Optimal reservoir operation for irrigation of multiple crops using elitist-mutated particle swarm optimization

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Abstract To achieve social and economic sustainability in arid and semi-arid areas under water scarce situations, it is vital to promote efficient use of water through improved management of water resources. This paper presents a swarm optimization based solution to a detailed operational model for short-term reservoir operation for irrigation of multiple crops. The model integrates the dynamics associated with the water released from a reservoir to the actual water utilized by crops at farm level. It takes into account the nonlinear relationship of root growth, soil heterogeneity, soil moisture dynamics for multiple crops, yield response to water deficit at various growth stages of the crops and economic benefits from the crops. As the developed model is a nonlinear one, it is solved using a novel global optimization technique, namely elitist-mutation particle swarm optimization (EMPSO). The model's applicability is demonstrated through a case study of Malaprabha Reservoir system in Southern India. The performance of the model is examined for different water deficit conditions and the sensitivity of the crop yield is analysed for water shortage at various growth stages. Also, the consideration of economic benefits in the objective function and its effect on the water allocation decisions for multiple crops are studied. Consequently, the output from the model includes initial storages, releases, overflows and evaporation losses for each 10-day period on the reservoir side; and allocation of water, actual evapotranspiration and initial soil moisture for each crop for each 10-day period on the field side, thus facilitating decision making for optimal utilization of the available water resources.

Keywords crop water allocation; crop yield response; irrigation of multiple crops; particle swarm optimization; elitist mutation; reservoir operation; soil moisture dynamics

Gestion optimale de réservoir pour l'irrigation de cultures multiples en utilisant l'optimisation par mutation élitiste par essaim de particules

Résumé Pour atteindre la durabilité sociale et économique dans des zones arides et semi-arides en situation de rareté de l'eau, il est essentiel de favoriser l'utilisation efficace de l'eau par la gestion améliorée des ressources d'eau. Cet article présente une solution basée sur l'optimisation par essaim d'une modélisation opérationnelle détaillée de la gestion à court terme de réservoir pour l'irrigation de cultures multiples. Le modèle intègre la dynamique entre l'eau lâchée d'un réservoir et l'eau réellement utilisée par des cultures au niveau de l'exploitation agricole. Il tient compte de la relation non-linéaire entre la croissance racinaire, l'hétérogénéité du sol, la dynamique de l'humidité de sol pour des cultures multiples, la réponse en rendement au déficit en eau à différents stades de croissance des cultures, et les avantages économiques des cultures. Le modèle développé étant non-linéaire, il est résolu en utilisant une nouvelle technique d'optimisation globale par mutation élitiste par essaim de particules (EMPSO). L'applicabilité du modèle est démontrée avec l'étude de cas du système de réservoir de Malaprabha en Inde méridionale. La performance du modèle est examinée pour différents états de déficit en eau, et la sensibilité du rendement cultural à la pénurie d'eau est analysée à diverses étapes de la croissance. En outre la prise en compte des bénéfices économiques dans la fonction objectif et son influence sur les décisions d'allocation de l'eau pour les cultures multiples sont étudiées. Par conséquent, les sorties du modèle couvrent les stockages initiaux, les lâchures, les déversements et les pertes par évaporation du réservoir pour chaque période de dix jours; ainsi que l'allocation de l'eau, l'evapotranspiration réelle et l'humidité initiale du sol pour chaque récolte pour chaque période de dix jours, ce qui facilite la prise de décision pour une utilisation optimale des ressources en eau disponibles.

Mots clefs allocation de l'eau agricole; réponse en rendement cultural; irrigation de cultures multiples; optimisation par essaim de particules; mutation élitiste; gestion de réservoir; dynamique de l'humidité du sol.

INTRODUCTION

As there is a continuously growing demand for water for various purposes, the shortage of freshwater is causing serious problems worldwide. Agriculture is the major consumer

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Open for discussion until 1 February 2008

of available water resources and therefore efficient use of water for irrigation is critical to social and economic sustainability of a country. This requires efficient management of the limited water resources to increase crop productivity. To achieve this, an effective reservoir operation model for irrigation should consider decisions both at reservoir level and at farm level. As it is a usual practice in many farming communities to grow several crops within a season, the actual model should also consider the multicrop situation (Rao *et al.*, 1990; Vedula & Nagesh Kumar, 1996). In water scarce conditions, for irrigation of multiple crops, there is always competition among the crops for the limited amount of water available. A multicrop water allocation model must therefore include a crop model that encodes the sensitivity of crop yield to moisture stress during various physiological growth stages of the plants. This can provide the structure for prioritizing the allocation of irrigation water during critical growth stages of the crop in order to maximize final crop yield. Hence, modelling for optimal allocation of scarce water resources is a challenging task in irrigation water management, more so in the case of multicrop farming.

For optimal allocation of irrigation water, models were developed based on stochastic dynamic programming (SDP) for a single crop situation (for example, Dudley *et al.*, 1971; Dudley & Burt, 1973; Bras & Cordova, 1981) and for a multicrop situation (for example, Rao *et al.*, 1990; Vedula & Mujumdar, 1992; Vedula & Nagesh Kumar, 1996). Rao *et al.* (1990) developed a model for optimal weekly irrigation scheduling policy for two crops, considering both seasonal and intraseasonal competition for water. Vedula & Mujumdar (1992) formulated a model to obtain a steady state optimal reservoir operating policy for irrigation of multiple crops with stochastic inflows by first using dynamic programming (DP) to optimally allocate the available water among all crops within a given period, and then evaluating the system performance using SDP to optimize the benefits over a full year. This model is constrained by coarse discretization and fewer numbers of state variables due to the curse-of-dimensionality of the DP approach. Vedula & Nagesh Kumar (1996) developed an improved model using a two-stage linear programming(LP)-SDP approach considering the soil moisture balance independently for each crop, and treating the rainfall in the irrigated area as stochastic for obtaining the steady-state optimal operating policy and the associated crop-water allocations to multiple crops. Due to the limitation of LP, various functional relationships are assumed to be linear in the model. But this will not reflect the actual situation in the field. The use of an intraseasonal allocation model in the case of two seasonal crops is also an additional limitation, since the model gives the same priority for two seasonal crops of few growth stages as that for complete growth and yield formation of a single seasonal crop.

It is also noted that SDP-based steady state operating policies are useful for maximizing the long-term benefits from an irrigation system (Oliveira & Loucks, 1997; Mujumdar & Ramesh, 1998). For real time reservoir operation an adaptive optimization model will be more useful. Mujumdar & Ramesh (1998) developed a short-term yearly reservoir operation model based on LP for irrigation of multiple crops. This model optimizes the annual crop production, starting from the current period in real time. The solution specifies the reservoir release and optimal irrigation allocations to individual crops during an intraseasonal period. However, this model also has similar types of limitations as mentioned above for the LP approach, since it assumes many relationships as linear. Also, the models were applied to the crops grown on a single soil type irrigated area.

In this study a nonlinear reservoir operation model is proposed for irrigation of multiple crops. The model can handle all types of nonlinear relationships. At the reservoir level, it considers the water availability in the reservoir and makes decisions on releases to meet the existing irrigation demands to the maximum extent possible through optimization. At field level, the model considers periodical competition for water among multiple crops, the soil moisture dynamics in each cropped area and crop response to the level of irrigation in each stage of its growth, and then allocates the limited water among multiple crops.

Evolutionary algorithms for optimization

Recently, evolutionary algorithms (EA) have become popular tools for solving real-world optimization problems. These algorithms use simulated evolution to search for complex problems. Under the evolutionary paradigm, the genetic algorithm (GA) is a search method based on the principles of Darwinian natural selection and survival of the fittest (Goldberg, 1989). GA has been applied to solve several single objective reservoir operation problems (Oliveira & Loucks, 1997; Wardlaw & Sharif, 1999; Cai *et al.,* 2001; Raju & Nagesh Kumar, 2004; Akter & Simonovic, 2004; Nagesh Kumar *et al.* 2006). Multi-objective evolutionary algorithms (MOEA) were also applied for analysing operational policies for multi-objective reservoir operation problems (Janga Reddy & Nagesh Kumar, 2006, 2007). Other meta-heuristic techniques applied for reservoir operation include, simulated annealing (Teegavarapu & Simonovic, 2002), ant colony optimization (ACO) (Nagesh Kumar & Janga Reddy, 2006) and particle swarm optimization (PSO) (Nagesh Kumar & Janga Reddy, 2007).

More recently, Nagesh Kumar & Janga Reddy (2007) proposed a novel optimization algorithm for reservoir operation by incorporating an elitist mutation mechanism into the standard PSO method. The efficiency of the developed elitist mutated PSO (EMPSO) method was tested on two problems in reservoir operation and it was found that EMPSO yields robust global optimal solutions while taking fewer functional evaluations, compared to standard PSO and GA techniques. In this study, it is proposed to use the EMPSO technique to solve the model developed for integrated reservoir operation for multi-crop irrigation system.

RESERVOIR OPERATION MODEL FOR IRRIGATION

A nonlinear optimization model is formulated for reservoir operation by integrating the dynamics of water availability at reservoir level with crop water requirements at field level. The model aims to maximize the total relative yield from the crops, subject to various constraints. The constraints conceptually consist of two components, the first component is reservoir level constraints dealing with reservoir release decisions and the storage related constraints, and the second is farm level constraints dealing with crop water allocation decisions and the soil moisture balance related constraints.

Objective functions

In crop water allocation modelling, dated crop-water requirement functions incorporate effects of both timing and quantities of water applications on crop yield. Generally, evapotranspiration is used as the water related independent variable. Following Stewart *et al.* (1974), the expression used for crop yield is a function of the ratio of actual evapotranspiration (AET) and potential evapotranspiration (PET), i.e. (AET/PET). A multiplicative dated crop-water requirement function, derived from sensitivity factors for water stress in various physiological growth stages of crops (Doorenbos & Kassam, 1979), is considered in this study. As the formulation is for multiple crops, the sum of the relative yields of all the crops in a year is taken as the total relative crop yield. It includes *kharif* season crops (June–October), *rabi* season crops (November–March) and two seasonal crops. The objective is to maximize total relative yield over a year. The expression used for the total relative yield from multiple crops is:

$$\max \mathrm{OF}_1 = \sum_{c=1}^{\mathrm{NC}} \left[\prod_{t=T0_c}^{\mathrm{NTP}_c} \left(1 - \mathrm{ky}_c^t \left(1 - \frac{\mathrm{AET}_c^t}{\mathrm{PET}_c^t} \right) \right) \right] \tag{1}$$

where c is a crop index; t is a time period index; $\mathrm{AET}_c{}^t$ and $\mathrm{PET}_c{}^t$ are actual evapotranspiration (mm) and potential evapotranspiration (mm), respectively; NC is the total number of crops grown in a year; $T0_c$ and NTP_c are the starting and ending times, respectively, for the crop c; and $\mathrm{ky}^t{}_c$ is the yield stress sensitivity factor for the time period t of crop c, which is assumed to be the same in every period in a growth stage.

Yield sensitivity factors If the water deficit during the critical growth periods is severe, it may cause a large decrease in the final yield of crops. The crop yield sensitivity factor (ky) enables the model to allocate water, depending on the crop growth sensitivity and its influence on final yield. The objective function will be at its maximum when the allocation of available water to individual crops is such that AET = PET for each crop in each period. As such an ideal condition is not always possible, for a water deficit situation, the irrigation allocation is made such that the total relative yield is maximum. Yield sensitivity factors for different crops were adopted from Doorenbos & Kassam (1979).

From experimental results it is observed that, for multiple crop situations with varying irrigated area sizes, the objective function given in equation (1) has a biased influence on water allocation to the small area irrigated crops. This objective function is a plain crop water production function that always favours water allocation to the crops which demand less water and with small irrigated area. Also it does not consider any economic value associated with the crop yield.

Equivalent benefit coefficient To reduce the influence of size of the area and to incorporate economic benefits from the crops, the following objective function is chosen:

$$\mathrm{OF}_2 = \sum_{c=1}^{\mathrm{NC}} \left[B_c \prod_{t=T0_c}^{\mathrm{NTP}_c} \left(1 - \mathrm{ky}_c^t \left(1 - \frac{\mathrm{AET}_c^t}{\mathrm{PET}_c^t} \right) \right) \right] \tag{2}$$

where B_c is the equivalent benefit coefficient for crop c.

This objective function, in addition to considering crop yield sensitivity to irrigation, also incorporates crop related economic benefits in the water allocation decisions. The benefit coefficient values are derived from the crop area and its associated economic benefits. The equivalent benefit coefficient for crop c with respect to a reference crop, rc, is defined as:

$$B_c = (\mathrm{CY}_c \ \mathrm{PR}_c \ A_c)/(\mathrm{CY}_{\mathrm{rc}} \ \mathrm{PR}_{\mathrm{rc}} \ A_{\mathrm{rc}}) \tag{3}$$

where CY_c is crop yield (kg/ha); PR_c is the price of crop (Rs/kg); A_c is the area of crop (ha); and rc denotes reference crop.

The objective functions in equations (1) and (2) are subjected to the following constraints.

Reservoir level constraints

Reservoir water balance This is governed by the reservoir storage continuity equation:

$$S^{t+1} = S^t + Q^t - R^t - L^t - \mathrm{OVF}^t \quad \forall\ t \tag{4}$$

where S^t is reservoir storage at the beginning of period t in hm^3 (million cubic meters); Q^t is inflow into the reservoir during period t in hm^3; R^t is release from the reservoir in period t in hm^3; L^t is evaporation losses during period t in hm^3 (a nonlinear function of initial and final storages of period t); and OVF^t is overflow from the reservoir in period t in hm^3.

Storage bounds The reservoir storage is restricted by:

$$S^{\min} \le S^t \le S^{\max} \qquad \forall\ t \tag{5}$$

where $S^{\min}$ and $S^{\max}$ are the minimum and maximum storage limits of the reservoir in hm^3.

Farm level constraints

Water available for irrigation The water release made from the reservoir (R^t), undergoes seepage, conveyance, application and other losses. The water actually available for irrigation at the farm level Q^t, is therefore a fraction of R^t, given by:

$$Q^t = \eta R^t \qquad \forall\ t \tag{6}$$

where η is the conveyance efficiency accounting for all the losses from the reservoir head regulator to the farm level.

Allocation constraints Total water available for irrigation (Q^t) in a period must be equal to the total water actually allocated to all crops in that period:

$$Q^t = \sum_{c=1}^{NC} q_c^t A_c \qquad \forall t \tag{7}$$

where q_c^t is water allocation for crop c in period t; A_c is the area of crop c under irrigation.

Soil moisture balance The root-zone water content decreases with crop transpiration and evaporation from soil, and it increases with rainfall, irrigation and deepening of the root zone as the crop grows. The general mass balance equation for soil moisture is:

$$\mathrm{SM}_c^{t+1}D_c^{t+1} = \mathrm{SM}_c^t D_c^t + \mathrm{RF}^t + q_c^t - \mathrm{AET}_c^t + \mathrm{SM}_c^{\max}(D_c^{t+1} - D_c^t) - \mathrm{DP}_c^t - \mathrm{SR}_c^t \quad \forall c,t \tag{8}$$

where SM_c^t is available soil moisture at the root zone for crop *c* in period *t* (mm/cm); D_c^t is root depth of crop *c* in period *t* (cm); RF^t is rainfall in period *t* (mm); q_c^t is water allocation for crop *c* in period *t* (mm); $\mathrm{SM}_c^{\max}$ is maximum available soil moisture at field capacity for crop *c* (mm/cm); DP_c^t and SR_c^t are deep percolation and surface runoff, respectively, in period *t* (mm); and the available soil moisture in any time period *t* is restricted to the maximum capacity of the soil:

$$\mathrm{SM}_c^t \le \mathrm{SM}_c^{\max} \qquad \forall c,t \tag{9}$$

In equation (8), various model variables are computed as follows:

Crop root depth The depth of the active soil reservoir from which the crops extract water depends on the effective depth of root penetration into the soil. This depth increases with the crop growth and attains a maximum value by the end of the flowering period for most crops. Root depth as a function of time after planting is obtained using the Borg & Grimes (1986) sinusoidal function:

$$D_c^t = D_c^{\max}\left(0.5 + 0.5\sin\left[3.03\left(\frac{(t - \mathrm{T0}_c) + 1}{\mathrm{NTP}_c - \mathrm{T0}_c + 1}\right) - 1.47\right]\right) \ \forall c,t \tag{10}$$

where $D_c^{\max}$ is the maximum possible depth of effective root zone for crop *c* (cm).

Actual evapotranspiration Actual crop evapotranspiration depends on the evaporative demand of the atmosphere, the crop growth stage, and the available soil moisture in the root zone. Among several methods available for determining the reference evapotranspiration (ET_0), the FAO Penman-Monteith method (Allen *et al.*, 1998) is considered to be more accurate and is used to compute the ET_0.

Potential evapotranspiration is given by:

$$\mathrm{PET} = K_c \mathrm{ET}_0 \tag{11}$$

where K_c is a crop coefficient.

The actual evapotranspiration in relation to its potential rate is determined by considering whether the available water in the root zone is adequate or whether the crop will suffer from stress induced by water deficit. The actual evapotranspiration in each period is computed as follows:

$$\mathrm{AET}_c^t = \begin{cases} 0, & \mathrm{SM1}_c^t \le \mathrm{WP} \\ \dfrac{\mathrm{PET}_c^t(\mathrm{SM1}_c^t - \mathrm{WP})}{(1-p)(\mathrm{FC} - \mathrm{WP})} & \mathrm{WP} < \mathrm{SM1}_c^t \le [\mathrm{WP} + (1-p)(\mathrm{FC} - \mathrm{WP})] \ \forall c,t \\ \mathrm{PET}_c^t & \mathrm{SM1}_c^t \ge [\mathrm{WP} + (1-p)(\mathrm{FC} - \mathrm{WP})] \end{cases} \tag{12}$$

where WP is the wilting point (mm/cm); FC is the field capacity (mm/cm); *p* is the crop water depletion factor and $\mathrm{SM1}_c^t = (\mathrm{SM}_c^t D_c^t + \mathrm{RF}^t + q_c^t)/D_c^t$.

Surface runoff After meeting field capacity requirements, any excess water applied to the irrigated area drains-out as surface runoff (SR) from the irrigated area.

$$\mathrm{SR}_c^t = \begin{cases} 0 & \mathrm{SM2}_c^t \le \mathrm{SM}_c^{\mathrm{sat}} \\ (\mathrm{SM2}_c^t - \mathrm{SM}_c^{\mathrm{sat}})D_c^t & \mathrm{SM2}_c^t > \mathrm{SM}_c^{\mathrm{sat}} \end{cases} \quad \forall c,t \tag{13}$$

where $\mathrm{SM}_c^{\mathrm{sat}}$ is the saturated soil moisture content (mm/cm); and $\mathrm{SM2}_c^t = \left(\mathrm{SM}_c^t D_c^t + \mathrm{RF}^t + q_c^t - \mathrm{AET}_c^t + \mathrm{SM}_c^{\max}(D_c^{t+1} - D_c^t)\right) / D_c^t$.

Deep percolation When irrigation is applied, any excess beyond its field capacity will drain down as deep percolation, which is also included in the soil moisture balance equation. The present study uses an empirical equation for calculating the deep percolation component (Rao *et al.*, 1990; Paul *et al.*, 2000), which is given by:

$$\mathrm{DP}_c^t = \begin{cases} 0\,, & \mathrm{SM3}_c^t \le \mathrm{FC} \\ \left(\dfrac{v(\mathrm{SM}_c^{\mathrm{sat}})(\exp[\mathrm{SM3}_c^t - \mathrm{FC}] - 1)}{(\exp[\mathrm{SM}_c^{\mathrm{sat}} - \mathrm{FC}] - 1)} \right) D_c^t & \mathrm{FC} < \mathrm{SM3}_c^t \le \mathrm{SM}_c^{\mathrm{sat}} \end{cases} \quad \forall c,t \tag{14}$$

where v is the pore connectivity index; and $\mathrm{SM3}_c^t = \left(\mathrm{SM}_c^t D_c^t + \mathrm{RF}^t + q_c^t - \mathrm{AET}_c^t + \mathrm{SM}_c^{\max}(D_c^{t+1} - D_c^t) - \mathrm{SR}_c^t\right) / D_c^t$.

The final model formulation involves maximization of the objective function given in equation (1) or (2) subject to various constraints given in equations (4)–(14).

PARTICLE SWARM OPTIMIZATION

Particle swarm optimization (PSO) is a new class of population based heuristic search technique in swarm intelligence (Kennedy & Eberhart, 2001). This technique is inspired by the social behaviour of bird flocking and shares many similarities with evolutionary computation techniques such as genetic algorithms (GA). Similar to GA, PSOs are initialized with a population of random solutions and conduct searches for optima by updating generations. However, in contrast to methods like GA, in basic PSO, no operators inspired by natural evolution are applied to extract a new generation of candidate solutions. Instead, PSO relies on the exchange of information among individuals (particles) of the population (swarm). In effect, each particle adjusts its trajectory towards its own previous best position and towards the current best position attained by any other member in its neighbourhood (Parsopoulos & Vrahatis, 2002).

Recently, various studies on the PSO algorithm have reported that this technique is providing quick convergence to global optimal solutions. It is also found that PSO is outperforming other heuristic search methods such as Genetic Algorithms (Fourie & Groenwold, 2002; Abido, 2002; Salman *et al.*, 2002) and can be used as an attractive alternative for numerical optimization problems. However, when it was applied to complex reservoir system problems, it was observed that the technique is converging to local optima solutions. To improve the performance of the algorithm, recently Nagesh Kumar & Janga Reddy (2007) incorporated a new elitist-mutation mechanism into the standard PSO algorithm. The efficiency of the developed elitist-mutated particle swarm optimization (EMPSO) algorithm is demonstrated through application to standard test problems in reservoir system optimization. The present study uses the EMPSO technique to solve the formulated optimization model.

The EMPSO algorithm

If the search space is D-dimensional, the ith individual (*particle)*, of the population (*swarm*), can be represented by a D-dimensional vector, $X_i = (x_{i1}, x_{i2}, \ldots, x_{iD})^T$. The *velocity* (position change) of this particle, can be represented by another D-dimensional vector $V_i = (v_{i1}, v_{i2}, \ldots, v_{iD})^T$. The position that is associated with the best fitness (the particle has achieved so far) is considered as its previous best position. This position for the ith particle is recorded and represented as $P_i = (p_{i1}, p_{i2}, \ldots, p_{iD})^T$ and its corresponding fitness value, called the individual best (Pbest$_i$), is also recorded. The overall best position of the population associated with the best fitness value among all the individual bests (Gbest), is recorded and represented as $P_g = (p_{g1}, p_{g2}, \ldots, p_{gD})^T$. During the iteration procedure, the velocity and position of particle i are updated according to the following equations:

$$v_{id}^{n+1} = \chi[w\, v_{id}^{n} + c_1 r_1^n (p_{id}^n - x_{id}^n)/\Delta t + c_2 r_2^n (p_{gd}^n - x_{id}^n)/\Delta t] \quad (15)$$

$$x_{id}^{n+1} = x_{id}^{n} + v_{id}^{n+1} \quad (16)$$

where $d = 1, 2, ..., D$; $i = 1, 2, \ldots, N$; N is the size of the swarm; χ is a constriction coefficient; w is inertia weight; c_1 and c_2 are positive constant parameters called acceleration coefficients; r_1 and r_2 are random numbers, uniformly distributed in [0,1]; Δt is the time step, usually set as 1; and n is the iteration number.

The elitist-mutation step is performed in each iteration after updating velocity and position vectors. First the fitness values of all the particles are sorted in ascending order and thus the index number for the respective particles is obtained. The elitist-mutation is performed on a pre-specified number of the worst fitness value particles in the swarm. In the elitist-mutation mechanism, a variable-wise mutation is performed on the position vector of P_g with some probability (P_{em}) and replaces the respective particle position. Such mutations are realized by replacing P_{id} with $P_{gd} + S_m \times VR_i \times N(0,1)$, where VR_i is range of the ith decision variable, S_m is mutation scale factor and $N(0,1)$ is a normal random number with zero mean and standard deviation one. The elitist-mutation mechanism provides good exploration (i.e. it tries to continuously search for better quality solutions by maintaining diversity among the population) and good exploitation (i.e. it provides neighbourhood search around the best solutions). More details of the procedure are given in Nagesh Kumar & Janga Reddy (2007), in which it was inferred that the best set of parameters for reservoir operation problems are: constriction coefficient $\chi = 0.9$; inertia weight = 1; acceleration coefficients $c_1 = 1.0$ and $c_2 = 0.5$.

Computational procedure of EMPSO for reservoir operation

The EMPSO approach for solving the optimal reservoir operation problem for irrigation of multiple crops involves the following steps:

1. *Input system data* Input the data required for reservoir system. Input the dimension of the problem (D); size of the swarm (N); maximum number of iterations

(MaxIter); constriction coefficient (χ) and control parameters c_1 and c_2.

2. *Initialize particle population* Randomly generate an initial population of particles with random positions and velocities on D dimensions in the solution space. Set the index of iteration n to 0.
3. For each time period t, first perform the calculations at reservoir level; then for each crop perform the calculations at farm level.
4. *Calculate the fitness value* For each particle, first calculate the total fitness value using equation (1), and then check for all the constraints of the model. If certain constraints are violated, add a suitable penalty term to the total fitness value and obtain the final fitness value of each particle.
5. If $n = 0$, then set the current fitness as its personal best (Pbest_i) for each particle and move to Step 6; otherwise, for each particle, compare its fitness value with the individual best. If this fitness value is better than the earlier Pbest_i fitness, set this value as the current best, and record the corresponding particle position.
6. Choose the particle associated with the maximum individual best of all particles, and set the corresponding Pbest_i as the current overall best (Gbest).
7. *Velocity and position updating* For each particle, calculate the new velocity using equation (15), and then update the particle position using equation (16).
8. For each new particle, check its position to ensure feasibility. If the position on a dimension exceeds the specified range, then the position on that dimension is limited to the corresponding bound.
9. *Elitist-mutation* Perform elitist-mutation on pre-specified number of particles.
10. *Check the stopping criterion* If the maximum number of iterations is reached or if any other termination criteria is reached, stop the iteration and output the optimal solution; otherwise, set the index $n = n + 1$, and reiterate steps 3–10.

CASE STUDY AND MODEL APPLICATION

The developed model is applied to a case study of Malaprabha Reservoir system in the Krishna Basin, Karnataka State, India. The Malaprabha Dam is located in Belgaum District of Karnataka State at 15°49′N latitude and 75°6′E longitude. The reservoir has a gross storage capacity of 1070 hm^3 and a live storage capacity of 830 hm^3; the 75% dependable annual yield for the reservoir is 1205 hm^3. The reservoir is a single-purpose irrigation reservoir and has been in operation since 1973. The project provides irrigation in Dharwad, Belgaum, and Bijapur districts of Karnataka state. The reservoir has a major portion of irrigated area (71%) in black cotton soil and the remaining in red soils under right bank and left bank canal command areas. The major crops grown are cotton, wheat, sorghum, maize, safflower and pulses. The details of inflows for a period of 53 years (June 1951–May 2004) and other details of the study area were collected from Water Resources Development Organization (WRDO), Bangalore. The daily crop water requirements are calculated using FAO Penman-Monteith method (FAO-56) and totalled to obtain 10-day crop water demands and are used in the model application.

Figure 1 shows the principal crops and the crop calendar for the command area of the Malaprabha Reservoir system. The crop growth stages and yield response factors

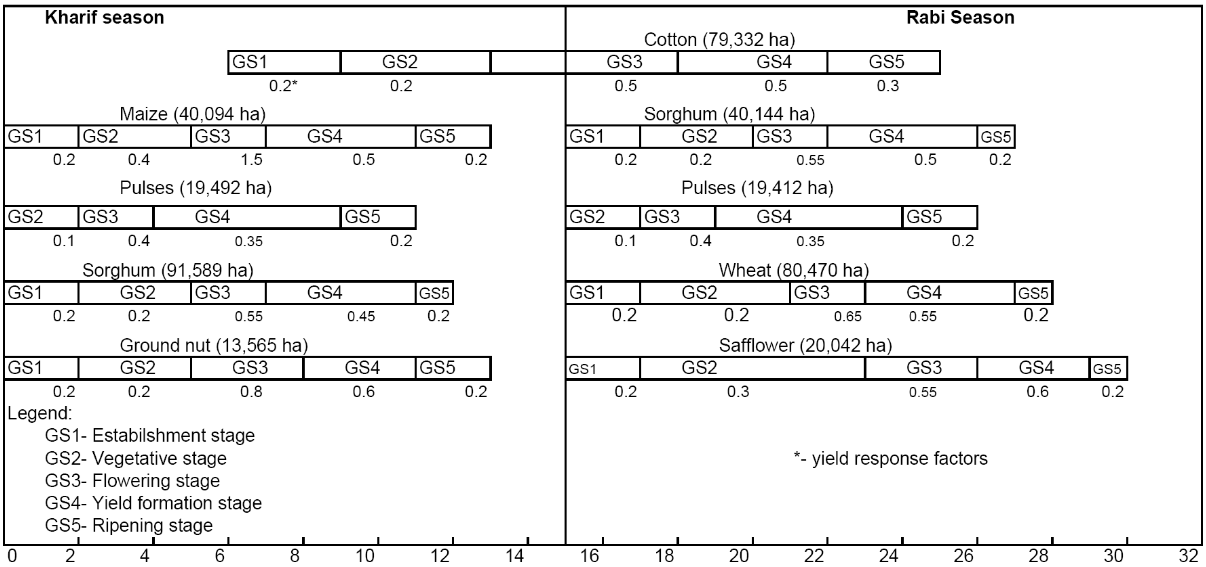

Fig. 1 Crop calendar used for command area of Malaprabha Reservoir system. Numbers on the *x*-axis are 10-day periods. The irrigated area of each crop is given in parentheses.

are taken from Doorenbos & Kassam (1979). In a year, there are two principal cropping seasons: *kharif* (monsoon season: June–October) and *rabi* (non-monsoon season: November–March). Under reservoir irrigation, nine crops are considered for the study, four in *kharif*, four in *rabi*, and one two-seasonal crop. It should be noted that, for modelling purposes, if the same crop is planted at a different time in the year, it is treated as a separate crop. Similarly if the same crop is grown in different types of soil, it is considered as a different crop. The water year begins on 1 June and ends on 31 May next calendar year. Each month is divided into three periods: the first two being 10-day periods, and the third consisting of the remaining days in the month. Thus a time interval of 10 days is chosen for reservoir operation and irrigation allocation decisions. The growth stages of the crops were adjusted to be multiples of the decision intervals (10-day) and are modelled accordingly in the study.

For model application, each crop is considered to have five growth stages: establishment, vegetative, flowering, yield formation and ripening (Fig. 1). The crop root growths are nonlinear functions of the time period (equation (10)). The inputs to the model include the initial storage of the reservoir at the starting period of optimization, the inflows into the reservoir, rainfall in the command area, the potential evapotranspiration values for the crops and crop yield response factors for each growth stage. The soil moisture values at the beginning of crop growth were assumed to be at the field capacity of the soil for all the crops. The field capacity (FC) and wilting point (WP), respectively, are adopted as 3.5 and 1.7 mm/cm for black cotton soil, and 2.0 and 1.0 mm/cm for red soils; the crop water depletion factor (p) is taken as 0.4 and 0.5 for black cotton and red soils respectively; irrigation efficiency is taken as 50%. To apply the EMPSO technique, the following parameters are used: size of population = 200; maximum number of iterations = 500; constant parameters c_1 = 1.0 and c_2 = 0.5; constriction coefficient χ = 0.9; size of elitist-mutated particles is 20; probability of elitist-mutation (P_{em}) = 0.1; and the value of S_m decreases from 0.1 to 0.01 over the iterations. The model is applied for three different inflow scenarios:

– *Scenario 1*: Average hydrological conditions, i.e. average monthly inflows into the reservoir (INF_{avg}) and average monthly rainfall in the command area ($RAIN_{avg}$)
– *Scenario 2*: Below average hydrological conditions: $0.8 \times INF_{avg}$, and $0.8 \times RAIN_{avg}$
– *Scenario 3*: Far below average hydrological conditions: $0.6 \times INF_{avg}$ and $0.6 \times RAIN_{avg}$

It is assumed that these three scenarios can provide a sufficient insight into the model under water deficit conditions. The model performance is evaluated for two types of objective functions, OF_1 and OF_2. The first objective function (OF_1), maximizes the total relative yield of multiple crops, without considering the economic benefit (equation (1)). The second objective function (OF_2), in addition, considers the value of equivalent benefit coefficient (B_c), i.e. the model objective function integrates area-related economic benefits with crop growth sensitivity. To compute equivalent benefit coefficients, wheat is chosen as the reference crop. Table 1 shows the data related to irrigated area, crop productivity, equivalent economic benefits and total crop water requirement (CWR) for all the crops that are considered in the study.

Table 1 Details of the data used in the study for *kharif*, *rabi* and two-seasonal crops.

Crop	Area (ha)	Crop yield [a] (kg/ha)	Price [b] (*Rs/100 kg)	Equivalent benefit coefficient (B_c)	Total CWR (mm)
Kharif season					
Maize	40 094	1820	540	0.2798	384.30
Pulses	19 492	600	1435	0.1192	291.10
Sorghum	91 589	803	525	0.2742	335.40
Ground nut	13 565	970	1520	0.1420	327.57
Rabi season					
Sorghum	40 144	803	525	0.1202	428.40
Pulses	19 412	600	1435	0.1187	374.00
Wheat	80 470	2692	650	1.0000	620.65
Safflower	20 042	596	1760	0.1493	602.41
Two-seasonal					
Cotton	79 332	500	1760	0.4958	683.70

Sources: [a] Dept of Agriculture and Cooperation Annual Report, 1999–2000, Govt of India.
[b] Dept of Agriculture and Economics, minimum support price for 2005 season, Govt of India.
* US$1 = Rs 46.

RESULTS AND DISCUSSION

The developed reservoir operation model for irrigation of multiple crops is solved using the EMPSO technique for optimal utilization of available water resources to maximize the relative yield from all the crops. The model considers nine crops in a year, viz., in the *kharif* season: maize, pulses, sorghum and groundnut; in the *rabi* season: sorghum, pulses, wheat, safflower; and a two-seasonal crop: cotton. If there is enough water to meet the demands of all the crops, it will allocate the water accordingly. However, if there is competition for limited water among the multiple crops, the allocation of irrigation water to individual crops depends upon crop growth stage, sensitivity to water stress and its effect on final yield. For the Malaprabha Reservoir system, water deficit exists mainly in the *rabi* season, since the *kharif* crops benefit to a large extent from the monsoon rains.

The multicrop reservoir operation model was run for the three hydrological scenarios described above. The model output gives detailed results, such as: decisions at reservoir level, including reservoir water release, storage, evaporation losses and overflow losses in each time period; and decisions at farm level including the water allocation, soil moisture status, actual evapotranspiration, deep percolation and surface runoff for each crop and for each time period. Table 2 gives a typical output from the model run using the EMPSO technique for the *rabi* season for Scenario 1. Figure 2 shows the releases obtained from the reservoir operation model for all three scenarios. It can be seen from the figure that, during the *kharif* season (time periods 1–15), the releases are very small; since, during most of the time periods, the crop water demands were met from the monsoon rainfall in the irrigated area. So in the following, the analysis is presented for *rabi* season crops.

Figure 3 shows the water allocated by the model for different crops grown for three scenarios using two types of objective functions. It can be seen that the results obtained for the two objective functions allocate water differently, resulting in differences in the crop response pattern. It is interesting to note that the first objective function (OF_1) allocates more water to the plants which have lesser water demands in the field and allows deficit to crops demanding larger amount of water. The wheat crop has a larger area and water demand. This model (with OF_1) allots the deficits to that particular crop and provides adequate water for the other crops in order to maximize the total relative yield. The model with second objective function (OF_2) always gives priority to the crop which is yielding the highest economic benefits. From Fig. 3, it can be seen that, with objective function OF_2, crop water demands are fully met for the wheat crop and it allots deficits to other crops. The main reason is that the wheat crop provides the highest economic benefits, so more water is allocated to that crop. Similar results are observed for below average and far below average hydrological conditions (scenarios 2 and 3). Also it is noticed that due to high competition for water, some of the crops just survive with available rainfall that is received in the command area, but the yield (benefits) of the crops will be drastically reduced. This is compensated by other crops producing higher yield (benefits), resulting in overall maximum benefit of the optimization model.

Figure 4 shows the crop response (AET/PET) patterns over various time periods for the three scenarios using two types of objective function during the *rabi* season. It can be seen that the OF_2 model gives maximum preference for wheat throughout its growth periods and allocates deficits to other crops, whereas OF_1 allots water deficit to wheat, and provides healthy crop conditions for other crops to maximize the total relative yield.

So the best possible alternative to overcome crop failure under water-scarce conditions is that: if the seasonal forecast is available well in advance (seasonal rainfall forecasts are available for different meteorological divisions from Indian Meteorological Department, IMD, while any conventional/advanced forecasting technique can be used for inflow forecasts), the cropping pattern can be fixed depending on the total water availability. Then the developed reservoir operation model with the economic benefit related objective function (OF_2) can provide suitable crop water allocation decisions to achieve maximum benefit from the available water resources.

Table 2 Typical model result for *rabi* season for Scenario 1 (average hydrological conditions) with objective function OF_1.

Crop	Intra-seasonal 10-day time period, *t*: 16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Reservoir storage at the beginning of each period (hm^3):															
	826.77	815.67	743.13	643.82	533.82	419.65	342.81	233.69	151.99	86.04	80.25	23.24	23.52	12.90	1.42
Release from the reservoir (hm^3):															
	23.41	82.45	107.09	114.54	117.32	78.93	112.40	85.19	69.68	8.44	59.80	2.40	14.47	13.61	0.87
Evaporation losses from the reservoir (hm^3):															
	5.266	5.189	4.598	4.18	3.243	3.151	2.166	1.673	1.375	1.072	0.93	0.592	0.743	0.718	0.783
Irrigation allocation for each period for each crop (hm^3):															
Sorghum	1.85	0.00	6.41	13.75	13.78	23.74	20.61	15.50	10.72	0.91	13.17	0.00			
Pulses	1.53	0.03	5.00	5.36	6.67	7.49	0.00	3.26	7.55	0.06	4.59				
Wheat	0.00	4.30	2.08	0.00	2.87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		
Safflower	2.02	0.86	1.26	7.16	7.34	8.24	5.49	7.74	12.48	3.25	12.14	1.20	7.23	6.80	0.43
Cotton	6.32	36.04	38.80	31.00	28.00	0.00	30.10	16.10	4.09	0.00					
AET value for each period for each crop (mm):															
Sorghum	16.78	17.69	33.61	34.48	34.33	59.12	51.34	38.61	41.50	38.13	40.38	22.43			
Pulses	14.69	15.47	26.89	27.58	34.33	38.56	33.48	36.19	38.91	52.42	55.52				
Wheat	16.78	17.69	26.10	17.65	21.07	23.82	29.03	25.64	13.47	11.08	9.32	5.64	2.15		
Safflower	16.78	17.69	35.85	36.78	36.62	41.13	35.71	38.61	62.25	57.19	60.56	28.55	36.09	37.20	14.65
Cotton	48.25	50.84	51.53	39.08	38.91	43.70	37.95	31.37	33.72	24.28					
Soil moisture at the beginning of each period for each crop (mm/cm):															
Sorghum	3.50	2.80	2.41	2.39	2.77	2.98	3.09	3.16	3.20	3.10	2.82	2.77	2.59		
Pulses	3.50	3.46	3.03	3.26	3.35	3.40	3.42	3.13	3.00	3.00	2.58	2.32			
Wheat	3.50	2.41	2.35	2.19	2.30	2.32	2.28	2.17	2.07	2.03	1.96	1.89	1.85	1.86	
Safflower	3.50	3.45	2.87	2.14	2.59	2.85	2.99	2.99	3.06	3.11	2.78	2.81	2.64	2.67	2.68
Cotton	3.38	2.97	3.07	3.14	3.17	3.20	2.83	2.86	2.78	2.54	2.34				

Note: reservoir overflow, deep percolation and surface runoff losses are zero or negligible during these time periods.

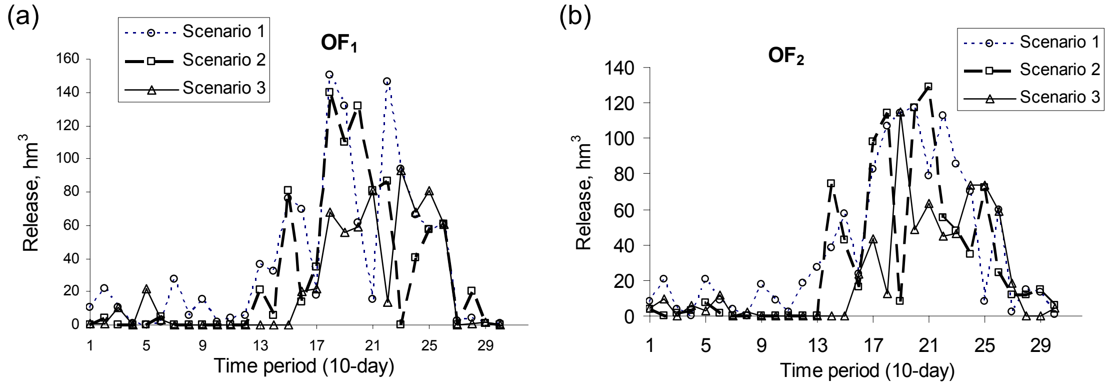

Fig. 2 Release policy obtained from the reservoir operation model for different hydrological conditions: (a) OF$_1$ model results with $B_c = 1$; and (b) OF$_2$ model results, with actual B_c, i.e. the equivalent benefit coefficient for each crop is considered in the objective function.

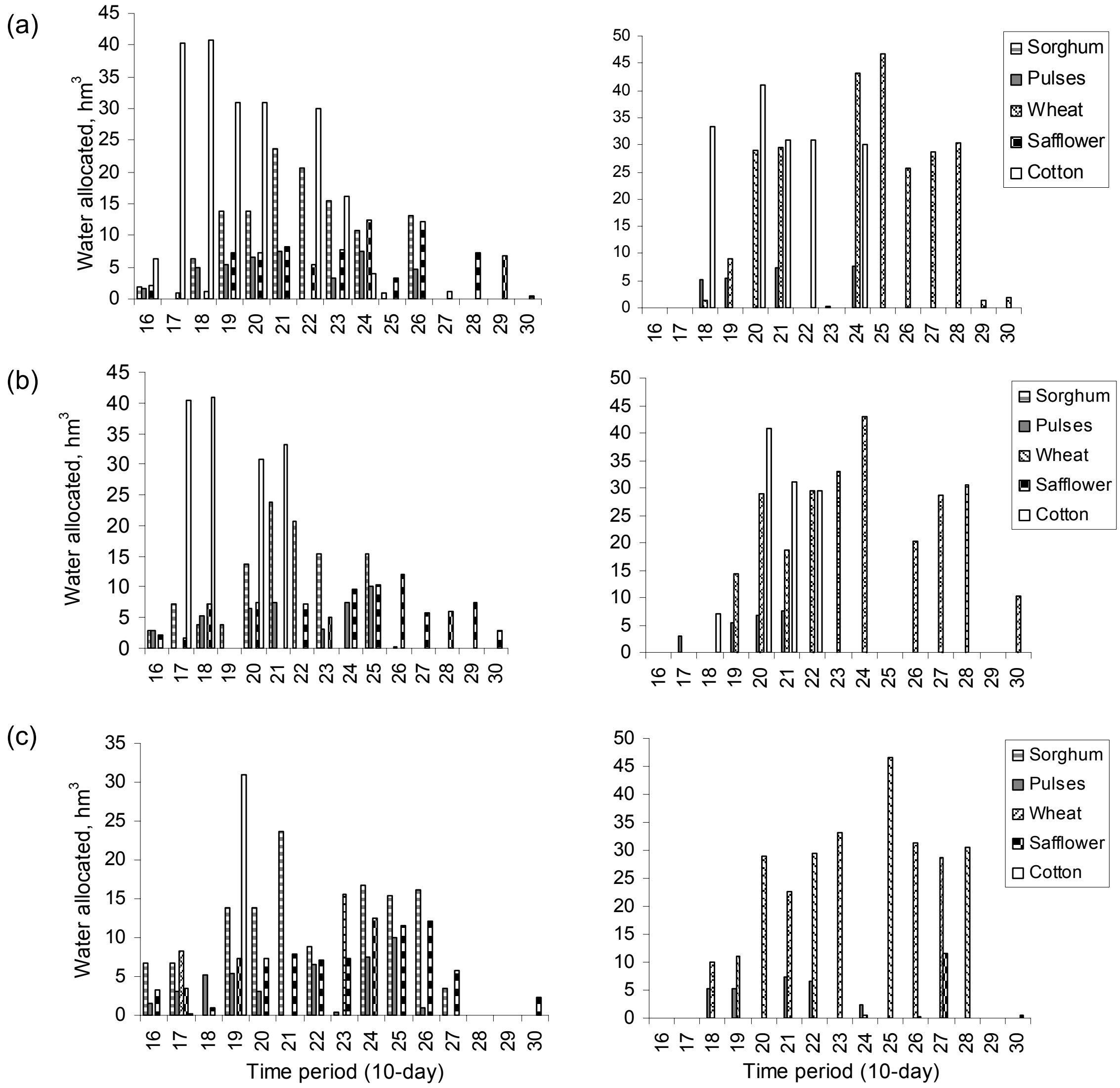

Fig. 3 Crop water allocated to different crops in the *rabi* season using two types of objective function (left: OF$_1$ and right: OF$_2$) for different hydrological conditions: (a) Scenario 1, (b) Scenario 2, and (c) Scenario 3.

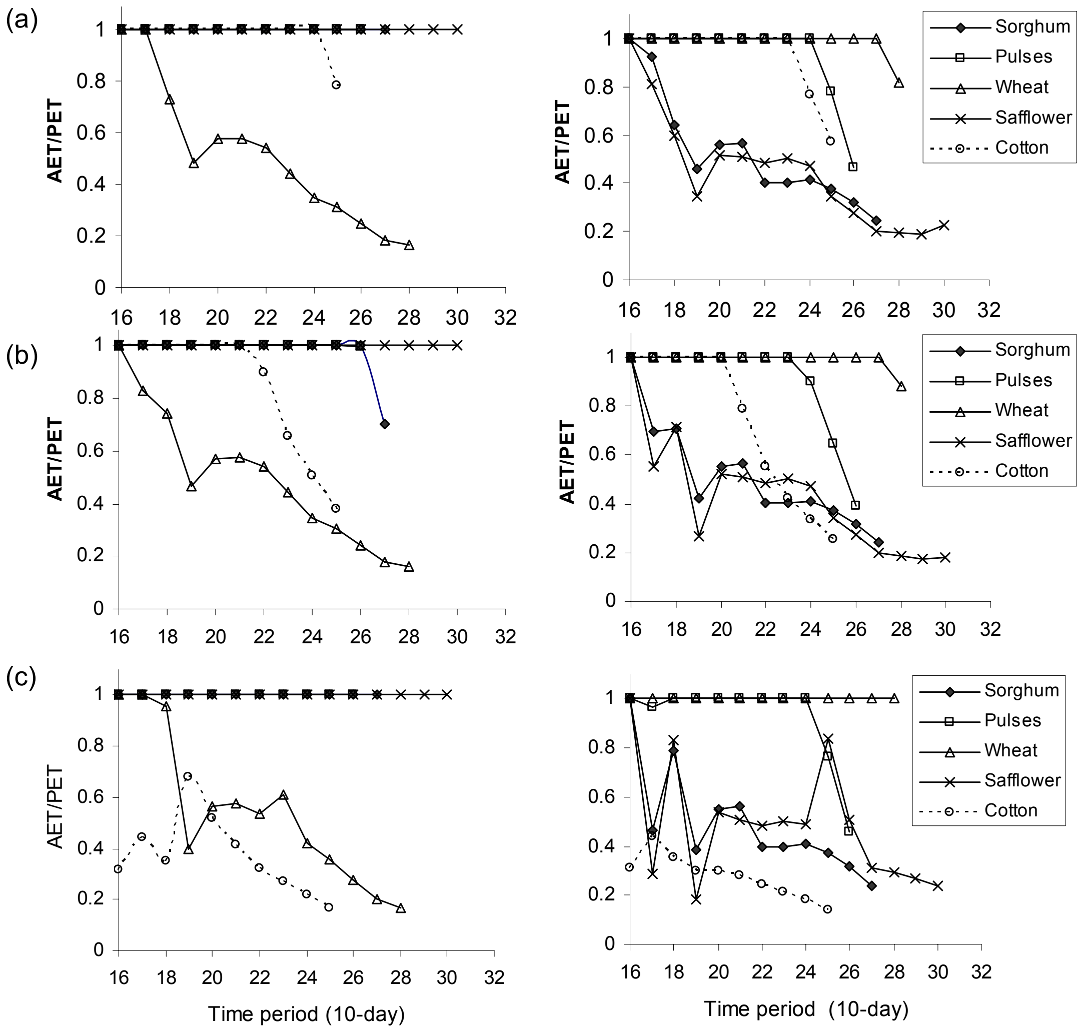

Fig. 4 Response of crops in the *rabi* season using two types of objective function (left: OF_1 and right: OF_2) for different hydrological conditions: (a) Scenario 1, (b) Scenario 2, and (c) Scenario 3.

CONCLUSIONS

This paper presents particle swarm optimization-based solutions to an integrated operational model for short-term reservoir operation for the irrigation of multiple crops. The model integrates the dynamics associated with the water released from a reservoir to the actual water utilized by crops at farm level. It takes into account the nonlinear relationships of root growth, soil heterogeneity, soil moisture dynamics for multiple crops, actual AET–PET relationships, yield in response to water deficit at various growth stages of the crops, and economic benefits from the crops. To solve the model, an efficient stochastic search technique, namely elitist-mutated particle swarm optimization (EMPSO) is used. The application of the model is illustrated through a case study of an existing irrigation reservoir system, the Malaprabha Reservoir in India. The model provides decisions on reservoir releases and crop water allocations for 10-day periods for each crop over a year. The model is run for different water-deficit conditions and the sensitivity of the crop yield to water shortages is analysed.

Also, the effect of economic benefits on the objective function is studied and the results are presented. Thus the proposed model can be effectively used for optimal utilization of available water resources of a reservoir for multicrop irrigation.

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Received 3 January 2006; accepted 22 January 2007