OPTIMAL DESIGN OF WATER DISTRIBUTION SYSTEM USING LINEAR PROGRAMMING GRADIENT (LPG) METHOD

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Introduction

Water Distribution Systems (WDS) are interconnected networks of pipe lines, valves of various types, hydrants, pumps, and reservoirs. The distribution system is an essential part of all water supply schemes. The cost of this portion of any sizable water supply scheme amount to as much as 70% of the entire cost of the project. Therefore, the cost of the distribution system must be reduced through optimization in the analysis and design. Water distribution systems can be designed either by conventional methods or by using mathematical models.

In conventional design the engineer has to initially specify the dimensions of the system based on his knowledge and experience. By mathematical calculations he then checks whether the velocities, flows and other system parameters are within the allowable limits. If not he will modify the system and repeat the procedure until all parameters are within the stated limits. In a model-based design the engineer can formulate by mathematical equations simultaneously the limits and objectives of the system. The optimization model then will produce the dimensions by which the objective at best is met within the limits of constraints. In other words, whereas conventional design calculation is a more a check up of a defined or assumed system, model-based design employs a mathematical model for defining and checking the system. Principally, two types of models can be developed for water distribution systems viz., simulation models and optimization models.

Many techniques were used to develop optimization models for the design of pipe networks. Linear Programming (LP), Non-linear programming, and more recently genetic algorithm technique, are some of the important techniques used for developing optimization models.

Significant works in LP based models were carried out by Karmeli[1], Kally[2], Quindry et al[3], Morgan and Goulter[4] and Alperovits and Shamir[5], Works by Watanatada[6], Shamir[7], Su et al[8] Lansey and Mays[9], and Cullinane et al[10] were important contributions in Non-linear programming models. Recently, Simpson et al[11] developed a new model using genetic algorithm technique for pipe optimization Deb and Sarkar[12] developed a technique called "Equivalent Diameter Method".

The model used in the present work which is called as "Linear Programming Gradient" (LPG) method, is based on LP and was developed by Alperovits and Shamir[5]. This method is applied for looped networks. The basic component of this method is Linear Programming (LP) formulation used for optimization of branched networks. The LPG method deals with looped networks and decomposes the optimization problem into a hierarchy of two levels. The LP is solved for an initially assumed flow distribution in the network. The results of LP are used to change the flows in such a way that it reduces the overall cost of the network. LP is solved with new flow distribution and the iterative procedure is continued until convergence i.e. no improvement in cost, is reached. The method can be applied for networks with pumps, valves, storages and optimization can be obtained by considering multiple loading patterns.

Model formulation

Consider a water distribution network with a number of sources supplied by gravity. At each of these nodes of the network j=1........N, a given demand d_j has to be satisfied. The head at each node H_j, is to be between a given minimum head, $Hmin_j$ and a maximum head, $Hmax_j$. As the layout of the network is given, let the length of the link connecting nodes i and j be L_{ij}. The LP design procedure is based on a special selection of decision variables namely, the lengths of the segments of constant diameter within the link. Let the length of the pipe section of the mth diameter in the link connecting nodes i and j be denoted by x_{ijm}. Then the relation between L_{ij} and x_{ijm} is given by

$$\sum_m x_{ijm} = L_{ij} \quad \text{...(1)}$$

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and the relation has to hold for all links, where the group of candidate diameters(m) may be different for each link. The flows in all links will be known in a branched network once the consumptions at nodes are known. But in a looped network, an initial flow distribution is assumed satisfying the demands at nodes.

The hydraulic gradient, J, is calculated using Hazen-William's equation,

$$J = \propto (Q/C)^{1.852} D^{-4.87}$$

where Q is discharge, C is Hazen-William's coefficient, D is the pipe diameter and $\propto$ is a coefficient whose value depends on the units used (162.7 for Q in m^3/h and D in inches). The head loss, then can be calculated as,

$$\Delta H_{ijm} = J_{ijm} x_{ijm}$$

where J_{ijm} is the hydraulic gradient of the mth diameter of link (i,j).

Starting from any node, s. in the system at which the head is known in advance (at a reservoir or source), the constraints on head at node can be written as.

$$Hmin_n \leq H_s \pm \sum_{i,j} \sum_{m} J_{ijm} x_{ijm} \leq Hmax_n \text{.......(2)}$$

where the first summation is over all links (i,j) and the second summation is over all the sections, m. in each link. The sign of term depends on the direction of flow i.e., if the direction of flow in the link is along the direction of the path considered, then the sign is taken as positive and it is negative otherwise. In a looped network, the head losses along certain paths in the network should satisfy the following type of constraints :

$$\sum_{i,j} \sum_{m} J_{ijm} x_{ijm} = b_p \text{....................................(3)}$$

where b_p is the head difference between the end nodes of the path p. Above equation has to hold for all closed paths i.e., loops, where b=0. For each pair of nodes at which the heads are fixed, above equation is formulated proceeding along any path which connects two nodes.

The cost of pipe line is assumed to be linearly proportional to its length, a reasonable assumption in most of the circumstances. Thus the total cost of the network is given by :

$$\sum_{i,j} \sum_{m} c_{ijm} x_{ijm} \text{....................................(4)}$$

where c_{ijm} is the cost of the m th diameter section in link (i,j). Hence the objective function is minimization of this cost function (eq. 4) subject to constraints and to non-negatively requirements,

$$x_{ijm} \geq 0. \text{ ....(5)}$$

The first step in developing the LPG method is to consider optimization of the design when the distribution of flows in the network is assumed to be known. The formulation consisting of the objective function given by equation (4) and the set of constraints of the type given in equations (1), (2), (3) and (5) is adopted in which the length of the sections of constant diameter in each link (x_{ijm}) are the decision variables. By solving the LP, a set of optimal segments of pipe lengths are obtained such that the network is hydraulically balanced and the cost is minimized for the specific flow distribution.

The next stage is to develop a method for systematically changing the flows in the network with the aim of minimizing the cost. The method for changing the flows is based on the use of dual variables of the constraints of type in eq. (2) and (3), which aid in defining a gradient move. We can solve for dual variables for the constraints of any LP formulation. The dual variable represents the unit worth of the quantity on the right hand side of the constraint on the objective function. For example, if we take a loop head loss constraint.

$$\sum_{i,j \in p} \sum_{m} J_{ijm} x_{ijm} = 0$$

the dual variable of this constraint indicates how the value of objective function i.e., cost, changes due to change in head loss from zero to some other value. So, in this model dual variable concept is used to change the flow in loops so that the cost is improved in each iteration until it converges. The gradient calculated using the dual variables is used to change the flow in such a way that the cost of the network is reduced with respect to the previous flow distribution.

The gradient expression for the path, p, derived by Alperovits and Shamir[5] is given as follows :

$$G_p = \frac{\partial (\text{cost})}{\partial (\Delta Q_p)} = W_p \sum_{i,j \in p} (1/Q_{ij}) \sum_{m} \Delta H_{ijm} \qquad (6)$$

Quindry et al[13] made changes in this expression and they considered interactions of the path with each other since a change in flow in one path affects the flow in other paths in the network. They modified the expression as,

$$G_p = \frac{\partial (\text{cost})}{\partial (\Delta Q_p)} = W_p \sum_{i,j \in p} (1/Q_{ij}) \sum_{m} \Delta H_{ijm} + \sum_{r \in R} \pm W_r \sum_{\substack{i,j \in r \\ i,j \in p}} (1/Q_{ij}) \sum_{m} \Delta H_{ijm} \qquad (7)$$

where R is used to denote all the paths other than p in the network and the sign of each additional term depends on whether or not path 'r' uses link i, j in the same direction as path 'p'. The sign is negative if the direction is same and positive otherwise.

After calculating the gradients for each path, the next step is to change the flow in the links of the paths. A heuristic approach was adopted for this purpose. A step size, given in terms of change in flow, is flexed at the start of program. The flow component or path which has the largest value of the gradient C_p is given a flow change of the specified step size, and the flow in the other paths are changed by quantities reduced by the ratio of their gradient to the largest gradient component. The sign of the flow change in the path can be decided as follows.

$$G_p = \frac{\partial (\text{cost})}{\partial (\Delta Q_p)}$$

We have to make the flow change in such a way that the overall cost is reduced i.e. ∂ (cost) is negative. So, for this to be negative the flow change should be positive if the gradient G is negative and vice versa.

In this way, the flow distribution in the network is changed at each iteration so as to reduce the overall cost of the network. The optimality is said to have reached when there is no improvement in the cost for the specified step size. It means that after a particular number of iterations, the cost starts increasing with respect to the previous iteration. At this stage the initially specified step size is reduced at the iteration, where the cost started increasing, so as to make finer changes in the flows and hence to get a better optimum value.

Extension of LPG method to complex systems

Pumps

The location at which pumps may be installed are selected but since the program can set certain pump capacities to zero, if that is the optimal solution, the program actually selects the locations at which pumps can be installed. The decision variables associated with each location at which the specified pump may be located are the heads to be added by the pump for each loading. The maximum of these specifies the pump capacity which has to be installed. If we denote by $H_p(t,1)$, the head added by pump number, t, and load, 1, then the set of head constraints of type given in eq. (2) for paths with pumps becomes,

$$Hmin_n(1) \leq H_s(1) \pm \sum_{t} H_p(t,1) \pm \sum_{i,j} \sum_{m} j_{ijm}(1)\, x_{ijn} \leq Hmax_n(1) \qquad (8)$$

Similarly, the set of constraints of type (3) becomes,

$$\sum_{i,j} \sum_{m} J_{ijm}(1)\, x_{ijm} \pm \sum_{t} H_p(t,1) = b_p(1) \qquad (9)$$

where the sign of terms depends on the direction of flow.

The decision variables for the pumps have to be introduced linearly into the objective function if the problem is to remain a LP problem. Even though the cost curve for pump as a function of its capacity is not exactly linear, it can be assumed linear and in the present study cost function is assumed linear. The power (Horse Power) needed to operate the pump is given by,

$$HP = \Upsilon \ Q H_p / \eta$$

where 'ϒ' is unit weight of water, Q is the flow, H_p is the head added by pump, and 'η' is its efficiency. If we assume a fixed value of efficiency, then for a fixed discharge through, pump the above equation becomes,

$$HP = K_t \cdot H_p$$

where K_t is a constant.

The equation shows that the power is linearly proportional to the head added by the pump. As the operating costs add only a constant value to the total cost, the optimum design will be same as that obtained without considering the operating costs. Thus, in this work, the operating costs of pumps are not considered. Only capital cost of the pump is considered. Since the curve is assumed linear, cost per horse power is assumed, and by multiplying is with K_t another constant is obtained. This value is introduced into the objective function as the coefficient of the decision variable for pump i.e., head added by the pump.

If the original cost curve is considered which is slightly non-linear, an iterative procedure is applied as follows: (1) Assume cost per hp from the actual data for each pump (2) solve the LP with these values multiplied by K_t as the coefficient in the objective function (3). For the resulting H_p, after the LP has been solved, compute the cost per hp. If all values are close enough to those assumed, this step is complete and we can proceed for next floe iteration. Otherwise we assume a new cost and solves the LP again. This procedure is continued until the actual values are close enough to those assumed.

In the present work a linear cost curve is assumed for implementing the complex network.

Reservoirs

The decision variable for a reservoir or overhead tank is the elevation at which it is to be located. An initial elevation is assumed, then Hr is the additional elevation where the reservoir is to be located relative to its initially assumed elevation. Path equations have to be formed between the

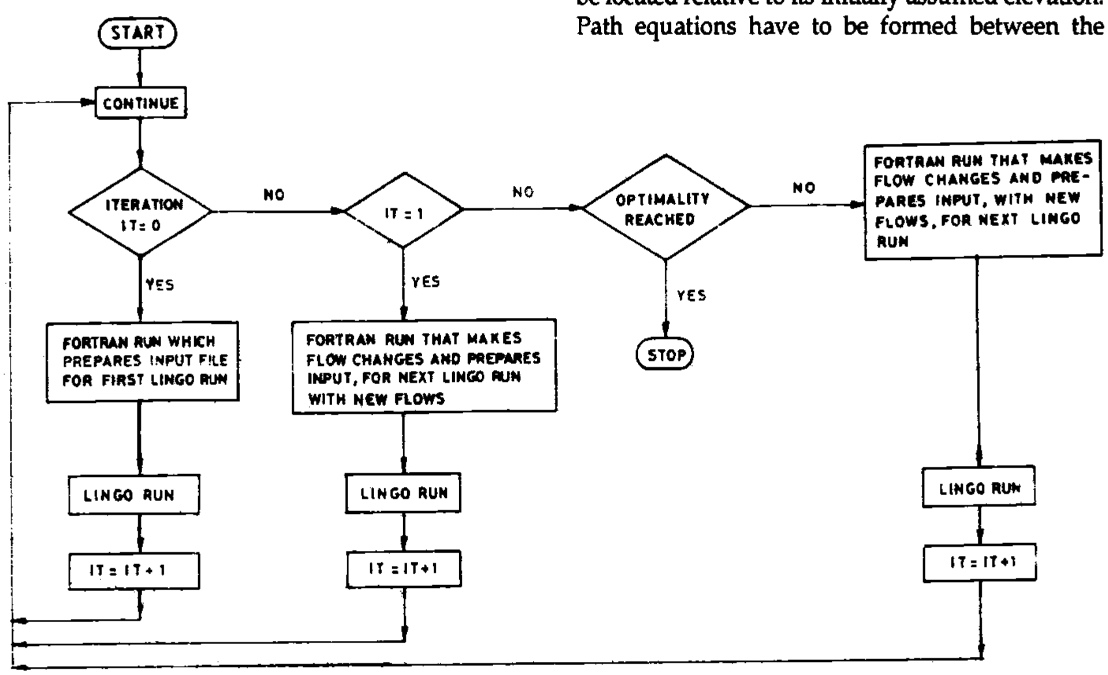

Fig. 1 Schematic representation of modal implementation

For node, n,

$$Hmin_n (1) \leq (Ho_s + Hr_s) \pm \sum_{i,j} \sum_{m} j_{ijm}(1) \; x_{ijm} \leq Hmax_n(1) \qquad (10)$$

reservoir at node 's' and nodes in the network. where Ho_s is the initial elevation of the reservoir at node 's' and Hr_s is the additional elevation to be selected by the program. The coefficient of Hr_s in the objective function is the cost of raising the location of the reservoir by one unit i.e., one metre.

Additions to an existing system

When parts of the network already exists and only new parts are to be designed, the existing components are specified as being fixed, and the program solves for the rest.

Two case studies are considered to illustrate the use of the model for optimizing Water Distribution Systems which are explained in the following sections.

Model Implementation

Some of the aspects of model implementation for the two case studies are described below.

Selection of candidate diameters

The list of candidate diameters for each link is based on a minimum and a maximum value of the hydraulic gradient, which is set in the design of WDS for the velocity to be maintained in a desired range in the system. For the initial assumed flow in each link a range of diameters can be found, which are commercially available, by substituting the minimum and maximum dydraulic gradient in the equation (11).

$$v = 4\, Q / \pi\, D^2$$

According to Hazen-William's equation,

$$v = 0.849\, C\, R^{0.63}\, j^{0.54}$$

where

R = hydraulic radius = D/4 for circular pipes

C = Hazen-William's coefficient

$$0.849\, C\, (D/4)^{0.63}\, j^{0.54} = 4\, Q/\pi\, D^2$$

or $$D^{2.63} = 0.349\, Q/C\, j^{0.54} \qquad (11)$$

The range of diameters in the candidate list should be neither too narrow, which may cause infeasibility nor too long, which increase the number of LP variables thus increasing the computing time.

FORTRAN 77 language along with a latest and efficient programming language called Language for Interactive General Optimization (LINGO) are used for model implementation. LINGO is used to solve the LP problem using Revised simplex method. A FORTRAN program was linked with the LINGO program to perform the iterations by calculating the gradients and check for optimality.

Setting up node pressure constraints

A minimum pressure must be maintained at all nodes of the network. The related pressure constraint should theoretically be established for all nodes. However, in practice the pressure becomes critical only at some high points or points remote from the source. These points are called critical nodes. Critical node is a node where the pressure constraint may become effective. In the implementation of the present work, such critical nodes were located and pressure constraints were set up only at those points thus simplifying the model considerably.

Implementation

The schematic representation of the implementation is shown in Fig. 1. At iteration = 0, the FORTRAN run takes candidate diameters, cost, minimum node pressures, flows in links and the total length of each link as the input and calculates the hydraulic gradient. The output from this run would be cost, hydralic gradients, minimum node pressures and length of each link. This output from the FORTRAN run forms the input for first LINGO run. The LINGO run optimizes the cost, and the decision variables and dual variables obtained from this LINGO run are used for making flow changes in the network for next iteration. At iteration 1 the output from the LINGO run and the basic data forms the input for FORTRAN run. This FORTRAN run calculates the gradient (G) value and makes flow changes in the network. It prepares the input for next LINGO run with these new flows. Again the LINGO run was made. At iteration greater than 1 the progra checks for optimality and if the optimality is reached the program stops otherwise the FORTRAN run is made similar to that at iteration 1.

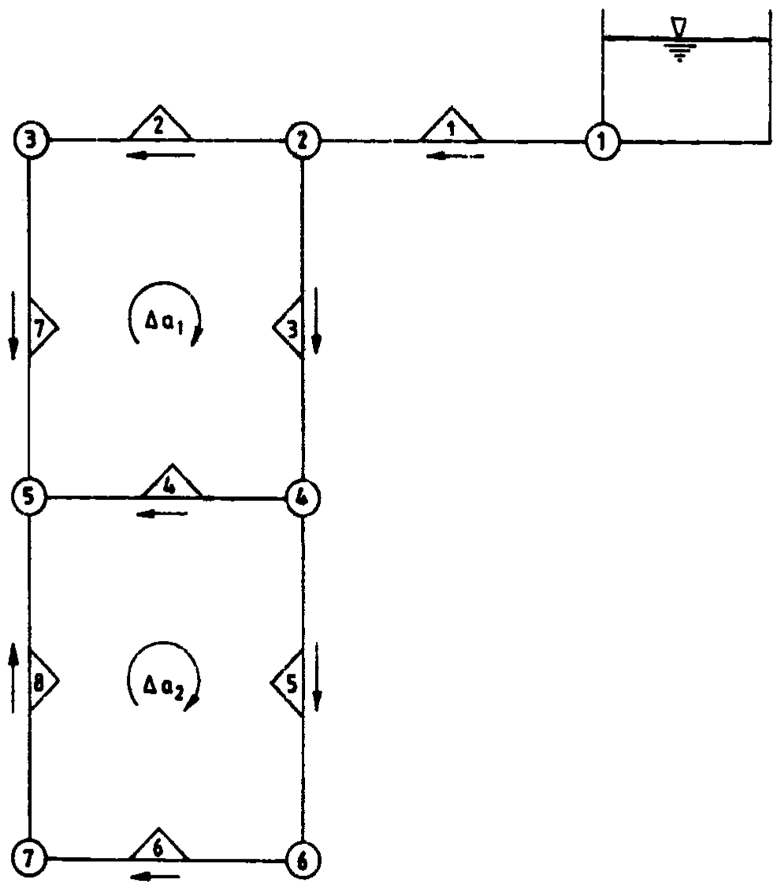

Fig. 2 Network of case study 1

Case studies

In both the case studies, costs are kept in arbitrary units as given in Alperovits and Shamir[5], so that the results obtained can be compared with the results given in the work.

Case study 1

The network taken for the first case study is shown in Fig. 2. The network has 7 nodes, and 8 links and the water is supplied from a constant head reservoir. It has two loops. The pipe cost data, is shown graphically in Fig. 3. The node data and link data are shown in Table 1 and 2. The remaining relevant data is available in the M.Tech. project report[14]. Five candidate diameters are assumed for each link based on the hydraulic gradient range of 0.0005 to 0.05. The results obtained for this network are outlined below.

In this network the cost reduced from 4.73,880 to 4,48,799 units, a reduction of 25,081 units. The optimum cost was reached in 17 iterations. Initially, a step size of 5 m^3/h was taken and at iteration number 16, where the objective function started increasing, the step size was reduced to a value of 1 m^3/h to facilitate finer changes in flows and hence better optimum cost. The iteration results are presented graphically in Fig. 4. The pipe diameters and section lengths at optimum are shown in Table 3.

Case study 2

The network taken for case study 2 is shown in Fig. 5. It is a complex network with 52 nodes, 65 links, and 15 loops. It is a combined gravity and pumping system. There are two pumps and a

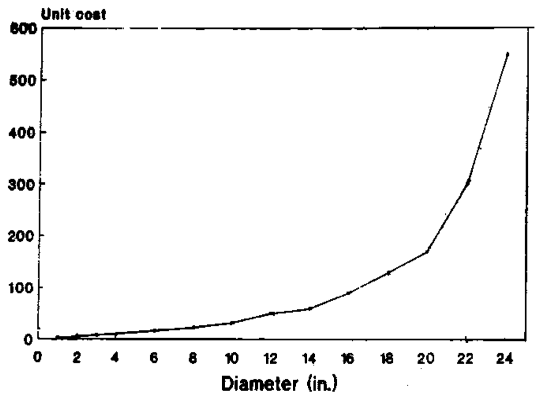

Fig. 3 Cost data graph for case study 1

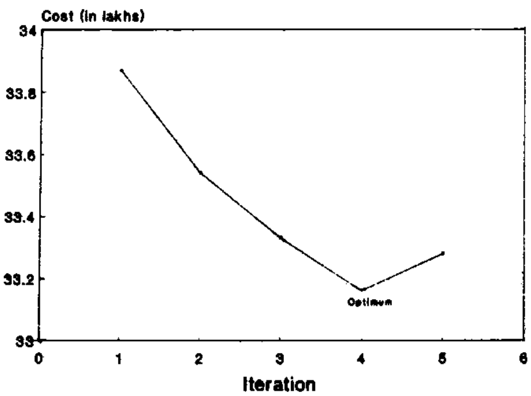

Fig. 4 Change of objective function value with iteration for case study 1.

balancing reservoir. Candidate diameter list was assumed based on the hydraulic gradient range of 0.025 to 0.005. The results of this network are outlined below.

The cost reduced from an initial value of 33,86,977 to an optimum value of 33,16,183, a reduction of 70,794 units. The optimum was reached in 4 flow iterations. The pump capacities obtained at optimum are 45.1 hp for pump 1 and 40.1 hp for pump 2. No additional elevation was required for the reservoir at optimum. So, the reservoir can be placed at its initially assumed elevation. The pattern of cost change with each iteration is shown in Fig. 6.

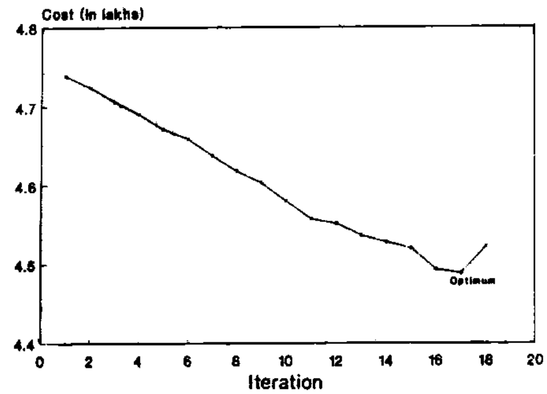

Fig. 6

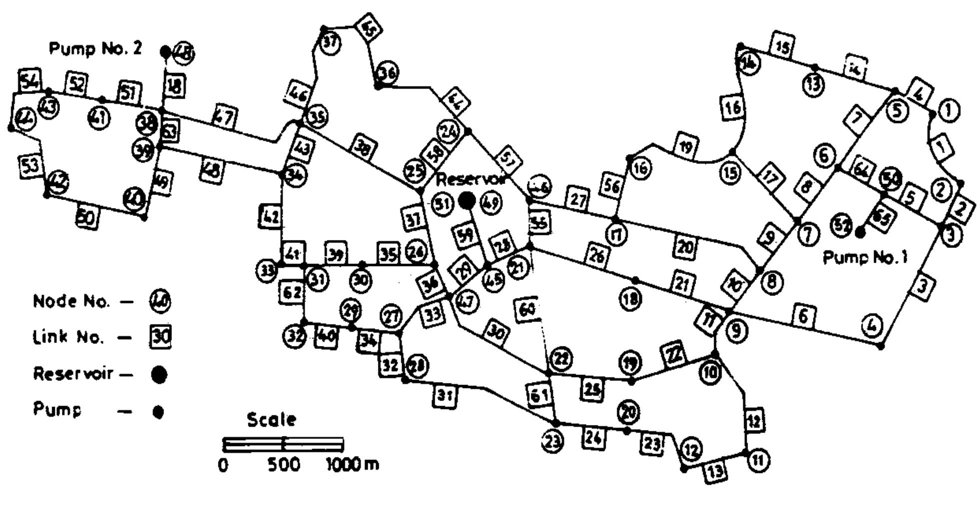

Fig. 5 Network of case study 2.

Table 1 Link data for case study 1

Section	Length	C	Initial flow Distribution (m^3/h)	Candidate Diameters (inches)
1	1000.0	130.0	1120.0	12,14,16,18,20
2	1000.0	130.0	220.0	6,8,10,12,14
3	1000.0	130.0	800.0	10,12,14,16,18
4	1000.0	130.0	30.0	3,4,6,8,10
5	1000.0	130.0	650.0	10,12,14,16,18
6	1000.0	130.0	320.0	8,10,12,14,16
7	1000.0	130.0	120.0	6,8,10,12,14
8	1000.0	130.0	120.0	6,8,10,12,14

Table 2 Node data for case study 1

Node	Elevation (metres)	Minimum Pressure Allowed (m)	Consumption (m^3/h)
1	210.0	0.0	−1120.0*
2	150.0	30.0	100.0
3	160.0	30.0	100.0
4	155.0	30.0	120.0
5	150.0	30.0	270.0
6	165.0	30.0	330.0
7	160.0	30.0	200.0

*negative sign indicates supply from the node

Table 3 network configuration for case study 1 at optimum

Link	Diameter (inches)	Length (m)
1	18	612.55
	20	387.45
2	8	75.26
	10	924.74
3	16	1000.00
4	3	858.42
	4	141.58
5	16	1000.00
6	10	302.61
	12	697.39
7	8	1000.00
8	6	1000.00

Conclusions

Linear Programming Gradient method was used for the optimal design of water distribution systems. The method was an iterative method. For a given flow distribution, the water distribution system is optimized by using Linear Programming (LP) and the flow was changed iteratively so as to reduce the overall cost of the system, using the gradients calculated from the results of LP (dual variables).

Two case studies were considered, one a simple network and the other a complex network. These were implemented in a computer program. An efficient optimization language Language for Interactive General Optimization (LINGO) was used for solving LP problem along with FORTRAN 77 programming language, to perform the iterations and check for optimality.

From the solutions obtained for the networks it was observed that improved results were obtained compared to the original work. In the first network the optimum was obtained in 17 flow iterations compared to 31 flow iterations in the original work. In the second network the optimum was obtained in 4 flow iterations. This method can be easily extended to multiple loadings case. This method can be applied for real, complex systems.

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