Optimal Irrigation Allocation: A Multilevel Approach

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Abstract: Optimal resources allocation strategies for a canal command in the semiarid region of Indian Punjab are developed in a stochastic regime, considering the competition of the crops in a season, both for irrigation water and area of cultivation. The proposed strategies are divided into two modules using a multilevel approach. The first module determines the optimal seasonal allocation of water as well as optimal cropping pattern. This module is subdivided into two stages. The first stage is a single crop intraseasonal model that employs a stochastic dynamic programming algorithm. The stochastic variables are weekly canal releases and evapotranspiration of the crop that are fitted to different probability distribution functions to determine the expected values at various risk levels. The second stage is a deterministic dynamic programming model that takes into account the multicrop situation. An exponential seasonal crop-water production function is used in this stage. The second module is a single crop stochastic dynamic programming intraseasonal model that takes the output of the first module and gives the optimal weekly irrigation allocations for each crop by considering the stress sensitivity factors of crops.

INTRODUCTION

Irrigation scheduling deals with two questions, when and how much to irrigate. When adequate water supply is assured, irrigation can be scheduled to saturate the crop root zone depth up to the field capacity, and the timing of irrigation may be estimated by the time it takes for the soil moisture to deplete to the critical level. Such an irrigation schedule ensures that the crop will grow at its potential rate provided all other agricultural inputs are supplied at optimal levels. When the available water is not adequate to meet the crop-water demands for the season, water deficits during some periods in the season cannot be avoided. Because crop-water response to water deficits at different periods is not uniform, it becomes critical for irrigation managers to decide how best to distribute the deficits among the intraseasonal periods for a crop. The problem becomes more complicated when a given amount of water is to be allocated to a number of crops over the intraseasonal periods. A deficit occurring in a certain stage of crop growth may cause a greater reduction in yield than the same amount of deficit occurring in some other growth stages.

The limited supply of canal water and scanty rainfall in the semiarid regions of Indian Punjab have forced reservoir managers and farmers to decide on an irrigation schedule and optimum cropping pattern for each crop-soil condition so as to maximize profit. This particular study is aimed at helping both reservoir managers and farmers in decision making. In these regions, a rotational system of canal regulation is adopted, based on grouping of canal outlets and sequences of priorities. The first priority outlets are supplied with full supply level for a fixed duration, followed by second priority outlets that may be supplied with half supply level. The duration of supply in these outlets is based on the cropped area but not according to the crop-water requirement. Based on canal-water supply, farmers allocate their lands to various crops. The canal discharge is also dependent on storage available in the reservoir but not on irrigation demand at the command sea level. Hence, evapotranspiration and canal-water releases are characterized with a great level of uncertainty in semiarid regions.

Rhenals and Bras (1981) developed a model based on stochastic dynamic programming (SDP) to maximize net benefits from a crop facing uncertain evapotranspiration demands. Bras and Cordova (1981) were concerned with optimal temporal allocation of irrigation water and considered stochastic crop-water requirements and the dynamics of the soil moisture depletion process. All the aforesaid models dealt with the single crop situation.

Limited numbers of works have dealt with irrigation scheduling under a multicrop situation. Rao et al. (1990) developed a model for optimal weekly irrigation scheduling policy for two crops by considering both seasonal and intraseasonal competition for water. They used dynamic programming (DP) for solving both seasonal as well as intraseasonal problems based on a heuristically derived seasonal crop-water production function, considering the area for a particular crop as fixed a priori. Vedula and Nagesh Kumar (1996) developed a mathematical programming model to determine the steady-state optimal operating policy and the associated optimal crop-water allocations to all the crops for a single purpose irrigation reservoir, combining linear programming in the intraseasonal period and SDP in the interseasonal period. Optimal seasonal multicrop irrigation water allocation and optimal stochastic intraseasonal (daily) irrigation scheduling were carried out by Sunantara and Ramírez (1997) using a two-stage decomposition approach based on SDP methodology. Optimal seasonal water and acreage allocation among several crops were defined using deterministic dynamic programming (DDP) with the objective of maximizing total benefits from all the crops.

Most farming situations are concerned with several crops grown in the same season. Both allocation of land and water resources under a multicrop situation in a season should be considered (Paul 1998).

In this paper, a seasonal allocation model is developed to maximize net profit from crop-water production by optimally allocating land and water resources to different crops using DDP. This is followed by development of an intraseasonal irrigation allocation model to maximize relative yield using SDP for determining the optimal weekly irrigation allocation for a single crop. The stochastic parameters are evapotranspiration and canal-water releases. These models are applied to a canal command in the semiarid region of Indian Punjab for large scale adoption.

MODEL FORMULATION

The model is developed for determining the optimal cropping pattern and irrigation scheduling for all the crops grown

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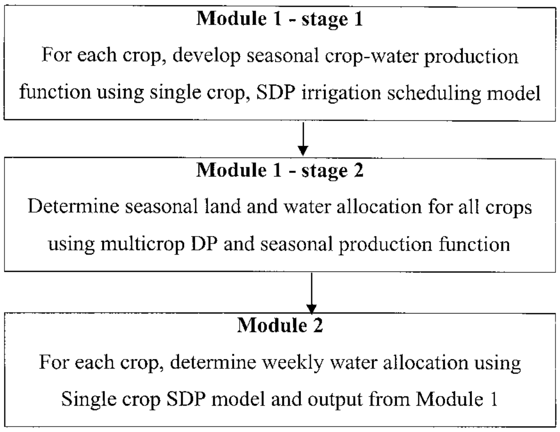

FIG. 1. Schematic of Multilevel Decomposition Procedure

in the study area. In a single-level approach, all the complexities taken in the present study cannot be considered. For example, if a single stochastic optimization technique like SDP is used, it will not be possible to solve the problem as stated, due to the ''curse of dimensionality.'' Therefore, a multilevel approach comprising two modules, seasonal and intraseasonal levels, is adopted in the present study. This multilevel approach definitely leads to optimum. The basic elements of the multilevel approach are (1) a DDP solution for the seasonal allocation of land and water resources for all crops; and (2) a SDP solution for the intraseasonal weekly allocation for a single crop. Fig. 1 shows the schematic representation of the whole procedure.

Module 1

The first module, the seasonal allocation of land and water for a number of crops, is carried out in two stages. The seasonal crop-water production functions for each crop are obtained in the first stage using a SDP approach such that the expected relative yield of the crop for a given seasonal water availability is maximized. At this stage, the model takes into account the uncertainty of weekly crop evapotranspiration and canal-water release. This particular stage is performed repeatedly for several values of seasonal water availability, ranging between zero and the maximum feasible irrigation requirement for every crop under consideration. Thus, a relationship between the depth of seasonal water applied and total expected value of relative yields for each crop is developed. This relationship is modeled as an exponential function (Hexem and Heady 1978). The second stage is the determination of optimal seasonal water allocation for each crop and the optimal cropping pattern so that total benefits from all crops are maximized. This is done using DDP, where each crop corresponds to a decision stage in the DP algorithm.

Module 2

The second module is the intraseasonal allocation model, solved by SDP where inputs will be the outputs from the first module. Thus, the resulting optimal weekly water allocation for single crops, which will be the final result of the entire problem, is conditional upon the optimal seasonal land and water allocations from the first module.

Seasonal Allocation Model (Module 1)

The objective of the problem is to maximize the sum of the net benefits from all the crops. The optimal seasonal allocation of irrigation water and acreage for each crop is a resource allocation problem among multiple competing users. Each stage of the DP algorithm represents a different crop.

Seasonal Crop-Water Production Function

Several forms of the mathematical relationship between relative yield of crop and water supply have been suggested in the literature. In this study, an exponential function is used following Hexem and Heady (1978) and is given for the crop season by

$$y_k - y_{0k} = (a_k - y_{0k})\exp\left[\frac{-b_k(x_{mk} - x_k)}{x_k}\right] \tag{1}$$

where y_k = seasonal relative yield for crop k corresponding to x_k depth of water applied; y_{0k} = relative yield corresponding to zero irrigation allocation; a_k = maximum relative yield obtainable for the initial soil and climatic conditions; x_{mk} = minimum depth of water required to give potential relative yield a_k, mm; and b_k = coefficient for a particular crop.

DP Formulation

The DP formulation determines the optimal seasonal allocation of water to multiple crops and also determines the optimal cropping pattern. Here, the objective is to maximize the total net benefits from all the crops. The decision variables in DP are the area and water to be allocated to each crop in a particular stage. State variables are the area and water available for the remaining crops. A backward recursive equation is used and is given by

$$B^*(x_k, A_k) = A_k PRO_k Y_{k_{\max}} y_k(x_k) \tag{2}$$

$$f_k^*(x_k, A_k) = \max_{\text{feasible}(x_k, A_k)} [B^*(x_k, A_k) + f_{k+1}(Q - x_k, A - A_k)] \tag{3}$$

$$f_N^*(x_N, A_N) = \max_{\text{feasible}(x_N, A_N)} [B^*(x_N, A_N)] \tag{4}$$

where A_k = area allocation to crop k (ha); PRO_k = profit per yield of crop k [rupees (Rs.)/100 kg]; $Y_{k\text{-max}}$ = maximum obtainable yield corresponding to crop k (100 kg/ha); $y_k(x_k)$ = relative yield for x_k depth of irrigation water; $B^*(x_k, A_k)$ = net profit for the allocated amounts of water and area (Rs.); $f_k^*(x_k, A_k)$ = measure of performance in DP; Q = total seasonal water available (mm); and N = total number of crops.

Area Constraint in DP

The DP solution should be such that the total area allocated to all crops is limited to the total area available to them. And it should also take into consideration that the maximum area that can be allocated to a particular crop is limited to a predetermined value that is fixed according to an earlier study in the area (Panda et al. 1996). Thus the two constraints can be given by

$$\sum A_k \le A \tag{5}$$

$$A_k \le A_{k_{\max}} \tag{6}$$

where A = total area available (ha); and $A_{k_{\max}}$ = maximum area under crop k (ha).

Intraseasonal Allocation Model (Module 2)

This model determines the weekly allocation of water for the given amount of available water in the season and is carried out for a single crop. The model consists of different components that are explained in the following section.

Dated Crop-Water Production Function

Dated crop-water production functions incorporate effects of both timing and quantities of water applications on crop yield, and generally evapotranspiration is used as the waterrelated independent variable (Doorenbos and Kassam 1981). Out of a number of dated crop-water production functions, a multiplicative dated crop-water production function derived from sensitivity factors for water stress in physiological growth stages of crops is used (Rao et al. 1990)

$$\frac{Y_a}{Y_{\max}} = \prod_{t=1}^{NP} \left[1 - k_t \left(1 - \frac{\text{AET}}{\text{PET}} \right)_t \right] \tag{7}$$

where Y_a = actual yield obtained (100 kg/ha); $Y_{\max}$ = maximum obtainable yield (100 kg/ha); k_t = yield stress sensitivity factor for period t; AET = actual evapotranspiration (mm); PET = potential evapotranspiration (mm); t = period index; and NP = total number of periods for the crop.

Actual Evapotranspiration

The actual crop evapotranspiration depends on the evaporative demand of the atmosphere, the crop growth stages, and the available soil moisture in the root zone. From among various methods for determining the reference evapotranspiration, the Penman-Monteith method is chosen for this study, because of its wide applicability, and is given by

$$\text{ET}_0 = \frac{\delta(R_n - G)}{(\delta + \gamma^*)\lambda} + \frac{936\gamma U_2(e_a - e_d)}{(\delta + \gamma^*)(T_{avg} + 276)} \tag{8}$$

where ET_0 = reference evapotranspiration (mm/day); R_n = net radiation (MJ/m^2 day); G = soil heat flux (MJ/m^2 day); γ = adiabatic psychometric constant (kPa/°C); γ^* = modified psychometric constant (kPa/°C); U_2 = wind speed measured at 2-m height (m/s); δ = slope of vapor pressure curve (kPa/°C); T_{avg} = average temperature (°C); $(e_a - e_d)$ = vapor pressure deficit (kPa); and λ = latent heat of vaporization (MJ/kg).

Potential evapotranspiration is given by

$$\text{PET} = K_c \text{ET}_0 \tag{9}$$

where K_c = crop coefficient.

The actual evapotranspiration in relation to its potential rate is determined by considering whether the available water in the soil is adequate or whether the crop will suffer from stress induced by water deficit. The actual evapotranspiration in each week is found as follows:

$$\text{AET}_t = \begin{cases} 0; \quad \text{SM}_t \le \text{WP}_t \\ \dfrac{\text{PET}_t(\text{SM}_t - \text{WP})}{(1 - p)(\text{FC} - \text{WP})}; \quad \text{WP} < \text{SM}_t \le (1 - p)(\text{FC} - \text{WP}) \\ \text{PET}_t; \ \text{SM}_t \ge (1 - p)(\text{FC} - \text{WP}) \end{cases} \tag{10}$$

where SM_t = soil moisture content in depth units per unit root depth in period t (mm/cm); WP = wilting point (mm/cm); FC = field capacity (mm/cm); and p = crop water depletion factor.

Soil Moisture Balance

As shown in Table 1, the rainfall in the study area is scanty. In addition, there are bunds around the field to prevent runoff from this scanty rainfall. Considering the general mass balance equation, and neglecting the runoff from the field, the soil moisture balance equation is stated as follows:

$$\text{SM}_{t+1} Z_{t+1} = \text{SM}_t Z_t + R_t + x_t + S_0(Z_{t+1} - Z_t) - \text{AET}_t - P_t \tag{11}$$

where Z_t and Z_{t+1} = root zone depths in the periods t and $t + 1$, respectively, (cm); R_t = rainfall in the period t (mm); x_t = water allocation in period t (mm); S_0 = initial soil moisture content (mm/cm); and P_t = deep percolation (mm).

The soil moisture in the incremental root growth during the time step is represented by $S_0(Z_{t+1} - Z_t)$. In the beginning of the season, soil moisture in the entire root zone is assumed to be at field capacity.

Rainfall

Since the study area is located in a semiarid region and the amount of weekly rainfall is insignificant (Table 1) compared to the canal-water supply and the evaporative demand of the

TABLE 1. Weekly Average Rainfall and Evapotranspiration at Different PEs during Winter Season

Standard week (1)	Mean rainfall (mm) (2)	REFERENCE EVAPOTRANSPIRATION (mm)								
						PE				
		Mean (3)	SD (4)	Skewness (5)	PD (6)	90% (7)	80% (8)	70% (9)	60% (10)	50% (11)
43	0.61	20.28	0.0769	0.711	LND	17.43	18.32	18.98	19.58	20.15
44	2.40	12.09	0.0575	1.22	LND	9.18	10.62	11.12	11.54	11.97
45	1.85	12.09	0.0575	1.22	LND	9.18	10.62	11.12	11.54	11.97
46	0.39	12.09	0.0575	1.22	LND	9.18	10.62	11.12	11.54	11.97
47	0.98	12.09	0.0575	1.22	LND	9.18	10.62	11.12	11.54	11.97
48	1.10	7.77	0.0683	2.18	LND	5.39	6.04	6.56	7.03	7.51
49	1.38	7.77	0.0683	2.18	LND	5.39	6.04	6.56	7.03	7.51
50	1.59	7.77	0.0683	2.18	LND	5.39	6.04	6.56	7.03	7.51
51	2.05	7.77	0.0683	2.18	LND	5.39	6.04	6.56	7.03	7.51
52	4.14	7.77	0.0683	2.18	LND	5.39	6.04	6.56	7.03	7.51
1	0.65	7.86	0.0542	2.25	LND	5.92	6.48	6.91	7.31	7.68
2	2.62	7.86	0.0542	2.25	LND	5.92	6.48	6.91	7.31	7.68
3	2.60	7.86	0.0542	2.25	LND	5.92	6.48	6.91	7.31	7.68
4	4.31	7.86	0.0542	2.25	LND	5.92	6.48	6.91	7.31	7.68
5	3.53	13.41	0.0653	1.93	LND	11.02	11.74	12.29	12.79	13.27
6	3.32	13.41	0.0653	1.93	LND	11.02	11.74	12.29	12.79	13.27
7	4.41	13.41	0.0653	1.93	LND	11.02	11.74	12.29	12.79	13.27
8	1.70	13.41	0.0653	1.93	LND	11.02	11.74	12.29	12.79	13.27
9	1.82	20.19	0.0943	0.63	LND	16.73	17.78	18.49	19.29	19.99
10	3.12	20.19	0.0943	0.63	LND	16.73	17.78	18.49	19.29	19.99
11	5.82	20.19	0.0943	0.63	LND	16.73	17.78	18.49	19.29	19.99
12	4.67	20.19	0.0943	0.63	LND	16.73	17.78	18.49	19.29	19.99
13	3.98	32.40	0.115	0.43	ND	27.98	29.99	30.57	31.57	32.4

Note: SD = standard deviation; PD = probability distribution; LND = lognormal distribution; ND = normal distribution; PE = probability of exceedence.

crops, the rainfall component is considered to be deterministic. Therefore, average rainfall of the historical data for each week (Table 1) is used in the soil moisture balance equation.

Deep Percolation

The present study uses an empirical equation for calculating the deep percolation component (Rao et al. 1990)

$$P_t = \frac{c(\mathrm{SM}_{sat})(\exp[\mathrm{SM}_t - \mathrm{FC}] - 1)}{\exp[\mathrm{SM}_{sat} - \mathrm{FC}] - 1} \qquad (12)$$

where SM_{sat} = saturated soil moisture content (mm/cm); and c = pore connectivity index.

Root Growth Model

The depth of the active soil reservoir from which the crops extract water depends on the effective depth of root penetration into the soil. This depth increases with the crop growth and attains a maximum value by the end of the flowering period for most crops. A sigmoidal root growth model (Borg and Grimes 1986) is used in the study

$$Z_t = Z_{\max}\left(0.5 + 0.5 \sin\left[3.03\left(\frac{t}{t_{\max}}\right) - 1.47\right]\right) \qquad (13)$$

where Z_t = depth of effective root zone at time t after sowing (cm); $t_{\max}$ = time for the full development of root zone (days); and $Z_{\max}$ = maximum possible depth of effective root zone (cm).

SDP

Since both the weekly canal release and evapotranspiration have a great amount of uncertainty, these variables are considered to be stochastic in the SDP model. The measure of relative yield is determined for each week corresponding to all levels of canal release and actual evapotranspiration. The objective of SDP is to maximize the expected value of relative yields in all the irrigation periods. The backward recursive equations used are given by

$$R_t^*(x_t, \mathrm{AET}_t) = 1 - k_t\left(1 - \frac{\mathrm{AET}}{\mathrm{PET}}\right)_t \qquad (14)$$

$$f_t^*(x_t, \mathrm{AET}_t) = \max_{\text{feasible} \sim x_t} \cdot \left[R_t(x_t, \mathrm{AET}_t) + \prod_{j=1}^{j_{\max}} \prod_{m=1}^{m_{\max}} TS_{ij}(t) TE_{lm}(t) f_{t+1}(x_{t+1}, \mathrm{AET}_{t+1})\right] \qquad (15)$$

$$f_{NP}^*(x_{NP}, \mathrm{AET}_{NP}) = \max_{\text{feasible} \sim x_{NP}} [R_{NP}(x_{NP}, \mathrm{AET}_{NP})] \qquad (16)$$

where $R_t^*(x_t, \mathrm{AET}_t)$ = relative yield corresponding to x_t irrigation depth and the given AET; $f_t^*(x_t, \mathrm{AET}_t)$ = measure of performance in SDP; $TS_{ij}(t)$ = transition probability of weekly canal release from level i in period t to level j in period $t + 1$; $TE_{lm}(t)$ = transition probability of weekly evapotranspiration from level l in period t to level m in period $t + 1$; $j_{\max}$ = number of class levels for canal release; and $m_{\max}$ = number of class levels for evapotranspiration.

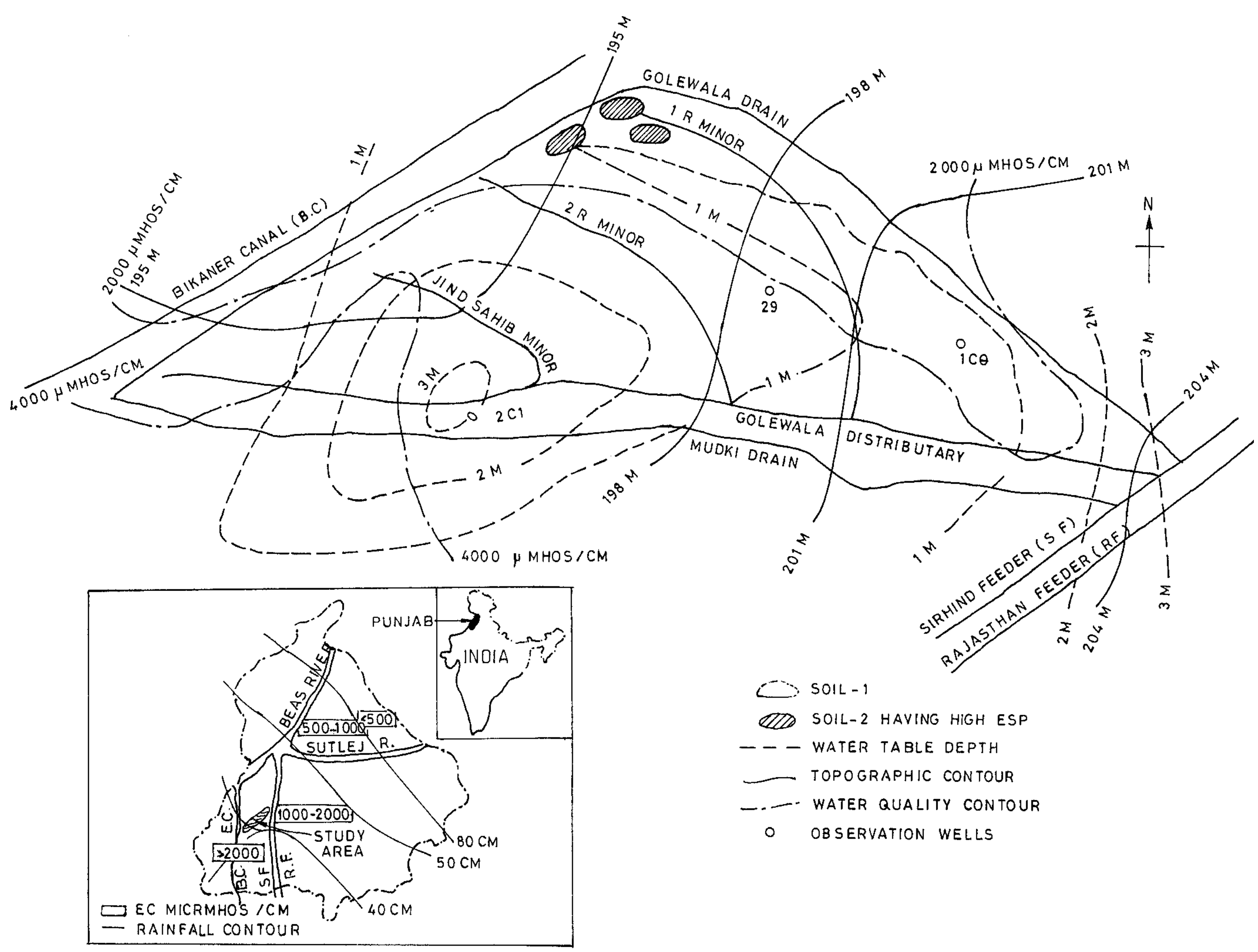

FIG. 2. Location Map of Study Area

TABLE 2. Detailed Crop Information

Crop (1)	Wheat (2)	Mustard (3)	Barley (4)	Gram (5)
Product price (Rs./100 kg)	215	700	185	600
Straw price (Rs./100 kg)	30	10	15	10
Variable cost of production (Rs./ha)	5,966.02	4,393.83	3,620.93	3,638.31
Average date of sowing	November 3	November 3	November 3	October 23
Length of growing season (days)	150	150	125	160
Length of crop development stages (days)	25:35:60:30	15:45:65:25	15:25:55:30	25:50:55:30
Crop coefficients at different stages	0.34:0.69:1.05:0.65	0.34:0.61:0.88:0.82	0.34:0.69:1.05:0.65	0.26:0.63:1.0:0.63
Yield response factors	0.2:0.6:0.6:0.5	0.3:0.6:0.6:0.3	0.2:0.6:0.45:0.2	0.2:0.9:0.7:0.2
Potential yield (100 kg/ha)	45.45	9.10	39.49	24.90
Average yield (100 kg/ha)	36.50	7.78	30.0	9.35
Maximum root depth (cm)	120	125	165	135
Maximum area (ha)	21,500	415	2,869	287

Note: U.S.$1 = Indian Currency [rupees (Rs.) 42].

APPLICATION

The models were applied to a canal command area under the Golewala distributary, which lies in the southwestern plain region of Punjab, India (Fig. 2). The study area lies between latitude 30°39′ N and 30°51′ N and longitude 74°34′ E and 74°50′ E. The region is semiarid in nature. The average annual rainfall is 440 mm with 2/3 of this occurring during June through September. The weekly average rainfall data is given in Table 1.

The parameters of the crop-water production functions for different crops are determined as follows. Assuming the entire cropped area and canal water are available to a given crop, the SDP model is run for different values of seasonal canal water varying from zero to the maximum value of expected canal release, with an increment of 200 ha-m. The value of x [(1)] is determined by dividing the total volume of canal release by the area under the given crop. It is plotted to determine the parameters of the seasonal crop-water production function that gives a relationship between the seasonal relative yield of the crop and seasonal depth of irrigation. The relative yield value corresponding to zero irrigation depth is y_{0k}, maximum value of relative yield in the plot is a_k, and the minimum depth of water for which the relative yield becomes a_k is x_{mk}. The value of b_k is found by putting the values of other parameters in the seasonal production function for a given depth of water. For accurate estimate of b_k we take the average of the values obtained for b_k for various depths of water application.

Daily reference evapotranspiration during the period 1960–1985 was found using the Penman-Monteith method [(8)]. These daily values were then summed to give the corresponding weekly values. The weekly values were fitted into either a normal or lognormal distribution (Haan 1979). The selection of distribution is based on whether the skewness of the original data approaches zero or that of log-transformed data. The class intervals or class limits for reference evapotranspiration values in each week were determined such that the probabilities of each class interval were the same. The mean, standard deviation, skewness, selected distribution, and the expected value of weekly reference evapotranspiration for different probabilities of exceedence (PEs) are also given in Table 1.

The transition probabilities $TE_{lm}(t)$ corresponding to each class interval, l in period t to m in period $(t + 1)$, was determined. Initially, the number of events of reference evapotranspiration during the period t in class l transforming into the class interval m in period $(t + 1)$ was calculated. The required probability value was then obtained by dividing this value by the total number of events of reference evapotranspiration data falling in the class interval l. These are maximum likelihood estimates of the transition probabilities.

The general slope of the land is from northeast to southwest and averages about 0.45 m/km. Soil types for most of the cropped areas are sandy loam to loamy sands. The field capacity, wilting point, saturated soil moisture, and pore connectivity indexes of the soil are 2 mm/cm, 1 mm/cm, 3.4 mm/cm, and 0.25, respectively. The main crops grown in the study area in winter season are wheat, barley, mustard (oil seed), and gram. The characteristics of these crops are listed in Table 2.

The head regulator of Golewala distributary releases 7.07 m^3/s of canal water. The cultivatable and gross command areas of the distributary are 28,700 and 29,800 ha, respectively. Daily canal releases during the period 1963–1987 were used for determining weekly water availability. The historical data of weekly canal releases at the head regulator were analyzed for 25 years and found out to be stochastic in nature, and in most of the weeks it conforms to a gamma distribution. The transition probabilities of the weekly canal releases were determined for all the levels of canal releases. The seasonal values of canal release were fitted to gamma distribution. The

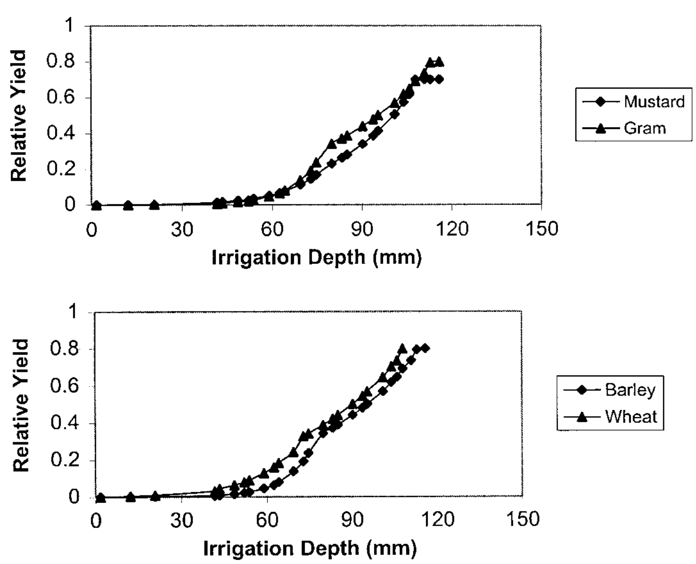

FIG. 3. Seasonal Production Function of Crops

TABLE 3. Parameters of Seasonal Production Function of Crops

Crop (1)	y_{0k} (2)	a_k (3)	b_k (4)	X_{mk} (mm) (5)
Gram	0.000019	0.8	1.480	115
Mustard	0.000347	0.7	1.670	109
Barley	0.00901	0.8	0.702	88.5
Wheat	0.00142	0.8	1.130	109

TABLE 4. Allocation of Water and Acreage to Crops at Different PEs

PE (%) (1)	Gram Water (ha-m) (2)	Gram Area (ha) (3)	Mustard Water (ha-m) (4)	Mustard Area (ha) (5)	Barley Water (ha-m) (6)	Barley Area (ha) (7)	Wheat Water (ha-m) (8)	Wheat Area (ha) (9)	Total Water (ha-m) (10)	Total Area (ha) (11)
90	94.5	200	0	0	3,055.5	2,800	0	0	3,150	3,000
80	109.5	200	0	0	1,350.5	2,800	2,190	20,000	3,650	23,000
70	121.8	200	0	0	1,339.8	2,800	2,598	20,800	4,060	23,800
60	133.5	200	0	0	1,379.5	2,800	2,937	21,200	4,450	24,200
50	96.2	200	0	0	1,443	2,800	3,222	21,400	4,810	24,400

scale and shape parameters were calculated using the maximum likelihood method (Haan 1979). The average seasonal canal release is 4,989.36 ha-m. Scale and shape parameters are calculated as 0.0021 and 10.51, respectively. The expected values of canal releases for the winter season correspond to different PEs of 90, 80, 70, 60, and 50% are 3,150, 3,650, 4,060, 4,450, and 4,810 ha-m, respectively.

RESULTS AND DISCUSSION

The single crop SDP model was performed for all feasible values of seasonal water availability starting from zero to the minimum depth required for obtaining the potential yield of the crop to get the relationship between the seasonal depth of irrigation water used and the relative yield. Graphs were plotted for all the four crops—mustard, gram, barley, and wheat—and are shown in Fig. 3. All the curves are exponential in nature. The parameters of the seasonal crop-water production function [(1)] for the crops are calculated as explained earlier and are given in Table 3.

The relative yield for all the crops found by the production function using these parameters were in good agreement with those presented in an earlier study in the area (Panda et al. 1996). The correlation coefficients corresponding to wheat, mustard, barley, and gram were found to be 0.97, 0.94, 0.98, and 0.95, respectively.

The multicrop DDP model was run for expected values of seasonal canal releases corresponding to different PEs to allocate land and irrigation water to crops like gram, mustard, barley, and wheat in a season (Table 4).

The allocation of irrigation water and the area for each crop depend upon factors such as net profit per unit yield, potential yield obtainable per unit area, and minimum water application needed for getting the maximum yield. Although the net profit per unit yield of gram is very high compared to all other crops, the maximum yield per unit area of cultivation is less than that of crops like barley and wheat. Compared to the other two crops, the net profit for crop mustard (oil seed) is higher, but maximum yield per hectare is very low. This resulted in zero allocation of both area and water for mustard (Table 4). Even if the profit and yield were higher for wheat, at 90% PE it was not allocated any amount of water or area. The primary reason for this was that the minimum depth of water required for getting the maximum yield in the case of wheat was much higher than that of barley.

As the risk factor increases, the expected value of seasonal canal-water release increases, and thus the allocation of water to different crops also increases. At 80% PE compared to 90%, the water allocation to crop barley decreases although the area remains the same. The reason is that the area allocation could not be increased further, because it attained the maximum possible value. Here, the net return from the crops plays a major role. Similar results were obtained for PEs at 70, 60, and 50%. In all these cases, there is no allocation to mustard. This is due to the very low value of maximum yield per unit area. Expected values of total net return were plotted against PEs (Fig. 4). The value of expected net return increases as the risk factor increases (i.e., when PE decreases). As the risk factor increases, the expected value of total water available in the winter season increases, and the optimum irrigation water and area available for the crops increases. Thus the net return rises with a decrease in the PEs.

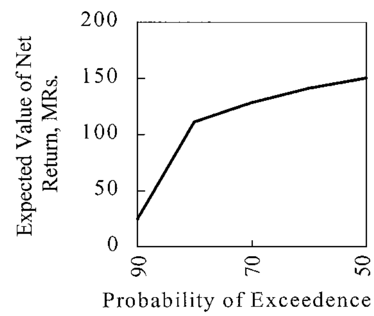

FIG. 4. Relation of Expected Value of Net Return (Measured in Millions of Rupees) with Different PEs

The seasonally allocated water to all the crops was redistributed into intraseasonal periods. The intraseasonal period comprises weeks. The weekly allocation of irrigation to gram, mustard, barley, and wheat at different PEs (90, 80, 70, and 50%) is shown in Fig. 5. The priority of weekly irrigation allocation was given to periods of higher stress sensitivity factor over periods of lower stress sensitivity factor.

Table 5 provides the comparison of existing cropping pattern and profit with that of model output. Considering the total net benefit from the entire area, it can be clearly seen that the cropping pattern obtained from the model is much more profitable than that of existing usage of the cultivable land. Moreover, the farmers of Indian Punjab are more inclined to maximize their profit from the cropped area.

SUMMARY AND CONCLUSIONS

A stochastic approach for solving the multicrop and multilevel optimal irrigation scheduling problem by the DP decomposition scheme has been presented. Optimal seasonal and intraseasonal allocation of land and water for several crops are developed that explicitly account for the physics of the soil moisture depletion process and the random nature of evapotranspiration and canal releases. Seasonal allocation of land and water resources depends on factors such as net profit per unit yield, potential yield of the crop, and the minimum depth of irrigation needed for obtaining the potential yield. At all the risk levels, the allocation of land and water for oil seed was found to be zero because of very low potential yield and relatively high value of cost of production. Another important factor that affects the resources allocation was the minimum depth of irrigation water needed to get the potential yield from a crop. Intraseasonal allocation of water for a single crop on a weekly basis was controlled by the stress sensitivity factors. The model was then applied to a canal command area in Indian Punjab.

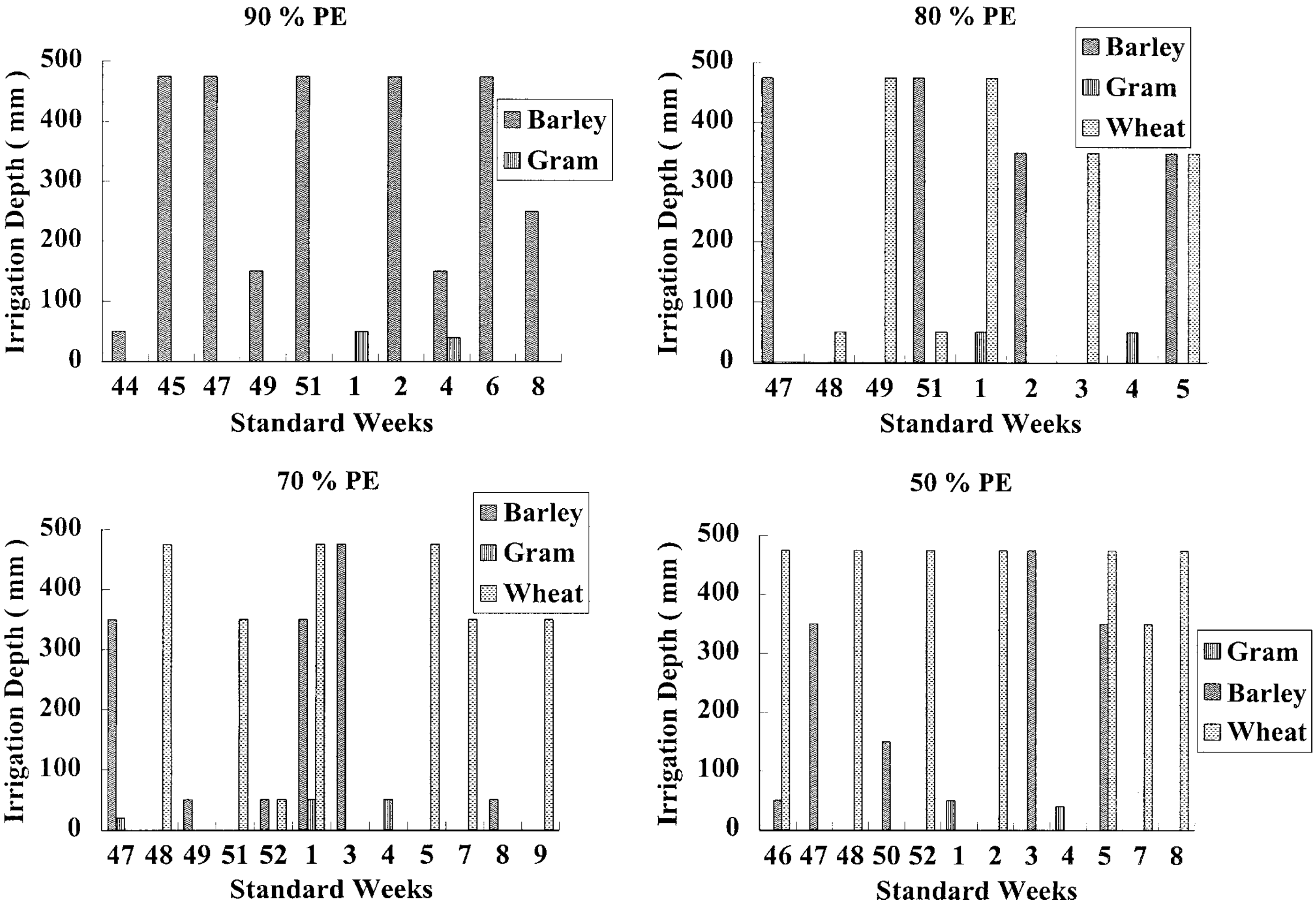

FIG. 5. Weekly Allocation of Irrigation Water at Different PEs

TABLE 5. Comparison between Existing Cropping Pattern and Model Result

Crop (1)	Area (ha) Existing[a] (2)	Area (ha) Model (3)	Average yield[b] (kg/ha) (4)	Net profit[c] (Rs./kg) (5)	Yield predicted (kg/ha) (6)	Profit (Million Rs.) Existing condition (7)	Profit (Million Rs.) Model result (8)
Gram	287	200	779	4.64	1,992	0.49	1.85
Mustard	415	0	773	2.27	0	0.71	0
Barley	283	2,800	1,561	1.08	3,159.2	0.49	9.58
Wheat	22,422	21,400	2,845	1.14	3,636	38.43	88.50
Total	23,407	24,400				40.12	99.93
Difference in Profit						59.81	

[a]Panda et al. (1996).
[b]"Deputy" (1988).
[c]Panda (1992).

APPENDIX I. REFERENCES

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APPENDIX II. NOTATION

The following symbols are used in this paper:

A = total area available (ha);
AET = actual evapotranspiration (mm);
A_k = area allocated to crop k (ha);
$A_{k_{\max}}$ = maximum area under crop k (ha);
a_k = maximum relative yield obtainable for initial soil and climatic conditions;
$B^*(x_k, A_k)$ = net profit for allocated amounts of water and area (Rs.);
b_k = coefficient for particular crop;
c = pore connectivity index;
FC = field capacity (mm/cm);
$f_k^*(x_k, A_k)$ = measure of performance in DP;
$f_t^*(x_t, \mathrm{AET}_t)$ = measure of performance in SDP;
$j_{\max}$ = number of class levels for canal release;
K_c = crop coefficient;
k_t = yield stress sensitivity factor for period t;
$m_{\max}$ = number of class levels for evapotranspiration;
N = total number of crops;
NP = total number of periods for crop;
PET = potential evapotranspiration (mm);
PRO_k = profit per yield of crop k [rupees (Rs.)/100 kg];
P_t = deep percolation (mm);
p = crop-water depletion factor;
Q = total seasonal water available (mm);
R_t = rainfall in period t (mm);
$R_t^*(x_t, \mathrm{AET}_t)$ = relative yield corresponding to x_t irrigation depth and given AET;
S_0 = initial soil moisture content (mm/cm);
SM_{sat} = saturated soil moisture content (mm/cm);
SM_t = soil moisture content in depth units per root depth in period t (mm/cm);
$TE_{lm}(t)$ = transition probability of weekly evapotranspiration from level l in period t to level m in period $t + 1$;
$TS_{ij}(t)$ = transition probability of weekly canal release from level i in period t to level j in period $t + 1$;
$t_{z\max}$ = time for full development of root zone (days);
WP = wilting point (mm/cm);
X_{mk} = minimum depth of water required to give relative yield (mm);
x_t = water allocation in period t (mm);
Y_a = actual yield obtained (100 kg/ha);
$Y_{k_{\max}}$ = maximum obtainable yield corresponding to crop k (100 kg/ha);
$Y_{\max}$ = maximum obtainable yield (100 kg/ha);
y_k = seasonal relative yield for crop k corresponding to x_k depth of water applied;
$y_k(x_k)$ = relative yield for x_k depth of irrigation water;
y_{0k} = relative yield corresponding to zero irrigation allocation;
Z = depth of effective root zone at t_z time after sowing (cm);
$Z_{\max}$ = maximum possible depth of effective root zone (cm); and
Z_t and Z_{t+1} = root zone depths in the periods t and $t + 1$, respectively (cm).

Subscripts

k = index for crop; and
t = index for period.