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An integrated model for optimal reservoir operation for irrigation of multiple crops

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Abstract. An integrated model is developed, based on seasonal inputs of reservoir inflow and rainfall in the irrigated area, to determine the optimal reservoir release policies and irrigation allocations to multiple crops. The model is conceptually made up of two modules. Module 1 is an intraseasonal allocation model to maximize the sum of relative yields of all crops, for a given state of the system, using linear programming (LP). The module takes into account reservoir storage continuity, soil moisture balance, and crop root growth with time. Module 2 is a seasonal allocation model to derive the steady state reservoir operating policy using stochastic dynamic programming (SDP). Reservoir storage, seasonal inflow, and seasonal rainfall are the state variables in the SDP. The objective in SDP is to maximize the expected sum of relative yields of all crops in a year. The results of module 1 and the transition probabilities of seasonal inflow and rainfall form the input for module 2. The use of seasonal inputs coupled with the LP-SDP solution strategy in the present formulation facilitates in relaxing the limitations of an earlier study, while affecting additional improvements. The model is applied to an existing reservoir in Karnataka State, India.

Introduction

The objective of the present study is to develop a mathematical programming model to determine the steady state optimal operating policy and the associated optimal crop water allocations to each crop for a single-purpose irrigation reservoir. The model should take into account the stochasticity of reservoir inflow and rainfall in the irrigated area, intraseasonal competition for water among multiple crops, soil moisture dynamics for each cropped area, the heterogeneous nature of the soil, and crop response to the level of irrigation applied. The model should be applicable to making reservoir release and irrigation allocation decisions in real time. The present paper is relevant in this context, and the issues involved pose a challenge even in the limited scope of the problem.

In a series of articles *Dudley et al.* [1971a, b, 1972] and *Dudley* [1972] dealt with modeling for irrigation planning with a hierarchy of short, intermediate, and long-run decisions to maximize the net benefits from the use of irrigation water. A combination of stochastic dynamic programming (SDP) and simulation was used as a solution technique. An integrated intraseasonal and interseasonal SDP model was developed by *Dudley and Burt* [1973]. All these models are essentially single-crop models. Relaxing the assumption of a single decision maker in communicating the stochastic nature of supplies and demands between the reservoir and farm managers is addressed in two different approaches, namely, volume sharing of reservoir [*Dudley*, 1988] and capacity sharing of reservoir [*Dudley and Musgrave*, 1988].

Dudley et al. [1976] developed a hierarchy of models to aid management and planning decisions in multicrop water resources systems. By using the reservoir level transition probabilities and the functional relationship between net revenues and reservoir releases derived by linear programming, optimal annual reservoir releases as functions of beginning-year reservoir level were derived by dynamic programming. Crop water requirements were assumed to be deterministic. Recently, *Dudley and Scott* [1993] developed methods and models for determining how large a farm ought to be under the institutional arrangement known as "capacity sharing." In these models it is presumed that the irrigator has a large tract of land with choices of cropping and of "abandoning" a part of it to rain-fed status, if doing so is found to be more profitable. The situation, however, in developing countries is quite different in the sense that the land holdings are relatively very small, and there is little choice of the cropping pattern, as this is by and large fixed or imposed by the project authorities. In addition, in the tropics, even the crop seasons are fixed because of the monsoon climatology. What are most necessary in such situations are multicrop models which optimize the crop output through the allocation of reservoir water. A literature search reveals that very little work has been done in this area. The uncertainties in reservoir inflow, rainfall, irrigation demand, and soil moisture together have not been considered in a single model thus far. This paper addresses these issues and presents an improvement of the recent work of *Vedula and Mujumdar* [1992].

Vedula and Mujumdar [1992] developed a model to obtain an optimal steady state reservoir operating policy for irrigation of multiple crops with stochastic inflows and crop water demands (implicitly stochastic) using stochastic dynamic programming. The model considers reservoir inflow, storage, and soil moisture in the irrigated area as state variables. The study, the first of its kind reported in the literature, has two phases. In the first phase, their model uses deterministic dynamic programming (DP) and allocates a given amount of water among all crops to optimize the impact of the allocation within a given

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period. This allocation is determined for all possible supplies in a given period and for all periods in a year. In the second phase an SDP model evaluates the system performance over all periods to optimize the overall impact of the allocations over a full year. The main contribution of the paper lies in the integration of the decision-making mechanism at the reservoir level and the farm level. This study, however, has some limitations: (1) the averaging of the soil moisture among all crops at the beginning of the period; (2) rainfall in the irrigated area being considered deterministic; and (3) the amount of allocation to a particular crop in a given period, not explicitly taking into account the allocations received by the crop in the earlier periods, due to a limitation in the technique used.

The present study removes the limitations mentioned above and provides improved features over the earlier work of *Vedula and Mujumdar* [1992]. The improvement is brought out by a change in the model formulation and methodology. Seasonal values of inflow and rainfall are considered rather than 10-day values. A linear programming–SDP (LP-SDP) approach is used instead of a DP-SDP approach. This facilitated removing the first and third limitations, stated above. The second limitation is removed by considering the rainfall in the irrigated area as stochastic. Evapotranspiration, however, is considered deterministic in both studies.

There are other conceptual improvements in the present study as well. The soil moisture in module 1 is not restricted to any set of discrete values as in the earlier study [*Vedula and Mujumdar*, 1992]. The data requirements are made relatively simple, and the scope for bias in estimating the data needs for real time operation is reduced. The earlier study considered a year as a single season consisting of thirty-six 10-day periods; therefore that study's model requires forecasts of 10-day flows for model use in real time operation. The present model, however, considers a year consisting of two seasons, the monsoon (Kharif) season and the nonmonsoon (Rabi) season, with the same total of thirty-six 10-day intraseasonal periods. The advantage here is that only seasonal values of inflow and rainfall need be considered. This helps the irrigation managers in planning their operations in advance of a season. The SDP formulation in the earlier study is based on transitions of 10-day values of inflow (rainfall being deterministic), whereas the present study is based on transitions of seasonal values of inflow and rainfall (both being stochastic). Both studies derive the steady state operating policy while maximizing the expected annual sum of relative crop yields. Seasonal forecasts are considered relatively more reliable and more easily obtainable than short-term forecasts of 10-days in aiding model application in real time. The strategy of using seasonal inputs together with the LP-SDP formulation facilitates the accounting of soil moisture balance and allocation of the irrigation water individually for each crop in each period, looking at the entire crop season. This was not possible in the earlier study. Estimate of seasonal rainfall in advance of a season can usually be obtained from a meteorological office, while any conventional forecasting model may be used to estimate the seasonal inflow. The seasonal inflow and rainfall values are disaggregated into those of 10-day intraseasonal periods using a simple disaggregation model.

The model and its application are presented briefly in the following sections. The details are reported by *Nagesh Kumar* [1992].

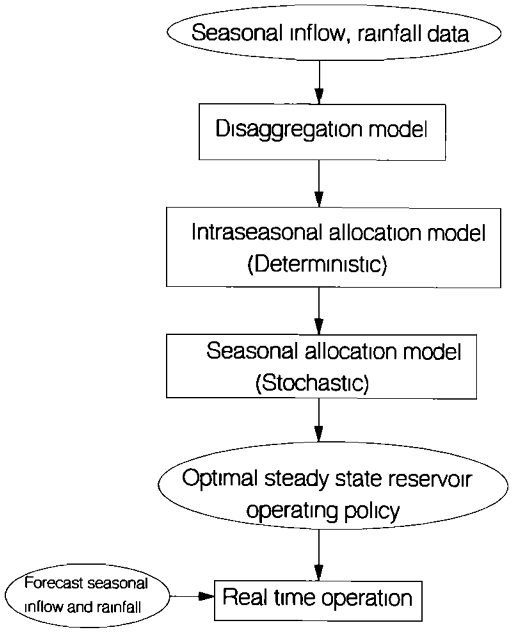

Figure 1. Schematic diagram of the reservoir operation model.

Reservoir Operation Model

The model formulation conceptually consists of two modules. Module 1 is intraseasonal modeling for allocation decisions within a season for given seasonal inputs. Module 2 is seasonal modeling for decisions over the seasons of a year resulting in the maximization of expected annual system performance. LP is used in module 1, and SDP is used in module 2 as optimization tools.

LP is used in module 1 to maximize the sum of relative yields of all crops for a given state of the system (defined by reservoir storages at the beginning and end of the season, seasonal inflow, and seasonal rainfall). Requirements of reservoir water balance in each period and soil moisture balance for each crop in each period form the constraints. SDP is used in module 2 to derive the steady state operating policy for the reservoir. The results of module 1 and the seasonal transition probabilities of reservoir inflow and those of rainfall form the input to module 2. Figure 1 shows the schematic diagram of the reservoir operation model developed in the present study.

The intraseasonal allocation model (module 1) has the capability to take into account (1) various crops with different crop durations staggered within the season; (2) the heterogeneity of soil types in the irrigation area; (3) soil moisture balance for each crop for each soil type; (4) crop root growth with time, as may be specified for each crop; and (5) specified irrigation policy for the crops. A linear crop root growth is assumed in the present model. The irrigation policy adopted is to allocate irrigation water to a crop to bring up the soil moisture in the cropped area as close to the field capacity as possible, whenever the actual soil moisture falls below the field capacity.

The seasonal allocation model (module 2) considers the stochasticity of seasonal inflow into the reservoir and rainfall over the irrigated area commanded by the reservoir, hereinafter referred to as the reservoir command. The reservoir inflow and the rainfall in the reservoir command are assumed to be independent, as the physiographic and meteorological characteristics of the watershed would very often be quite different from those of the reservoir command.

As for crop water demands, the stochastic nature of the demands is implicitly taken into account while determining the irrigation allocations for each crop through soil moisture balance in which rainfall stochasticity is considered. The crops, crop calendar, and the cropped areas are assumed fixed in the model. In modeling for multiple crops, the time period (intraseasonal period) is chosen such that the lengths of growth stages of all crops are integral multiples of the chosen time period, which is 10 days in the present case.

Intraseasonal Allocation Model

The intraseasonal model is solved for all possible combinations of input states. The inputs are the reservoir storages at the beginning and end of the season, seasonal inflow, and seasonal rainfall. The intraseasonal period values of inflow and rainfall are obtained from the seasonal inflow and rainfall, using a simple disaggregation model. The disaggregation model used is based on the conditional expectations of intraseasonal period values for a given seasonal value. These are derived from historical data or from synthetically generated data in case the historical data are not of sufficient length. The various aspects of the intraseasonal model are described next.

Crop yield response. The following additive form of the crop response function [*Doorenbos and Kassam*, 1979] is used for each crop in the present study:

$$\frac{y}{y_{\max}} = 1 - \sum_{g=1}^{\mathrm{NGS}} ky_g(1 - \mathrm{AET/PET})_g \qquad (1)$$

where y is the actual crop yield, $y_{\max}$ is the maximum yield, g is the growth stage index, NGS is the number of growth stages within the growing season of a crop, ky_g is the yield response factor for the growth stage g, AET is the actual evapotranspiration, and PET is the potential evapotranspiration.

Crop root growth. A crop is assumed to have five growth stages: establishment, vegetative, flowering, yield formation, and ripening. The root is assumed to grow linearly from zero depth at the beginning of the crop season to its full value at the end of the flowering stage and remain constant thereafter until the end of the crop season. The root depth in any period is taken as the average of the root depths at the beginning and end of the period.

Objective function. The sum of the relative yields of all crops in the season is taken as a measure of the relative crop yield for the season, as the formulation is for multiple crops.

The following objective function for the allocation problem is thus considered:

$$\mathrm{Max} \sum_{c=1}^{\mathrm{NC}} \left[1 - \sum_{g=1}^{\mathrm{NGS}} ky_g^c \left(1 - \sum_{t\in g} \mathrm{AET}_t^c \Big/ \sum_{t\in g} \mathrm{PET}_t^c \right)_g \right] \qquad (2)$$

where c is the crop index, ky_g^c is the yield response factor for the growth stage g of the crop c (which in reality varies within the growth stage with time but is assumed to be the same in each of the periods t within the growth stage g, following *Bras and Cordova* [1981], *Rao et al.* [1990], and *Vedula and Mujumdar* [1992]), and NC is the number of crops.

The objective function will be at its maximum possible value of 1 * NC when the allocation of available water to individual crops is such that AET = PET for each crop in each period. Whenever this is not possible, the irrigation allocation is made such that the total relative yield is maximized.

Potential evapotranspiration. PET, or the crop consumptive use, in any period t is determined by

$$\mathrm{PET}_t = k_t \mathrm{ET}_o^t \qquad (3)$$

where k_t is the crop factor, and ET_o^t is the reference evapotranspiration, determined by

$$\mathrm{ET}_o^t = k_{\mathrm{pan}} E_{\mathrm{pan}}^t \qquad (4)$$

where k_{pan} is the pan coefficient, and E_{pan}^t is the measured pan evaporation for the reservoir command for period t.

Actual evapotranspiration. In the present model it is assumed that AET = PET only when the soil moisture is at field capacity and that AET decreases linearly with the decrease in the soil moisture from the field capacity [*Doorenbos and Kassam*, 1979].

The various assumptions and the constraints of the intraseasonal model are presented in the following sections.

Reservoir water balance. The reservoir water balance is governed by the reservoir storage continuity equation:

$$S_t + Q_t - R_t - L_t - \mathrm{OVF}_t = S_{t+1} \qquad (5)$$

where S_t is the active storage at the beginning of the period t, Q_t is the reservoir inflow during the period t, R_t is the reservoir release (for irrigation) in the period t, L_t is the evaporation loss from the reservoir in period t, and OVF_t is the overflow from the reservoir during the period t.

The evaporation loss, L_t, in each period, t, may be approximated following *Loucks et al.* [1981]. The storage continuity equation (5) in the light of this approximation becomes

$$(1 - a_t)S_t + Q_t - R_t - \mathrm{OVF}_t - A_o e_t = (1 + a_t)S_{t+1} \qquad \forall\ t \qquad (6)$$

where

$$a_t = A_a e_t / 2 \qquad \forall\ t \qquad (7)$$

and A_o is the water spread area corresponding to the dead storage volume, A_a is the water spread area per unit active storage volume above the dead storage level, and e_t is the evaporation rate in period t.

X_t, the total amount of irrigation water available at the farm level, is given by

$$X_t = \eta R_t \qquad (8)$$

where η is the conveyance efficiency.

In (6), S_t, Q_t, R_t, and OVF_t are in volume units, whereas e_t is in depth units. Reservoir storage in any period should not exceed the capacity (active), $S_{\max}$,

$$S_t \leq S_{\max} \qquad \forall\ t \qquad (9)$$

The intraseasonal allocation model requires the storages at the beginning and end of the season T to be specified. The specified values are respectively equal to the representative values of the storage class intervals at the corresponding times.

Soil moisture balance. In the beginning of the season, soil moisture is assumed to be known. Here it is assumed to be at field capacity for all soils and crops:

$$\mathrm{SM}_1^c = \mathrm{SM}_{\max}^c \qquad \forall\ c \qquad (10)$$

where SM_1^c is the available soil moisture in depth units per unit root depth (L/L units) at the beginning of the first period ($t = 1$) for the crop c, and $SM_{\max}^c$ is the maximum available soil moisture at field capacity for crop c (L/L units).

It is also assumed that the soil moisture is at the field capacity in the incremental depth over which the crop root grows during each period. The soil moisture balance equation for a given crop c and period t is given by

$$SM_{t+1}^c D_{t+1}^c = SM_t^c D_t^c + IR_t + x_t^c - AET_t^c + SM_{\max}^c (D_{t+1}^c - D_t^c) - DP_t^c \qquad \forall\ c \text{ and } t \tag{11}$$

where SM_t^c is the available soil moisture at the beginning of the period t for the crop c (L/L units), D_t^c is the average root depth of crop c in period t, IR_t is the rainfall in period t in depth units, x_t^c is the irrigation water allocated to crop c in period t (depth units), AET_t^c is the actual evapotranspiration during period t for crop c (depth units), and DP_t^c is the deep percolation for crop c in period t (depth units).

The available soil moisture in any period t for crop c cannot exceed the value that corresponds to field capacity, $SM_{\max}^c$.

$$SM_t^c \le SM_{\max}^c \qquad \forall\ c \text{ and } t \tag{12}$$

The upper bound for AET is PET, and therefore

$$AET_t^c \le PET_t^c \qquad \forall\ c \text{ and } t \tag{13}$$

where PET_t^c is the potential evapotranspiration of crop c in period t.

The linear relationship between AET, PET, and the soil moisture is

$$AET_t^c \le \frac{SM_t^c D_t^c + IR_t + x_t^c}{SM_{\max}^c D_t^c} PET_t^c \qquad \forall\ c \text{ and } t \tag{14}$$

Allocation constraints. To assure that the reservoir release is properly allocated for different crops and periods within a growth stage, the following assumptions and constraints are used in the intraseasonal allocation model.

The crop yield function (1) is a function of AET. AET is a function of the irrigation allocation in a given growth stage, without regard to the distribution of this allocation among the time periods within the growth stage. To avoid possible undue concentration (of the allocations) in some of the periods, the irrigation water within a growth stage is assumed to be uniformly distributed among the periods of the growth stage.

$$x_t^c = RG_g^c / NP_g \qquad \forall\ c \text{ and } t, \tag{15}$$

except for t belonging to $g = 1$, where RG_g^c is the irrigation allocation for the growth stage g of the crop c (depth units), and NP_g is the number of periods in the growth stage g.

The uniform distribution assumption is relaxed for the first growth stage of all crops to avoid an anomaly. Because the soil moisture is assumed to be at field capacity at the beginning of the first period of the first growth stage, there will not normally be any irrigation requirement during the first period, whereas irrigation may be required in subsequent periods. The uniform distribution assumption for the first growth stage is relaxed to accommodate this situation.

In any period the total water allocated to all crops should equal the water available for allocation, X_t.

$$\sum_c x_t^c \, AREA^c = X_t \qquad \forall\ t \tag{16}$$

where $AREA^c$ is the area (assumed fixed) irrigated under crop c.

For any growth stage of any crop, the total allocation made should equal the sum of allocations made in all the periods of that growth stage.

$$\sum_{t \in g} x_t^c = RG_g^c \qquad \forall\ c \text{ and } g \tag{17}$$

The model should also take into account crops whose durations are longer than a season. In this case a convenient way is to specify that the end-of-season soil moisture should be at field capacity (to be consistent with the assumption of the soil moisture being at field capacity at the beginning of each season for all crops). The model forces irrigation to satisfy this requirement.

To ensure that soil moisture reaches field capacity before deep percolation occurs, and that the reservoir does not spill before reaching its capacity, a penalty term is added to the simple objective function (2) as follows:

$$\text{Max} \sum_{c=1}^{NC} \left[1 - \sum_{g=1}^{NGS} ky_g^c \left(1 - \sum_{t \in g} AET_t^c \Big/ \sum_{t \in g} PET_t^c \right)_g \right] - M \left(\sum_c \sum_t DP_t^c + \sum_t OVF_t \right) \tag{18}$$

where M is arbitrarily large.

The modified objective function (18) plus the constraints (6) through (17) constitutes the LP model. Figure 2 shows a block diagram of the intraseasonal allocation model with input and output details.

The model gives solution for each season (T) for a given reservoir storage class (k), seasonal inflow class (i), seasonal rainfall class (m), and final storage class (l). The maximized relative yield is denoted as $B(k, i, l, m, T)$. The model is solved for all feasible combinations of k, i, l, m, and T. The model solution gives for each period within the season, optimal irrigation allocation to each crop, the reservoir release, the reservoir storage at the beginning of the period, the soil moisture at the beginning of the period for each crop, deep percolation from each crop area, and evaporation loss from the reservoir.

Seasonal Allocation Model

The seasonal allocation model (module 2) gives the optimal steady state operating policy of the reservoir over the seasons. SDP is used for this purpose. The derived steady state operating policy specifies optimal end-of-season reservoir storage, for given conditions of initial reservoir storage, seasonal inflow, and seasonal rainfall. The policy implicitly specifies the optimal irrigation allocations in each of the intraseasonal periods for each of the crops.

State variables and discretization. Stochastic variables in the seasonal allocation model include seasonal inflow to the reservoir, seasonal rainfall in the reservoir command, and reservoir storage. Seasonal inflow and seasonal rainfall are each assumed to constitute a stationary Markov process. The model incorporates the stochasticity of the variables through their transition probabilities.

Each state variable is discretized into different classes (also called class intervals) in the SDP model. All values of the variable falling in a particular class (in model application) are

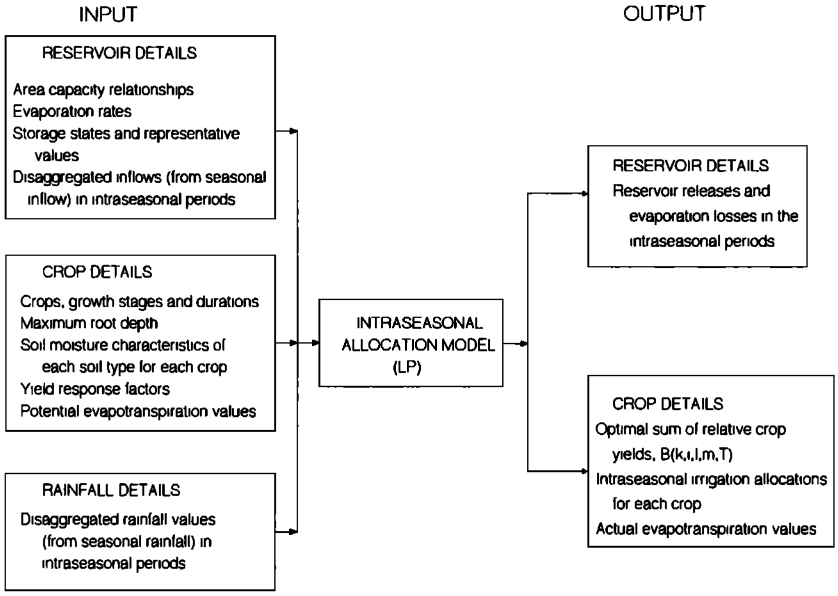

Figure 2. Block diagram for the intraseasonal allocation model (module 1).

represented by a single discrete value (within the class) which is taken to be its representative value (mean in the present case).

Recursive relation. Backward recursion is used in the present case to solve the SDP model. The study is set to start from the last season of some arbitrary year chosen long enough in the future to enable the derivation of a steady state operating policy from the model solution [*Loucks et al.*, 1981].

Let N define the number of seasons remaining and $f_T^N(k, i, m)$ represent the total expected value of the system performance with N seasons to go, including the current season T, given that the initial storage is S_k^T, the inflow is I_i^T, and the rainfall in the reservoir command is RAIN_m^T in the current season T. With only one season remaining ($N = 1$ and $T = TL$, the last season),

$$f_{TL}^1(k, i, m) = \underset{\text{feasible } l}{\mathrm{Max}} \, [B(k, i, l, m, TL)] \qquad \forall \; k, i, m \tag{19}$$

For $N = 2$ and $T = TL - 1$,

$$f_{TL-1}^2(k, i, m) = \underset{\text{feasible } l}{\mathrm{Max}} \left[B(k, i, l, m, TL - 1) + \sum_j \sum_n \mathrm{PI}_{ij}^{TL-1} \mathrm{PR}_{mn}^{TL-1} f_{TL}^1(l, j, n) \right] \qquad \forall \; k, i, m \tag{20}$$

where PI_{ij}^{TL-1} is the transition probability of seasonal inflow from class i in season $TL - 1$ to class j in season TL, and PR_{mn}^{TL-1} is the transition probability of seasonal rainfall from class m in season $TL - 1$ to class n in season TL.

In general, for stage N and season T, (20) can be generalized as

$$f_T^N(k, i, m) = \underset{\text{feasible } l}{\mathrm{Max}} \left[B(k, i, l, m, T) + \sum_j \sum_n \mathrm{PI}_{ij}^T \mathrm{PR}_{mn}^T f_{T+1}^{N-1}(l, j, n) \right] \qquad \forall \; k, i, m \tag{21}$$

In (21) T is reckoned $1, 2, \cdots, TL$ in the forward direction, and N, the stage number, is reckoned $1, 2, \cdots$ backward from the last season. The use of both indices, T and N, facilitates tracing the stage-by-stage movement in the dynamic program.

The recursive equations are solved for each season. The policy $l(k, i, m, T)$, gives the end-of-period storage class, l, as a function of k, i, m, and T. This policy will relatively quickly repeat itself in successive years. The steady state policy is reached when this occurs, implying that the expected annual performance $[f_T^{N+\mathrm{NS}}(k, i, m) - f_T^N(k, i, m)]$ is constant for all k, i, and m and for each season T, over a year, where NS is the number of seasons in the year.

The optimal final storage class, l^*, is thus obtained for given k, i, m, and T from the steady state operating policy. Associated with the optimal final storage class in a season are optimal intraseasonal crop allocations in each growth stage for each crop in each period. Figure 3 shows a block diagram of the seasonal allocation model.

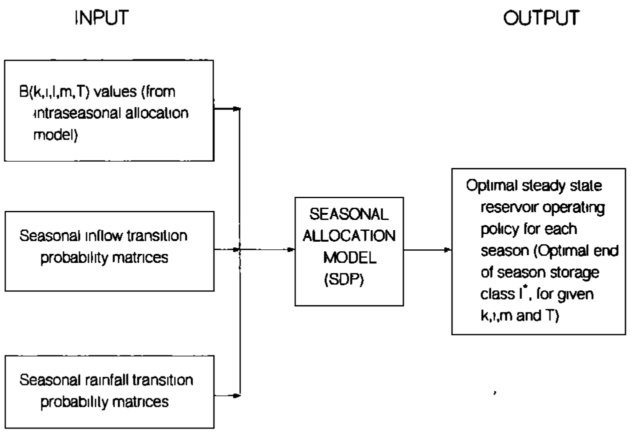

Figure 3. Block diagram of the seasonal allocation model (module 2).

Table 1. Typical Output From Intraseasonal Allocation Model for the Kharif Season

	Intraseasonal Period t														
Crop	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Release From the Reservoir in Each Period, M m³															
	0.0	0.0	1.0	1.0	1.1	27.6	27.6	0.2	0.2	0.2	0.2	1.3	0.0	10.1	10.1
Reservoir Storage at the Beginning of Each Period, M m³															
	17	30	46	96	209	312	378	457	588	672	718	759	805	852	870
Irrigation Allocation for Each Period for Each Crop, M m³															
Maize	0.0	0.0	1.0	1.0	1.0	10.9	10.9	0.0	0.0	0.0	0.0	1.3	···	···	···
Pulses	0.0	0.0	0.0	0.0	0.2	0.2	0.2	0.2	0.2	0.2	0.2	···	···	···	···
Sorghum	0.0	0.0	0.0	0.0	0.0	16.6	16.6	0.0	0.0	0.0	0.0	0.0	···	···	···
Cotton	···	···	···	···	···	···	0.0	0.0	0.0	0.0	0.0	0.0	0.0	10.1	10.1
AET Value for Each Period for Each Crop, mm															
Maize	13.1	13.1	44.3	37.7	38.8	56.6	52.1	42.6	44.0	39.4	39.4	23.7	···	···	···
Pulses	9.1	9.1	31.3	26.6	34.3	35.3	32.4	33.6	34.7	45.7	45.7	···	···	···	···
Sorghum	10.4	10.4	39.2	33.3	34.2	54.2	49.9	34.0	37.0	33.2	33.2	22.8	···	···	···
Cotton	···	···	···	···	···	···	21.7	22.4	23.1	33.2	33.2	33.2	25.4	51.1	52.7
Soil Moisture at the Beginning of Each Period for Each Crop, mm/cm															
Maize	2.5	2.5	2.5	2.2	2.2	2.1	2.2	2.3	2.1	2.2	2.4	2.4	2.5	···	···
Pulses	2.5	2.5	2.5	2.4	2.4	2.4	2.3	2.3	2.2	2.4	2.5	2.5	···	···	···
Sorghum	2.5	2.5	2.5	2.3	2.2	2.1	2.1	2.2	2.1	2.2	2.4	2.5	2.5	···	···
Cotton	···	···	···	···	···	···	2.5	2.5	2.5	2.5	2.5	2.5	2.4	2.5	2.5
Deep Percolation Value for Each Period for Each Crop, mm															
Maize	12.2	6.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	···	···
Pulses	16.1	10.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	···	···	···
Sorghum	14.8	8.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.2	6.3	0.0	···	···
Cotton	···	···	···	···	···	···	9.6	1.0	29.2	24.0	13.2	0.0	1.3	0.0	0.0
Overflow From the Reservoir in Each Period, M m³															
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Evaporation Losses From the Reservoir in Each Period, M m³															
	3.1	3.2	3.5	2.7	3.3	3.8	4.2	4.8	5.4	6.0	6.3	6.5	5.6	5.6	5.6

Here $k = 2$, $i = 2$, $l = 15$, $m = 4$, $T = 1$, $B(k, i, l, m, T) = 3.994$, $S_k^T = 17.0M$ m^3, and $S_l^{T+1} = 842.5M$ m^3.

Real time operation. The reservoir operation model developed can be used in real time operation. Although the storage at the beginning of a given season is known, the inflow and rainfall during the season are not known a priori. Therefore a forecast of the seasonal inflow and rainfall for the current season is required at the beginning of the season itself. With these inputs, optimal end-of-period storages and the associated releases for any given period are obtained from the steady state policy derived from the reservoir operation model. Details of the model application in real time are reported by *Nagesh Kumar* [1992].

Discussion of model features. The primary thrust of this work lies in the formulation of a stochastic multicrop model reflecting field conditions to the best possible extent. The existing models were examined, and a new mathematical programming model with conceptual improvements is presented. These improvements include consideration of (1) the stochastic nature of inflow and rainfall, (2) soil moisture balance for each crop taking crop root growth into account, (3) the heterogeneity of soils, and (4) irrigation allocation to each crop in each period, keeping the entire crop season in view. All of these features together are not considered in any of the existing models. The use of seasonal inputs coupled with the LP-SDP solution strategy in the present formulation facilitated in overcoming the limitations of the earlier study by *Vedula and Mujumdar* [1992], while effecting improvements. The model application presented below shows how it works and what results can be expected of it.

Model Application

Applicability of the developed model is demonstrated for the case of the Malaprabha single-purpose irrigation reservoir in the Krishna basin of Karnataka State, India (the same case used in the earlier study of *Vedula and Mujumdar* [1992]).

Ten-day periods were considered in the study. The water year, which begins on June 1 and ends on May 31, is divided into thirty-six 10-day periods, with three periods in each month. For modeling purposes, a year is divided into two seasons: season 1 (periods 1–15; Kharif (monsoon) season) and season 2 (periods 16–36; Rabi season, including summer). The last five periods of the year, that is, periods 31 through 36, have no irrigation activity.

Daily inflows for a period of 38 years, from June 1951 to May 1989, are available. Daily rainfall data are available for a period of 88 years from June 1901 to May 1989 at different locations in the reservoir command. The spatial averages of daily rainfall for the reservoir command were computed using Thiessen weights for the gauging stations within.

There are three crops within the kharif season, four within the rabi season, and a two-season crop starting in the Kharif season and ending in the Rabi season. The principal crops, their areas, and the crop calendar are as reported by *Vedula and Mujumdar* [1992].

Ten-day streamflows were generated using the Thomas-Fiering model and discretized into five class intervals. Seasonal inflow transition probabilities were obtained from the gener-

Table 2. Typical Output From Intraseasonal Allocation Model for the Rabi Season

Crop	Intraseasonal Period t															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Release From the Reservoir in Each Period, M m³																
	45.3	56.0	60.1	49.8	43.8	49.6	22.7	20.4	20.4	15.6	13.6	0.4	0.0	0.0	0.0	0.0
Reservoir Storage at the Beginning of Each Period, M m³																
	760	677	568	452	355	270	174	130	93	55	25	0	1	1	2	2
Irrigation Allocation for Each Period for Each Crop, M m³																
Sorghum	4.3	4.3	7.9	7.9	7.9	13.6	13.6	8.0	8.0	8.0	8.0	0.0	···	···	···	···
Pulses	1.9	1.9	2.5	2.5	4.7	4.7	4.7	4.7	4.7	0.0	0.0	···	···	···	···	···
Wheat	8.7	17.3	17.3	17.3	26.9	26.9	0.0	0.0	0.0	0.0	···	···	···	···	···	···
Safflower	2.2	4.4	4.4	4.4	4.4	4.4	4.4	7.7	7.7	7.7	5.7	0.4	···	···	···	···
Cotton	28.3	28.2	28.2	17.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
AET Value for Each Period for Each Crop, mm																
Sorghum	13.6	13.6	25.6	23.3	24.1	38.2	40.4	29.0	30.0	35.1	32.8	18.6	···	···	···	···
Pulses	11.9	11.9	21.1	18.7	24.1	24.9	26.3	27.2	28.1	48.3	37.8	···	···	···	···	···
Wheat	13.6	27.7	27.3	24.9	38.6	39.8	26.3	24.8	23.3	8.3	···	···	···	···	···	···
Safflower	13.6	27.3	27.3	24.9	25.7	26.5	28.1	43.5	44.9	52.7	28.7	26.6	···	···	···	···
Cotton	42.6	42.6	42.6	38.9	38.3	34.2	31.8	20.3	18.3	19.0	15.4	12.6	11.1	7.7	7.2	6.9
Soil Moisture at the Beginning of Each Period for Each Crop, mm/cm																
Sorghum	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.4	2.4	2.3	2.2	2.1	···	···	···
Pulses	2.5	2.5	2.5	2.4	2.4	2.4	2.5	2.5	2.5	2.5	2.1	1.8	···	···	···	···
Wheat	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.3	2.1	1.9	1.8	···	···	···	···	···
Safflower	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.4	2.5	2.3	···	···	···
Cotton	2.5	2.5	2.5	2.5	2.4	2.1	1.8	1.6	1.4	1.2	1.1	0.9	0.8	0.7	0.7	0.6
Deep Percolation Value for Each Period for Each Crop, mm																
Sorghum	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	···	···	···
Pulses	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	···	···	···	···
Wheat	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	···	···	···	···	···
Safflower	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	···	···	···
Cotton	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Overflow From the Reservoir in Each Period, M m³																
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Evaporation Losses From the Reservoir in Each Period, M m³																
	4.9	4.5	4.0	3.0	2.4	2.1	1.8	1.6	1.5	1.4	1.4	1.3	1.7	1.9	1.9	2.5

Here $k = 13$, $i = 1$, $l = 1$, $m = 1$, $T = 2$, $B(k, i, l, m, T) = 4.376$, $S_k^T = 760.5M$ m³, and $S_l^{T+1} = 2.5M$ m³.

Table 3. Optimal Steady State Operating Policy for the Kharif Season

Initial Storage State, k	Seasonal Rainfall State, m									
	1		2		3		4		5	
	l^*	OF	l^*	OF	l^*	OF	l^*	OF	l^*	OF
1	9	3.7020	11	3.8007	12	3.8703	13	3.8954	13	3.9907
2	9	3.7176	11	3.8163	12	3.8859	13	3.9124	14	3.9907
3	10	3.6948	12	3.7936	12	3.9118	13	3.9355	14	3.9907
4	10	3.7208	12	3.8195	13	3.8891	14	3.9153	15	3.9907
5	11	3.6981	12	3.8454	13	3.9150	14	3.9384	15	3.9907
6	11	3.7240	12	3.8712	14	3.8923	15	3.9181	15	3.9907
7	12	3.7013	13	3.8486	14	3.9182	15	3.9411	15	3.9934
8	12	3.7272	13	3.8744	15	3.8955	15	3.9596	15	3.9934
9	12	3.7531	14	3.8517	15	3.9212	15	3.9791	15	3.9934
10	12	3.7789	14	3.8776	15	3.9443	15	3.9861	15	3.9934
11	13	3.7562	14	3.9024	15	3.9657	15	3.9861	15	3.9934
12	13	3.7820	15	3.8808	15	3.9810	15	3.9871	15	3.9934
13	14	3.7594	15	3.9053	15	3.9810	15	3.9924	15	3.9934
14	14	3.7851	15	3.9286	15	3.9810	15	3.9943	15	3.9934
15	15	3.7851	15	3.9719	15	3.9937	15	3.9943	15	3.9934

Here l^*, optimal storage class at the end of the season; OF, objective function value. The seasonal inflow state, i, is 1.

Table 4. Optimal Steady State Operating Policy for the Rabi Season: Objective Function Values

Initial Storage State, k	Seasonal Rainfall State, m				
	1	2	3	4	5
1	2.4056	2.6348	2.5712	2.5745	2.8860
2	3.3908	3.5364	3.5024	3.5036	3.6760
3	3.7706	3.8790	3.8555	3.8562	3.9989
4	3.8460	3.9541	3.9306	3.9313	4.0734
5	3.9211	4.0291	4.0057	4.0064	4.1390
6	3.9962	4.0973	4.0772	4.0775	4.2012
7	4.0637	4.1606	4.1416	4.1420	4.2517
8	4.1263	4.2114	4.1954	4.1955	4.3017
9	4.1770	4.2616	4.2458	4.2459	4.3512
10	4.2273	4.3115	4.2959	4.2960	4.4003
11	4.2773	4.3610	4.3455	4.3456	4.4490
12	4.3269	4.4101	4.3948	4.3948	4.4973
13	4.3761	4.4587	4.4436	4.4436	4.5452
14	4.4249	4.5069	4.4921	4.4921	4.5926
15	4.4733	4.5548	4.5401	4.5401	4.6392

For all data, l^*, optimal storage class at the end of the season, is 1; the seasonal inflow state, i, is 1.

ated streamflow data. The seasonal rainfall was discretized into five class intervals. The seasonal rainfall transition probabilities were determined from the historical data of 88 years. The reservoir storage is discretized into 15 class intervals.

The disaggregation scheme for inflows was based on the conditional expectations derived from synthetic streamflows; that is, the inflow in an intraseasonal period (in a given season) is taken as its expected value in that period, given the seasonal inflow. Seasonal rainfall is disaggregated into its intraseasonal values, in a similar manner using the historical data.

Tables 1 and 2 show typical results of the intraseasonal allocation model for the Kharif and Rabi seasons, respectively. Each table shows the crops grown in the respective seasons. The results of the intraseasonal allocation model for each season are tabulated as per the output format shown in Figure 2. It can be seen in Table 3 that the end-of-period soil moisture reached field capacity (2.5 mm/cm) in those periods in which deep percolation was positive, as should be the case. This has been made possible by penalizing the deep percolation in the objective function (18).

The results of the intraseasonal model are fed into the seasonal allocation model. Values of l^*, the optimal storage class at the end of the season, for all combinations of k, i, m, and T were obtained and tabulated. Examples of this tabulation are given in Tables 3 and 4. Table 3 gives the optimal end-of-period storage classes and the objective function values (maximized relative crop yields) for $i = 1$ for all combinations of k and m for the Kharif season (season 1), and Table 4, for the Rabi season (season 2). It may be noted that the objective function value in the Rabi season is higher than that in the Kharif season for some combinations of state variables because the Rabi season has one additional crop in it. The storage at the end of the Kharif (monsoon) season tends to be as high as possible to accommodate the irrigation demands of the Rabi season, which has negligible inflows. To accommodate the high inflows and reduce the spills in the ensuing Kharif season, the storage at the end of Rabi season tends to be low. The optimal end-of-season storage (l^*) is in class 1 for all combinations of k, i, and m, (Table 4), thus enabling utilization of all available storage in the ensuing Rabi season. With l^* known, tracing back into the intraseasonal model solution gives the optimal 10-day irrigation allocations to each crop. In the present case study no provision was made for overyear storage.

Conclusions

An integrated model is developed for optimal reservoir operation for irrigation of multiple crops. The model consists of two modules: the intraseasonal allocation model (module 1) and the seasonal allocation model (module 2). The intraseasonal model (LP) solves for irrigation allocations for different crops within a season for a given state of the system, resulting in the maximized relative yield from all crops. The seasonal allocation model (SDP) solves for the steady state operating policy over the seasons for optimal expected relative crop yield over a year. Reservoir storage, inflow, and rainfall in the irrigated area are considered as stochastic state variables. The model overcomes the limitations of an earlier study [*Vedula and Mujumdar*, 1992] while providing improvements. The use of seasonal inputs coupled with the LP-SDP solution strategy facilitated this. The model is applied to the Malaprabha reservoir in Karnataka State, India.

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