IRRIGATION PLANNING OF SRI RAM SAGAR PROJECT USING MULTI OBJECTIVE FUZZY LINEAR PROGRAMMING

by

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ABSTRACT

Fuzzy Linear Programming (FLP) irrigation planning model is developed for the evaluation of management strategies for the case study of Sri Ram Sagar Project, Andhra Pradesh, India. Three conflicting objectives net benefits, crop production and labour employment are considered in the irrigation planning scenario. The present paper demonstrates how vagueness and imprecision in the objective function values can be quantified my membership functions in a fuzzy multi objective framework. Uncertainty in the inflows is considered by stochastic programming. Fuzzy Linear Programming (FLP) solution yields net benefits 1,633 million Rupees, 0.70 million tons of crop production, 42.89 million man-days with degree of truth (λ) 0.69. Analysis of results indicated that net benefits, crop production and labour employment in FLP have decreased by 2.38%, 9.6% and 7.22% as compared to ideal values in the crisp Linear Programming (LP) model. Comparison of results indicated that the methodology can be extended to other similar situations.

KEY WORDS : Irrigation planning, Linear programming, Fuzzy Linear Programming, Sri Ram Sagar Project.

INTRODUCTION

Increasing demands for agricultural products with limited water resources lead to irrigation planning and management problems. In addition, the conflicting objectives of individual monetary benefits, social benefits, inevitability of uneconomical crops and providing employment to labour make the problems rather more complex. For developing countries, like India, the later objectives are also important than mere maximization of net benefits. For efficient and scientific solutions of these problems ground water is also to be optimally extracted and combined with surface water to meet the requirements. At the same time it is necessary that the water quality standards are not seriously affected. In the above context it is not possible to maximize simultaneously net benefits, labour employment and crop production etc. Trade-offs are indeed necessary, when formulating plans that achieve appropriate compromise among

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the various conflicting irrigation development objectives (Loucks et al., 1981). On the other hand, uncertainty makes irrigation planning problems more complex in the form of unexpected droughts and floods, uncertainty in price of crops, uncertainty in yields, non-availability of labour at right time and variation of inflows from year to year. Fuzzy set theory is identified as an alternative approach to handle such vagueness of planning multiple objectives and imprecision involved in the parameter values since deterministic approaches are not sufficient to model such complex situations.

Chang et al. (1997) explained the advantage of fuzzy multi objective optimisation over deterministic approach as: 1) fuzzy uncertainties embedded in the model parameters can be directly reflected and communicated into the optimisation process; 2) the variation or vagueness of the decision maker's aspiration level in the model can be incorporated and there by generate a more confident solution set for decision maker; 3) regardless of the orientation of decision maker's aspiration level (maximization), each objective or goal have its own independent membership function and different aspiration levels. Application of Fuzzy Programming to reservoir operation have been presented by Fontane et al. (1997), Raj and Nagesh Kumar (1998), and Jairaj (1998). Similar analysis is reported for water quality management by Sasikumar and Mujumdar (1998).

The present study considered multi objective Fuzzy Linear Programming (FLP) frame work by incorporating three conflicting objectives net benefits, crop production and labour employment for selection of the compromise irrigation plan i.e., cropping pattern, surface and ground water allocation policies etc. for the case study of Sri Ram Sagar Project (SRSP), Andhra Pradesh, India. The culturable command area (CCA) of the project is 1,78,100 ha. Main crops grown in the command area are Paddy, Maize, Sorghum, Groundnut, Vegetables, Pulses, Chillies and Sugarcane. Map showing the location of the project is presented in Fig. 1. Brief description of multi objective fuzzy linear programming and its application to SRSP are explained in the following sections.
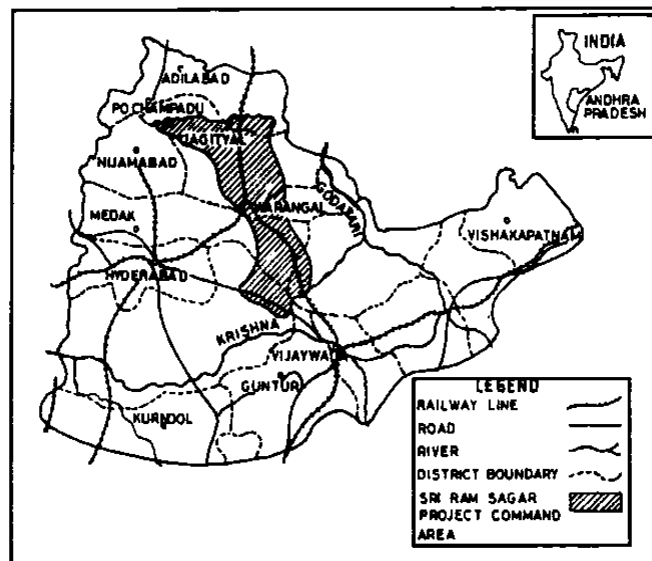

FIG. 1 LOCATION MAP OF SRI RAM SAGAR PROJECT

MULTI OBJECTIVE FUZZY LINEAR PROGRAMMING

Fuzzy Linear Programming (FLP) problem associates fuzzy input data by fuzzy membership functions. Fuzzy Linear Programming model assumes that objectives and constraints in an imprecise and uncertain situation can be represented by fuzzy sets. The fuzzy objective function can be maximized or minimized. In Fuzzy Linear Programming the fuzziness of available resources are characterised by the membership function over the tolerance range. In the present study objective functions are considered as fuzzy sets and inflows are considered in the form of stochastic programming. In conventional LP, the problem is defined as follows (Zimmermann 1996):

$$\text{Maximise } Z = C\,X \tag{1}$$

$$\text{Subject to } A\,X \leq B \tag{2}$$

$$X \geq 0 \tag{3}$$

where

A = (m × n) matrix of known constants

B = (m × 1) vector of constants

C = (n × 1) vector of known constants

X = (n × 1) vector of decision variables

Z = Objective function form

In the Fuzzy Linear Programming the problem can be restated as
Find X such that

$$C\,X \leq Z \tag{4}$$

$$A\,X \leq B \tag{5}$$

and $\quad X \geq 0 \tag{6}$

The membership function of the fuzzy set 'decision model' is

$$\mu_D(X) = \min_i \left\{ \mu_i(X) \right\} \; ; i = 1, 2, n' \tag{7}$$

$\mu_i(X)$ can be interpreted as the degree to which X fulfils inequality $C\,X \leq Z$ and n' is the number of objective functions. In the planning scenario, decision maker is not interested in a fuzzy set but in crisp optimum solution, maximizing Eq. (7) becomes

$$\text{Max}_{X \geq 0} \; \mu_D(X) = \text{Max}_{X \geq 0} \; \min_i \left\{ \mu_i(X) \right\} \tag{8}$$

Membership function $\mu_i(X)$ is represented as

$$\mu_i(X) = 0 \text{ for } Z < Z_L = \frac{Z - Z_L}{Z_U - Z_L} \text{ for } Z_L \leq Z \leq Z_U = 1 \text{ for } Z > Z_U \tag{9}$$

Z_U = Aspired level of objective

Z_L = Lowest acceptable level of objective

$\mu_i(X)$ reflects the degree of achievement. Value of $\mu_i(X)$ will be i for perfect achievement and 0 for no-achievement (worst achievement) of a given strategy and some intermediate values otherwise. The model can be transformed as follows:

$$\text{Max}_{X \geq 0} \ \min_i \ \frac{Z - Z_L}{Z_U - Z_L} \tag{10}$$

subject to

$$A \ X \leq B \tag{11}$$

$$X \geq 0 \tag{12}$$

Introducing new variable λ, the FLP problem can be formulated as equivalent LP model.
Max λ subjected to

$$\frac{Z - Z_L}{Z_U - Z_L} \geq \lambda \quad \text{for each objective function} \tag{13}$$

$$A \ X \leq B \tag{14}$$

$$0 \leq \lambda \leq 1 \tag{15}$$

$$X \geq 0 \tag{16}$$

In brief the FLP algorithm is divided into six steps :

1. Solve the problem as a Linear Programming problem by taking only one out of the objectives at a time.

2. From the results of step 1, determine the corresponding values of every objective at each solution derived.

3. From step 2, best (Z_U) and worst (Z_L) values for each objective can be calculated.

4. Formulate the linear membership function.

5. Formulate the equivalent Linear Programming model for the fuzzy multi objective problem.

6. Determine the compromise solution along with degree of truth (λ).

Mathematical modelling of the three conflicting objectives for irrigation planning problem of SRSP is briefly explained below.

Objective 1 : Maximization of Net Benefits

The net benefits (BEM) from the irrigated as well as un irrigated area under different crops are obtained by subtracting the costs of surface water, ground water, fertilizer and labour from the gross revenue for different crops. Maximization of net benefits can be expressed as

$$\text{Max BEM} = \sum_{i=1}^{16} B_i \ A_i - P_{sw} \sum_{t=1}^{12} V_{st} - P_{gw} \sum_{i=1}^{12} V_{gwt} - \sum_{f=1}^{3} \sum_{i=j}^{16} F_{f_i \ A_i P_r} - P_i \sum_{t=1}^{12} \sum_{i=1}^{16} L_{it} \ A_i \tag{17}$$

in which i = Crop index [1=Paddy(s), 2=Maize(s), 3=Sorghum(s), 4=Groundnuts(s), 5=Vegetable(s), 6=Pulses(s), 7=Paddy(srf), 8=Groundnut(srf), 9=Paddy(w), 10=Groundnut(w), 11=Pulses(w), 12=Maize(w), 13=Sorghum(w), 14=Vegetables(w), 15=Chillies(w), 16=Sugarcane(ts)]; s=Summer; w=Winter; ts=Two season; srf=Summer rainfed; t=Monthly index; f=Fertilizer index; A_i =Area of crop i (ha); B_i =Unit gross return from i th crop (Rs); P_{sw} =Unit surface water cost (Rs / Mm^3); V_{at} =Monthly canal water releases (Mm^3); P_{gw} =Unit ground water cost (Rs / Mm^3); V_{gwt} =Monthly ground water requirement (Mm^3); F_{fi} =Quantity of fertilizer of type f for crop i (tons/ha); P_f =Unit cost of fertilizer type f (Rs); P_t =Unit wage rate (rs); L_{it} =Man - days required for each hectare of crop i in month t; Rs=Rupees in Indian currency.

Objective 2: Maximization of Crop Production

Crop production (PRM) is maximized for meeting the demands and can be expressed as

$$\text{Max PRM} = \sum_{i=1}^{16} Y_i \ A_i \qquad (18)$$

where Y_i = Yield of i th crop (tons/ha).

Objective 3: Maximization of Labour Employment

The total labour employed (LAM) under all the crops for the whole year is maximized to increase the level of their economic status and can be expressed as

$$\text{Max LAM} = \sum_{t=1}^{12} \sum_{i=1}^{16} L_{it} \ A_i \qquad (19)$$

The constraints incorporated in the model are continuity equation, crop land requirements, water requirements, crops, ground water withdrawals, water quality, canal capacity restrictions, minimum and maximum reservoir storages, crop diversification considerations, downstream water requirements, labour and fertilizer availability etc. These are not presented due to space limitation. Cost coefficients, crop yields and other input parameters are obtained from Sri Ram Sagar Project reports.

RESULTS AND DISCUSSION

Stochastic Programming

The monthly inflows into the Sri Ram Sagar reservoir are assumed to follow the log-normal distribution (Yaqoob, 1986). Twenty three years of historical inflow data were used to obtain the various dependability levels of inflows. In the present study 90% dependability level inflows are considered. These are 132.10, 372.88, 798.50, 812.70, 352.02, 56.90, 36.00 Mm^3 respectively from June to December. The inflows of other months are not significant and are neglected.

Individual Optimization

Optimization of each individual objective (labour employment, crop production and net benefits) is performed with a Linear Programming (LP) algorithm that gave the upper and lower bounds for the multi objective analysis (Loucks et al. 1981). Results are presented in Table 1. Maximum (Z_U) and minimum values (Z_L) that can be obtained by each objective are denoted with symbol (+) and (-) respectively. The reason for more acreage of Paddy(s), Groundnut(srf) and Groundnut(w) in the case of net benefits maximization is due to the large net returns per unit area. In crop production maximization case, Maize and Sorghum (both in summer and winter) have large acreage compared to those in the remaining planning objectives because of their higher yield per unit area. In the irrigation planning model, there is no significant change in acreage of Groundnut, Vegetables, Pulses in summer season, Paddy, Pulses, Vegetables in winter season and Sugarcane for all the three planning objectives. Irrigation intensity in labour employment, crop production and net benefits maximization cases are 152.34%, 142.13%, 101.96% respectively. Cropping intensity is 197.92%, 173.87%, 154.33%. As can be seen from Table 1 that the three planning objectives conflict with one another. There is a need to develop a trade-off relationship to select the compromise cropping plan and the corresponding water allocation policies in the multi objective irrigation planning context to meet the chosen levels of satisfaction as would be demanded in the decision making process.

TABLE-1

CROP PLANS FROM THE PLANNING MODEL AND FUZZY LINEAR PROGRAMMING

Crops and related parameters		Units	Solution for maximization of			FLP
			Labour Employ.	Crop Produc.	Net Benefits	
1. Paddy	(s)	1000ha	2.000	2.000	62.93	24.46
2. Maize	(s)	1000ha	5.000	54.65	5.000	5.000
3. Sorghum	(s)	1000ha	22.87	50.00	1.900	1.900
4. Groundnut	(s)	1000ha	1.500	1.500	1.500	1.500
5. Vegetables	(s)	1000ha	2.000	2.000	2.100	2.000
6. Pulses	(s)	1000ha	4.200	4.200	4.200	4.200
7. Paddy	(srf)	1000ha	51.20	51.20	0.000	28.73
8. Groundnut	(srf)	1000ha	29.97	5.340	93.26	97.59
9. Paddy	(w)	1000ha	14.70	14.70	14.70	14.70
10. Groundnut	(w)	1000ha	5.700	5.700	15.81	21.94
11. Pulses	(w)	1000ha	39.85	39.85	39.85	39.85
12. Maize	(w)	1000ha	13.00	22.33	13.00	13.00
13. Sorghum	(w)	1000ha	40.00	40.00	4.500	40.00
14. Vegetables	(w)	1000ha	1.800	1.800	1.700	1.800

Crops and related parameters	Units	Solution for maximization of			FLP
		Labour Employ.	Crop Produc.	Net Benefits	
15. Chillies (ts)	1000ha	55.25	3.100	3.100	8.600
16. Sugarcane (ts)	1000ha	4.100	4.100	4.100	4.100
Irrigation Intensity	%	152.34	142.13	101.96	109.92
Cropping Intensity	%	197.92	173.87	154.33	180.85
Fertilizer	1000 Tons	44.89	43.33	36.81	39.99

Payoff Matrix

Net Benefits (Million Rupees)		1,418.60	1,084.00	1,672.90[+]	1,633.00
Crop Production (Million Tons)		0.55[-]	0.78[+]	0.68	0.70
Labour Employ. (Million Man-Days)		46.23[+]	35.16	40.43	42.89
					$\lambda = 0.69$

Multi Objective Fuzzy Linear Programming

Values of best (Z_U) and (Z_L) are substituted in the Eq. (13) which results Max λ, subjected to

$$-\lambda + 1.69 \times 10^{-9} \text{ BEM } \geq 2.84 \tag{20}$$

$$-\lambda + 4.47 \times 10^{-6} \text{ PRM } \geq 3.48 \tag{21}$$

$$-\lambda + 9.03 \times 10^{-8} \text{ LAM } \geq 4.18 \tag{22}$$

and all the existing constraints.

Three additional constraints (equations 20, 21, 22) are formulated corresponding to the three objectives functions in the planning model. Results of FLP are presented in Table 1. It is observed that FLP compromise solution favours both Groundnut (srf) and Groundnut (w) with an acreage of 97,590 ha, 21,940 ha. On the other hand solution is almost consistent for Groundnut (s), Vegetables (s & w), Pulses (s & w) and Paddy (w). Irrigation intensity and cropping intensity in the model are 109.92% and 180.85% respectively. Irrigation intensity is less than labour employment and crop production cases but more than the net benefits case by 8%. Significant change is observed for Chillies, where it is reduced from 55,250 ha in labour employment maximization to 8,600 ha in case of FLP. The FLP compromise solution (Table 1) yields net benefits 1,633 million Rupees, 0.70 million tons of crop production, 42.89 million man-days with $\lambda = 0.69$. Net benefits, crop production and labour employment per ha on average for the cultural command area (CCA) are 9,169 Rupees, 3.94 tonnes and 241 man-days respectively. It is observed that net benefits, crop production and labour employment in FLP have decreased by 2.38%, 9.6% and 7.22% as compared to ideal values in the crisp Linear Programming (LP) model since FLP solution is a compromise solution for three conflicting objectives. (Srinivasa Raju 1995).

CONCLUSIONS

Based on the application of multi objective Fuzzy Linear Programming (FLP) irrigation planning model for a real world irrigation planning problem of Sri Ram Sagar Project, Andhra Pradesh, India, the following conclusions are drawn.

1. Fuzzy Linear Programming (FLP) is simple and suitable tool for multi objective problems compared to other methods.

2. The FLP compromise solution yields net benefits 1,633 million Rupees, 0.70 million tons of crop production, 42.89 million man-days with degree of truth (λ) 0.69.

3. Net benefits, crop production and labour employment per hectare on average for the cultural command area (CCA) are 9,169 Rupees, 3.94 tonnes and 241 man-days respectively.

4. Analysis of results indicated that net benefits, crop production and labour employment in FLP have decreased by 2.38%, 9.6% and 7.22% as compared to ideal values in the crisp Linear Programming (LP) model.

5. The model can be extended to any number of objectives by incorporating only one additional constraint in the constraint set for each additional objective function.

6. Comparison of results indicated that the methodology can be extended to other similar situations.

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NOTATIONS

i	=	Crop index (1, 16)
f	=	Fertilizer index (f = 1, 2, 3 for Nitrogen, Phosphorous and Potassium)
s	=	Summer
srf	=	Summer rainfed
t	=	Monthly index (t = 1, 12)
ts	=	Two season
w	=	Winter
A	=	(m x n) matrix of known constants
A_i	=	Area of crop i (ha)
B	=	(m x 1) vector of constants
B_i	=	Unit gross return from i th crop (Rs)
BEM	=	Net benefits (Rs)
C	=	(n x 1) vector of known constants
CCA	=	Culturable Command Area (ha)
F_{fi}	=	Quantity of fertilizer of type f for crop i (tons/ha)
L_{it}	=	Man-days required for each hectare of crop i in the month t
LAM	=	Total man-days for the whole year
n'	=	Number of objective functions
P_f	=	Unit cost of fertilizer type f (Rs)
P_{gw}	=	Unit ground water cost (Rs/mm^3)
P_l	=	Unit wage rate (Rs)
P_{sw}	=	Unit surface water cost (Rs/mm^3)
PRM	=	Crop production (Tons)
V_{at}	=	Monthly canal water releases (mm^3)
V_{gwt}	=	Monthly ground water requirement (mm^3)
Rs	=	Rupees in Indian currency
X	=	(n x 1) vector of decision variables
Y_i	=	Yield of i th crop (tons/ha)
Z	=	Objective function form
Z_U	=	Aspired level of objective
Z_L	=	Lowest acceptable level of objective
λ	=	Degree of truth
$\mu_D(X)$	=	Membership for decision model
$\mu_i(X)$	=	Degree of achievement