K and for all cluster groups, K+1, respectively
f_j^*	:	ideal value of criterion j
$f_j(a)$	:	value of alternative a for criterion j
F_{fi}	:	quantity of fertilizer of type f for crop i, t/ha
GW	:	monthly ground water requirement, Mm^3
K	:	number of cluster groups
L_{it}	:	man-days required for each hectare of crop i in month t
$L_p(a)$	:	L_p- metric for alternative a
LAM	:	total man-days for the whole year
m_j, M_j	:	minimum and maximum values, respectively of criterion j in set A
N	:	number of alternatives
p	:	parameter reflecting the attitude of the decision maker
P_f	:	unit cost of fertilizer type f, Rs
P_{gw}	:	unit cost of ground water, Rs/Mm^3
P_l	:	unit wage rate, Rs
P_{sw}	:	unit cost of surface water, Rs/Mm^3
PCI, PCI_{max}, PCI_N	:	project concordance index , and its maximum and normalized values, respectively
PDI, PDI_{max}, PDI_N	:	project discordance index, and its maximum and normalized values, respectively
PRM	:	crop production, t
PSI, PSI_{max}, PSI_N	:	project satisfaction index; and its maximum and normalized values, respectively
Rs	:	rupees in Indian currency
R_t	:	monthly canal water releases, Mm^3
w_j	:	weight of the criterion j
Y_i	:	yield of i^{th} crop, t/ha
Z_U	:	aspired level of objective
Z_L	:	lowest acceptable level of objective

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INTRODUCTION

Multicriterion decision making (MCDM) methods are gaining importance because of their inherent ability to judge different alternative scenarios for the selection of the best, which may be further analyzed in depth for its final implementation. Many MCDM methods are employed for different studies in water resources, such as, river basin planning and development[2,3], hydropower operation[4] and ground water planning problems[5]. Compared to the earlier studies, the present study incorporates two new concepts: (a) cluster analysis to reduce the size of the nondominated alternatives to a manageable subset (b) involving irrigation management expert as a decision maker for the process of decision making. In the present study, two MCDM methods, compromise programming (CP) and multicriterion Q-analysis (MCQA) are employed for the irrigation planning problem. These methods[6] are discussed in brief here alongwith the irrigation system under consideration.

Cluster Analysis

Cluster analysis helps to reduce a large number of alternatives to a manageable number of groups, each containing relatively homogeneous alternatives, particularly when the efficient alternatives generated from multiobjective optimization (constraint method) are large in number. In clustering, alternatives within a cluster are more similar to each other than alternatives of different clusters. *K*-means clustering algorithm[7] is used to minimize the intra-cluster sums of squares of differences (error) based on initial partitions, to obtain final partitions. In this method, normalized alternatives are grouped, so that, each alternative is assigned to one of the fixed number of groups *K*. The sum of the squared differences of each criterion from its assigned cluster mean, is used as the criterion for the assignment. Alternatives are transferred from one cluster to another, so that, intra-cluster sum of squared differences (error) decreases. In a pass through the entire data set, if no transfer occurs, the algorithm stops[8]. The total square error value for cluster group K, E_K is given by

$$E_K = \sum_{k=1}^{12} e_k^2 \qquad (1)$$

K-means clustering, with more than one value of K, is performed and the value of K which best fits the data is used. Burn[9] proposed *F*-statistic value as bench mark to select optimal number of clusters. This value of F is measure of the reduction in variance from K to $K+1$ clusters and value of F greater than 10, justifies a transition from K to $K+1$ clusters[9]. The *F*-statistic can be defined as

$$F = [\ E_K / E_{K+1} - 1\ ]\ (N - K + 1) \qquad (2)$$

Compromise Programming (CP)

Compromise programming (CP) defines the 'best' solution as the one, whose point is at the least distance from an ideal point in the set of efficient solutions. The aim is to obtain a solution that is as 'close' as possible to some ideal[10]. The distance measure used in compromise programming is the family of L_p metrics and is given by

$$L_p(a) = \left[\sum_{j=1}^{J} w_j^p \left| \frac{f_j^* - f_j(a)}{M_j - m_j} \right|^p \right]^{\frac{1}{p}} \qquad (3)$$

For $p=1$, all deviations from f_j^* are taken into account in direct proportion to their magnitudes. For $2 < p < \infty$ the largest deviation has the greatest influence; while for $p = \infty$ the largest deviation is the only one taken into account (min-max criterion).

Multicriterion Q-Analysis (MCQA)

Multicriterion Q-Analysis (MCQA) approach is based on multidimensional graph theory[11]. In this technique the alternatives are compared simultaneously with the set of criteria. MCQA falls under mixed category and uses both value-type and outranking type indices. Comparisons are made based on the rankings derived from concordance and discordance indices. The MCQA method establishes a preference matrix derived from a payoff matrix that represents the decision maker's preference. The final ranking is based on the three indices so derived, namely, project satisfaction index (PSI), project concordance index (PCI) and project discordance index (PDI). PSI, PCI, PDI for each alternative a can be normalized by the highest values (PSI_{max}, PCI_{max}, PDI_{max}) that can be expected to be taken by PSI, PCI, PDI and denoted as PSI_N, PCI_N and PDI_N. The normalized values are combined through a L_p metric approach. The total priority of each alternative a is

$$L_p(a) = [\,|1 - PSI_N(a)|^p + |1 - PCI_N(a)|^p + |PDI_N(a)|^p]^{1/p} \qquad (4)$$

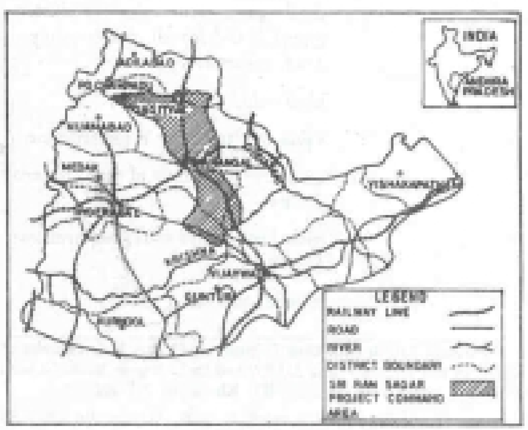

Figure 1 Location map of Sri Ram Sagar Project

IRRIGATION SYSTEM FOR STUDY

The described methodology is applied to the case study of Sri Ram Sagar Project (SRSP), Andhra Pradesh. The location map of SRSP is presented in Figure 1. The culturable command area (CCA) of the project is 178 100 ha. The main crops grown in the command area are paddy, maize, sorghum, groundnut, vegetables, pulses, chillies and sugarcane. Mathematical modelling of the three conflicting objectives with the corresponding constraints are briefly explained here.

Net benefits (BEM) from the irrigated as well as unirrigated area under different crops is obtained by subtracting the costs of surface water, ground water, fertilizer and labour from the gross revenue for different crops. Maximization of net benefits can be expressed as

$$\text{Max } BEM = \sum_{i=1}^{16} B A_i - P_{sw} \sum_{t=1}^{12} R_t - P_{gw} \sum_{t=1}^{12} GW_t$$

$$- \sum_{f=1}^{3} \sum_{i=1}^{16} F_{fi} A_i P_f - P_l \sum_{t=1}^{12} \sum_{i=1}^{16} L_{it} A_i \qquad (5)$$

in which i = crop index (1 = paddy(s), 2 = maize(s), 3 = sorghum(s), 4 = groundnut(s), 5 = vegetables(s), 6 = pulses(s), 7 = paddy(srf), 8 = groundnut(srf), 9 = paddy(w), 10 = groundnut(w), 11 = pulses(w), 12 = maize(w), 13 = sorghum(w), 14 = vegetables(w), 15 = chillies(w), 16 = sugarcane(ts)); t = month index (1 = January, 2 = February, 3 = March, 4 = April, 5 = May, 6 = June, 7 = July, 8 = August, 9 = September, 10 = October, 11 = November, 12 = December); f = fertilizer index (1 = nitrogen, 2 = phosphorous, 3 = potash);

Crop production (PRM) of crops is maximized for meeting the demands and can be expressed as

$$\text{Max } PRM = \sum_{i=1}^{16} Y_i A_i \qquad (6)$$

Total labour employed (LAM) under all the crops for the whole year is maximized to increase the level of their economic status and can be expressed as

$$\text{Max } LAM = \sum_{t=1}^{12} \sum_{i=1}^{16} L_{it} A_i \qquad (7)$$

The other constraints incorporated in the model includes continuity equation, crop land requirements, water requirements of crops, ground water withdrawals, canal capacity restrictions, minimum and maximum reservoir storages, crop diversification considerations, labour and fertilizer availability. These are not presented due to space limitation. Cost coefficients, crop yield and other input parameters are obtained from SRSP reports. In the planning model, stochastic nature of inflows (90% inflow level) is considered through chance constrained programming[12].

RESULTS AND DISCUSSION

Individual Optimization

Optimization of each individual objective (net benefits, labour employment and crop production) is performed with a linear programming (LP) algorithm that gave the upper and lower bounds for the multiobjective analysis[15]. Results are presented in Table 1. Maximum (Z_i) and minimum values (Z_i) that can be obtained by each objective are denoted with symbol (+) and (-), respectively. The reason for more acreage of paddy(s), groundnut(srf) and groundnut(w) in the case of net benefits maximization is due to the large net returns per unit area. In crop production maximization case, maize and sorghum (both in summer and winter) have large acreage compared to those in the remaining planning objectives because of their higher yield per unit area. In the irrigation planning model, there is no significant change in acreage of groundnut, vegetables, pulses in summer season, paddy, pulses, vegetables in winter season and sugarcane for all the three planning objectives. Irrigation intensity in net benefits, labour employment and crop production maximization cases are 101.96%, 152.34% and 142.13%, respectively. Cropping intensity is 154.33%, 197.92% and 173.87%. It is observed from this analysis that the three

Table 1 Crop plans from the planning model

Crops and related parameters		Unit	Solution for maximization of: Net benefits	Crop production	Labour employment	Solution for best plan (G4)
Paddy	(s)	1000 ha	62.930	2.000	2.000	2.000
Maize	(s)	1000 ha	5.000	54.650	5.000	5.000
Sorghum	(s)	1000 ha	1.900	50.000	22.870	49.650
Groundnut	(s)	1000 ha	1.500	1.500	1.500	1.500
Vegetables	(s)	1000 ha	2.100	2.000	2.000	2.100
Pulses	(s)	1000 ha	4.200	4.200	4.200	4.200
Paddy	(srf)	1000 ha	0.000	51.200	51.200	51.200
Groundnut	(srf)	1000 ha	93.260	5.340	29.970	55.250
Paddy	(w)	1000 ha	14.700	14.700	14.700	14.700
Groundnut	(w)	1000 ha	15.810	5.700	5.700	49.100
Pulses	(w)	1000 ha	39.850	39.850	39.850	39.850
Maize	(w)	1000 ha	13.000	22.330	13.000	13.000
Sorghum	(w)	1000 ha	4.900	40.000	40.000	27.920
Vegetables	(w)	1000 ha	1.700	1.800	1.800	1.700
Chillies	(ts)	1000 ha	3.100	3.100	55.250	3.100
Sugarcane	(ts)	1000 ha	4.100	4.100	4.100	4.100
Irrigation intensity %			101.960	142.130	152.340	126.400
Cropping intensity %			154.330	173.870	197.920	186.200
Net benefits		Million rupees	1672.900+	1084.000-	1418.600	1586.500
Crop production		Mt	0.680	0.78+	0.55-	0.740
Labour employment		Million man-days	40.430	35.16-	46.23+	40.330

Table 2 Proposed policies for Sri Ram Sagar Project

Policy	Net benefits, million rupees	Crop production, Mt	Labour employment, million man-Days
P01	1084.00	0.777 81	35.157
P02	1110.50	0.777 79	35.170
P03	1136.80	0.777 58	35.190
P04	1152.00	0.777 18	35.250
P05	1172.50	0.776 59	35.309
P06	1197.60	0.776 00	35.428
P07	1216.20	0.775 50	35.499
P08	1238.70	0.774 85	35.657
P09	1261.20	0.774 19	35.820
P10	1283.60	0.773 54	35.983
P11	1301.80	0.772 50	36.300
P12	1322.40	0.771 10	36.596
P13	1343.00	0.769 70	36.892
P14	1365.90	0.769 10	37.482
P15	1391.80	0.767 05	37.777
P16	1417.80	0.765 00	38.072
P17	1439.50	0.763 86	38.492
P18	1460.20	0.762 72	38.913
P19	1481.50	0.762 16	39.423
P20	1502.70	0.761 59	39.932
P21	1531.40	0.757 00	40.322
P22	1559.00	0.749 29	40.326
P23	1586.50	0.741 19	40.331
P24	1613.40	0.733 09	40.336
P25	1645.40	0.723 05	40.349
P26	1672.90	0.680 76	40.432
P27	1650.20	0.680 00	41.716
P28	1624.90	0.671 87	42.477
P29	1599.40	0.663 74	43.239
P30	1573.80	0.655 61	44.000
P31	1553.90	0.639 58	44.374
P32	1532.40	0.623 40	44.748
P33	1505.60	0.606 65	45.124
P34	1478.70	0.589 89	45.500
P35	1459.10	0.577 81	45.750
P36	1438.30	0.565 73	46.000
P37	1418.60	0.554 20	46.229

planning objectives conflict with one another. There is a need to develop a tradeoff relationship to select the compromise cropping plan and the corresponding water allocation policies in the multiobjective irrigation planning context to meet the chosen levels of satisfaction as would be demanded in the decision making process. Responses of farmers and officials are obtained through a survey conducted to assess the priority of each criteria (objective). Ninety two farmers and 18 officials are involved in the process. Geometric mean approach of Analytic Hierarchy Process is used to aggregate the views of farmers and officials (results are not presented due to space limitation). It is observed that net benefits are given higher importance (56.13%), followed by crop production (31.24%) and labour employment (12.63%).

Maximization of net benefits is selected as the main objective in the constraint method of multiobjective optimization formulation, because of the higher importance attributed to it. In this method, the other two objectives crop production and labour employment are placed as constraints in the constraint set. The nondominated set of alternatives are generated by parametrically varying the bounds of the constraints (crop production and labour employment) obtained from the individual optimal solutions. The number of nondominated alternatives are reduced to a chosen few, based on consultations between the analyst (first author) and the decision maker. Thirty seven policies labelled P01 to P37 are adopted by consultations and shown in Table 2. It is observed that with increase of labour employment and decrease of crop production, the net benefits gradually increased to a maximum level (corresponding to policy P26) and then decreased.

Cluster Analysis

Thirty seven policies (P01 to P37) are normalized by the difference between maximum and minimum values of each criterion. Selection of initial partitions is performed by Ward's method of hierarchical clustering[7]. Several runs are made by *K*-means algorithm for each clustering, with different numbers in the initial partitions, until no decrease in square error value is observed for that clustering. Figure 2 presents the square error values and corresponding *F*-statistic values, It is observed that the values of square error and *F*-statistic are decreasing with increase in number of clusters. The optimum number of cluster groups is taken as six corresponding to *F*-statistic value of ten[9] and then the representative policy for each cluster is

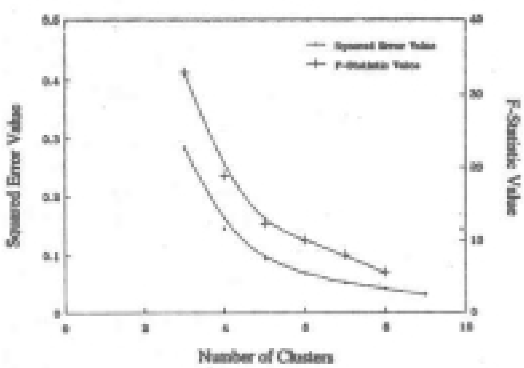

Figure 2 Squared error and *F*-statistic values for different clusters

Table 3 Alternative policies versus criteria array, L_p metric values and ranking pattern by compromise programming

Policy	Net benefits, million Rs	Crop production, Mt	Labour employment, million man-days	L_p metric value for $p = 1$	$p = 2$	$p = \infty$	Rank
G1	1152.00	0.777 18	35.250	0.623	0.512	0.497	6
G2	1301.80	0.772 50	36.300	0.474	0.372	0.354	5
G3	1439.50	0.763 86	38.492	0.330	0.240	0.223	3
G4	1586.50	0.741 19	40.331	0.201	0.118	0.083	1
G5	1599.40	0.663 74	43.239	0.264	0.177	0.159	2
G6	1478.70	0.589 89	45.500	0.456	0.321	0.263	4

Table 4 Preference matrix of multicriterion Q-analysis, L_p metric values and ranking pattern by multicriterion Q-analysis

Policy	Net benefits	Crop production	Labour employment	L_p metric value for $p = 1$	$p = 2$	$p = \infty$	Rank
G1	1.7	7.0	1.1	2.019	1.233	1.000	5
G2	3.2	6.9	1.6	1.455	1.090	1.000	4
G3	4.6	6.6	2.8	1.324	1.051	1.000	3
G4	6.1	6.0	3.8	0.929	0.564	0.415	1
G5	6.2	3.9	5.4	1.214	0.767	0.628	2
G6	5.0	2.0	6.6	2.641	1.608	1.174	6

determined. For this purpose the square error values between group mean and the weighted normalized proposed policy values for each criterion in that group are calculated. The summation of these square error values for all criteria gives the total square error value corresponding to each proposed policy in that group. The policy that gives the minimum total square error value is chosen as the representative policy for that group. The policies P04, P11, P17, P23, P29, P34 of Table 2 are found to be the representative ones of the six cluster groups (Results are not presented due to space limitation). The above groups are denoted as G1, G2, G3, G4, G5 and G6. Alternative policies *versus* criteria array (payoff matrix) is presented in Table 3.

FINAL RANKINGS

Interactive computer programs of MCQA and CP are developed. The programs have the capability to graphically display the ranking pattern in the form of bar chart. In CP the minimum and maximum values for each criteria for the 37 proposed policies are taken from Table 2. The L_p metric values (with respect to p) are computed using equation (3). The alternative having minimum L_p metric value is taken as the best compromise solution. Three compromise sets are calculated for $p = 1$, 2, and ∞. Table 3 presents L_p metric values and ranking pattern obtained by CP.

In MCQA, payoff matrix is transformed into a preference matrix based on a scale of 1 to 7, 1 representing the worst and 7 representing the best. The payoff matrix values are linearly interpolated to obtain the values of the preference matrix and presented in Table 4. The PSI, PCI and PDI are calculated for combined (optimum) threshold values of the preference matrix[11]. Table 4 presents L_p metric values and ranking pattern obtained for three different values of $p = 1,2$ and ∞ for combined (optimum) threshold values as per equation (4). All the three sets $p = 1,2$ and ∞ gave the same rank for each alternative. Alternatives G4, G5 and G3 occupied the first, second and third positions, respectively. Similar ranking pattern is observed both for CP and MCQA. There is a slight difference between the ranks of the other alternatives between the two methods. Extensive sensitivity analysis indicated that alternative policy G4 referring to policy P23 of Table 2 is the most suitable one for further investigation and implementation. It is observed from Table 1 that the best compromise solution yields net benefits of 1586 million Rs, 0.74 Mt of crop production and 40.33 million man-days. In the best plan more importance is given to sorghum(s), paddy(srf), groundnut(srf) and groundnut(w) by the model. Irrigation intensity has reached 126.4%. Net benefits, crop production and labour employment per hectare on average for the culturable command area (CCA) are Rs 8907.9, 4.16 t and 226 man-days, respectively. With the present inflow scenario, groundnut has emerged as the best suitable crop both for summer and winter seasons with paddy as the second best.

CONCLUSIONS

Two interactive MCDM methods CP and MCQA are implemented and applied to the case study of Sri Ram Sagar Project, Andhra Pradesh, India, for the selection of best irrigation planning strategy. Best compromise solution yields net benefits of 1586 million Rs, 0.74 Mt of crop production and 40.33 million man-days. In the best plan, more importance is given to the crops sorghum (summer), paddy(summer-rainfed), groundnut(summer-rainfed) and groundnut (winter) by the model. Net benefits, crop production and labour employment per hectare on average for the culturable command area (CCA) are Rs 8907.90, 4.16 t and 226 man-days, respectively with irrigation intensity of 126.4%. The ranking pattern is quite robust in both the methods to small parameter changes. Comparison of the results indicate that the methodologies are quite versatile and can be used in similar situations.

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