Quantum Correlation Without Classical Correlations

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We show that genuine multiparty quantum correlations can exist on its own, without a supporting background of genuine multiparty classical correlations, even in macroscopic systems. Such possibilities can have important implications in the physics of quantum information and phase transitions.

Quantum and classical correlations lie at the heart of sciences and technologies. The emerging quantum technology crucially depends on correlations that are different and more subtle than the ones in classical physics. This quantum form of correlations, known as entanglement [1], is currently being exploited to achieve higher levels of security in cryptography [2] and faster rates of information processing in computers [3]. Since quantum physics contains the classical one as a special case, we may think that reducing entanglement will in the same way ultimately lead to the reduction of all correlations to classical correlations. We will however show that this intuition is completely false; surprisingly, genuine multiparty entanglement can exist on its own without the need of genuine multiparty classical correlations, even for macroscopic systems. This result has potentially fundamental implications in the physics of quantum information and phase transitions, and in the current characterization of the quantum to classical transition.

It is widely known that quantum correlations are of a different kind than classical ones, and that quantum correlations can be vanishing even when classical correlations are present. However, the fact that even the opposite is true has so far eluded us. We present a general method to obtain that multiparty quantum and classical correlations are independent for a certain class of multiparty states, shared between an arbitrary odd number of two-dimensional systems (qubits). This phenomenon cannot happen for quantum systems with two subsystems; neither can it happen for pure multiparty states.

Quantum and classical correlations. We begin by discussing in generality how to distinguish quantum correlations and specifically genuine multiparty correlations, as well as the analogous classical notions. For an n-party state ρ, it is established usage to call it separable (or, precisely, *fully* separable) if it is a probability mixture (i.e., convex combination) of n-party product states [4]; otherwise the state is (somehow) entangled. A more stringent notion is that of genuine n-party entanglement [5], which demands that the state is not in the convex hull of tensor products over any bipartition of the n systems. [See [5] for a whole hierarchy of notions, k-separability and the complement, genuine $(n+2-k)$-party entanglement, for $k=2,\dots,n$.]

All this is fairly canonical – though one might consider the idea of defining quantum correlations via the violation of some Bell inequality [6], which is a strictly different concept, already in the bipartite case [4, 7]. We shall come back to this issue, but for the present paper, we stick with this more encompassing notion. On the other hand, defining correlations in general, and in particular genuinely multiparty *classical* correlation in a state, seems more contentious.

We propose the following point of view. First of all, classical correlations are to be about the values of (local) observables. Furthermore, taking the bipartite case as a model, it is easy to see that a state ρ_{AB} is correlated (i.e., not equal to a product state $\rho_A \otimes \rho_B$) if and only if there are local observables X and Y such that the *classical* variables X and Y are correlated. This in turn can be determined by looking at covariances $\mathrm{Cov}(X,Y) = \big\langle (X-\langle X\rangle)(Y-\langle Y\rangle)\big\rangle$: it is again not hard to see that if these are zero for all choices of local observable then the state must be a product, and vice versa. Unless mentioned otherwise, we shall restrict here to traceless observables X, Y, and if the states under consideration are such that $\langle X\rangle = \langle Y\rangle = 0$, then the covariances mentioned reduce to correlators $\langle XY\rangle = \mathrm{Tr}\,\rho(X\otimes Y)$, well-known quantities in statistical physics.

Encouraged by these observations, we take as our (at least sufficient) criterion of genuine n-party correlation, in a state $\rho_{12\dots n}$, that for some choice of local observables X_j, the "covariance" $\mathrm{Cov}(X_1,\dots,X_n) = \big\langle (X_1-\langle X_1\rangle)\cdots(X_n-\langle X_n\rangle)\big\rangle$ is nonzero. The question how to *define* genuine n-party correlations has been considered before; e.g. Zhou *et al.* [8] have proposed a set of axioms for n-party correlation measures. It can be shown that, at least for three parties and states with maximally mixed marginals, our criterion implies that of Ref. [8]. We shall only have occasion to look at states with maximally mixed marginals, so that it is enough if we stick to traceless observables, in which case this expression reduces to the higher correlator $\langle X_1\cdots X_n\rangle = \mathrm{Tr}\,\rho(X_1\otimes\cdots\otimes X_n)$. It is in this sense that we shall examine the presence of genuine n-party classical correlations in a state. We do not claim to have a well-established definition of genuine n-party correlations. All we want to say is that if all n-party covariances of an n-party state vanish, then the state should be considered to have no genuine n-party correlations.

It is curious to remark that the majority of Bell's inequalities, including multiparty ones, is actually expressed in terms of n-party correlators [9, 10, 11, 12]. Also, note that for an n-qubit state, as we shall look at, the absence of genuine n-party (more generally k-party) correlations means just that in the Pauli basis expansion all terms of n (k) proper Pauli ($\{\sigma_x, \sigma_y, \sigma_z\}$) operators are vanishing.

The example. For an integer $n \geq 3$, consider the W-state [13, 14] of n two-dimensional quantum systems (qubits) $|W\rangle = \frac{1}{\sqrt{n}}\big(|00\ldots001\rangle + |00\ldots010\rangle + \ldots + |10\ldots000\rangle\big)$, where $|0\rangle$ and $|1\rangle$ are eigenstates of the σ_z Pauli operator, and its "complement" $|\overline{W}\rangle = \frac{1}{\sqrt{n}}\big(|11\ldots110\rangle + |11\ldots101\rangle + \ldots + |01\ldots111\rangle\big) = \sigma_x^{\otimes n}|W\rangle$, and form the state

$$\rho_p = p|W\rangle\langle W| + (1-p)|\overline{W}\rangle\langle\overline{W}|,$$

which is just a mixture of $|W\rangle$ and $|\overline{W}\rangle$ with probabilities p and $1-p$. We must of course have $0 \leq p \leq 1$.

Quantum correlations exist. We first show that this state is genuinely multiparty entangled; in fact, we show that the subspace $\mathcal{S}$ spanned by $|W\rangle$ and $|\overline{W}\rangle$ contains no product vector $|\varphi\rangle_A|\psi\rangle_B$ for any partition $A \cup B$ of the sites $\{1, \ldots, n\}$. The subspace $\mathcal{S}$ is the support of the state ρ_p: Any decomposition of ρ_p must be a probabilistic mixture of pure states from $\mathcal{S}$. Therefore if we are able to show that $\mathcal{S}$ does not contain any product state, ρ_p cannot be written as a probabilistic mixture of product pure states in any bipartite splitting. Consequently, ρ_p will be genuinely multiparty entangled.

The subspace $\mathcal{S}$ has a lot of symmetry, which we will exploit: it is invariant under permutations of the sites, and under bit flip at all the sites (i.e. under $\sigma_x^{\otimes n}$). On the other hand, it is only two-dimensional, which will eventually result in a contradiction if we assume the existence of a product unit vector $|\varphi\rangle_A|\psi\rangle_B \in \mathcal{S}$.

By the permutation symmetry, we may assume, without loss of generality, that $A = \{1, \ldots, k\}$ and $B = \{k+1, \ldots, n\}$. Now, we first focus on the "weights" of the vectors involved, i.e., the numbers of $|1\rangle$'s in the expansion in the computational basis (the $\sigma_z^{\otimes n}$ eigenbasis). Observe that in $\mathcal{S}$, only weights 1 and $n-1$ occur. Let the set of weights occurring in $|\varphi\rangle$ be F, and G the corresponding set for $|\psi\rangle$; then the weights in the tensor product $|\varphi\rangle_A|\psi\rangle_B$ are precisely all pairwise sums, $F+G$. Since this has to be a subset of $\{1, n-1\}$, only $|F| = 1$, $|G| \leq 2$ and symmetrically $|G| = 1$, $|F| \leq 2$ are possible, and we may, without loss of generality, restrict to the former possibility. ($|\Lambda|$ denotes the cardinality of the set Λ.) Therefore only one weight, say r, occurs in $|\varphi\rangle$.

Now consider the projectors P_1 and P_{n-1} in the n-qubit system, where P_k projects onto the subspace spanned by basis vectors of weight k. Clearly, $P_1\mathcal{S}$ is just the line spanned by $|W\rangle$, and $P_{n-1}\mathcal{S}$ is that by $|\overline{W}\rangle$.

The first observation is that

$$P_1|\varphi\rangle_A|\psi\rangle_B = |\varphi\rangle_A|\psi'\rangle_B, \tag{1}$$

$$P_{n-1}|\varphi\rangle_A|\psi\rangle_B = |\varphi\rangle_A|\psi''\rangle_B, \tag{2}$$

with vectors $|\psi'\rangle$ and $|\psi''\rangle$, which have only one weight occurring, namely $1-r$ and $n-1-r$, respectively. This is because $|\varphi\rangle$ is already in a constant weight subspace, so the projection only ends up affecting $|\psi\rangle$.

But we observed already that the vectors in Eqs. (1) and (2) must be proportional to $|W\rangle$ and $|\overline{W}\rangle$, respectively, which are not product across any cut, so the right hand sides above must both be zero, and we arrive at the desired contradiction. This concludes the proof of the statement that the state ρ_p has genuine multiparty entanglement, whenever the state is composed of three or more qubits, and for any p in $[0,1]$.

Before proceeding further, let us note that the key ingredients in the above demonstration were the symmetries of the state ρ_p, and the low dimensionality of the support $\mathcal{S}$ of ρ_p. We expect to be able to relax the second ingredient – other small, but more than two-dimensional subspaces will have the same property of being "completely entangled". The first property should also not be strictly necessary, since it is clear that with $\mathcal{S}$, also a sufficiently small perturbation $\mathcal{S}'$ will contain no product vectors.

No classical correlations. We now show that for any *odd* number n (greater than one) of qubits, the state $\rho := \rho_{1/2}$ does *not* have any genuine n-partite classical correlations. More precisely, we show that the average value of *any* tensor product of traceless observables in the state is vanishing. First, we observe that ρ can be written as an equal mixture of the states $|V_\pm\rangle = \frac{1}{\sqrt{2}}(|W\rangle \pm |\overline{W}\rangle)$. We have $\sigma_x^{\otimes n}|V_\pm\rangle = \pm|V_\pm\rangle$. Thus

$$\begin{aligned}\langle V_\pm|\sigma_{k_1}^{A_1} \otimes \cdots \otimes \sigma_{k_n}^{A_n}|V_\pm\rangle &= \langle V_\pm|\bar{\sigma}_{k_1}^{A_1} \otimes \cdots \otimes \bar{\sigma}_{k_n}^{A_n}|V_\pm\rangle \\ &= (-1)^{n_2+n_3}\langle V_\pm|\sigma_{k_1}^{A_1} \otimes \cdots \otimes \sigma_{k_n}^{A_n}|V_\pm\rangle,\end{aligned} \tag{3}$$

where n_1, n_2, and n_3 are respectively the numbers of occurences of the Pauli matrices σ_x, σ_y, and σ_z, in the sequence $\sigma_{k_1}^{A_1} \otimes \cdots \otimes \sigma_{k_n}^{A_n}$, with $\sigma_{k_i}^{A_i}$ being a Pauli operator at the ith site ($i = 1, 2, \ldots, n$, $k_i = x, y, z$), and $\bar{\sigma}_{k_i} = \sigma_x \sigma_{k_i} \sigma_x$. It is clear that if $n_2 + n_3$ is an odd number, we have $\mathrm{Tr}(\rho\sigma_{k_1}^{A_1} \otimes \cdots \otimes \sigma_{k_n}^{A_n}) = 0$. If $n_2 + n_3$ is an even number, we have

$$\mathrm{Tr}(\rho\sigma_{k_1}^{A_1} \otimes \cdots \otimes \sigma_{k_n}^{A_n}) = \langle W|\sigma_{k_1}^{A_1} \otimes \cdots \otimes \sigma_{k_n}^{A_n}|W\rangle. \tag{4}$$

Due to the permutation symmetry of the state $|W\rangle$, the above expression is equal to

$$\langle W|\bigotimes_{i=1}^{n_1}\sigma_x^{A_i}\bigotimes_{i=n_1+1}^{n_1+n_2}\sigma_y^{A_i}\bigotimes_{i=n_1+n_2+1}^{n}\sigma_z^{A_i}|W\rangle, \tag{5}$$

with, as above, $n_2 + n_3$ being an even number. For convenience, let us define $\Sigma(n_1, n_2, n_3) = \bigotimes_{i=1}^{n_1} \sigma_x^{A_i} \bigotimes_{i=n_1+1}^{n_1+n_2} \sigma_y^{A_i} \bigotimes_{i=n_1+n_2+1}^{n} \sigma_z^{A_i}$. The state $|W\rangle$ is a superposition of n pure n-qubit states having only one qubit in the state $|1\rangle$ and the rest in the state $|0\rangle$. This means that $\Sigma(n_1, n_2, n_3)|W\rangle$

$$= i\delta_{n_1+n_2,2} \frac{(-1)^{n-2}}{\sqrt{n}} (|100\ldots0\rangle - |010\ldots0\rangle) + (1 - \delta_{n_1+n_2,2})|W(n_1, n_2, n_3)\rangle, \quad (6)$$

where $\langle W(n_1, n_2, n_3)|W\rangle = 0$. Clearly, this implies that $\langle W|\Sigma(n_1, n_2, n_3)|W\rangle = 0$, which ends the proof.

Discussion. Consequently, for odd n, the state

$$\rho = \frac{1}{2}|W\rangle\langle W| + \frac{1}{2}|\overline{W}\rangle\langle\overline{W}|$$

has no genuine n-partite classical correlations, despite the fact, as we have already shown, that it has genuine n-partite quantum correlations. Note, as an aside, that it *does* have $(n-1)$-party correlations, in the sense specified initially. Note also that if instead of looking at correlators, we look at the covariances $\mathrm{Cov}(X_1', \ldots, X_n')$, of arbitrary observables $X_1', \ldots, X_n'$ (which are not necessarily traceless), then such covariances vanish for ρ.

The states of rank unity (pure states) do not lend themselves to the phenomenon under study, while already rank-two states are shown to be eligible. Also, while bipartite systems cannot show this phenomenon, already a three-qubit system is qualified; for three qubits, an example of a state like ours here has previously been found [16].

It is interesting to find whether the state has an underlying local realistic model [6]. Since the classical correlations of the state ρ are vanishing, we cannot apply the existing multiparty Bell inequalities [9, 10, 11, 12] to test for violation of local realism, as they are based on genuine n-partite classical correlations. For example, the necessary and sufficient conditions, for existence of underlying local realistic models, in Ref. [9, 10, 11, 12], can only predict that ρ has a local realistic description for certain numbers of measurement settings of the observers. The existence of such multiparty states requires the derivation of multiparty Bell inequalities that are based on probabilities, à la Clauser-Horne inequalities [17], or on the concepts of mutual information, entropy, etc. If it happens that ρ does have a local realistic model, then we will have a curious scenario, of a "classical state" (in the sense that it has a local realistic description even with general tests of local realism that not necessarily involves correlators (cf. [18])) which has no genuine classical correlations.

Another approach to address the problem of local realism (at least numerically), as presented in the Ref. [19], is to numerically check if the full set of *probabilities* involved in an experiment on the n-particle state ρ, where the k-th observer ($k = 1, 2, \ldots, n$) chooses from a set of m_k von Neumann measurements on their qubit, admits a local realistic description. This powerful method gives a sufficient and necessary conditions for the existence of a local realistic description. We have performed numerical simulations for the state ρ, for three and five qubits, using the above approach. The simulation was made under the assumption that each observer chooses up to four measurement settings on their qubit. In all the cases, a local realistic description exists. Interestingly, the state $\rho_\epsilon = (\frac{1}{2} + \epsilon)|W\rangle\langle W| + (\frac{1}{2} - \epsilon)|\bar{W}\rangle\langle\bar{W}|$ violates local realism for $\epsilon > 10^{-3}$ (numerical precision used in the simulation was 10^{-5}), indicating that even a small perturbation to the state ρ, drastically changes its behavior, from the point of view local realistic theories. Note that for any $\epsilon > 0$ the state ρ_ϵ has genuine n-partite classical correlations.

It is important to mention here that multipartite W-states have been prepared in the laboratory by several groups in different systems (see e.g. [20, 21]). Therefore, it is reasonable to hope that the effect discussed in this paper can be seen in the laboratory, especially because the effect can already be seen for three qubits. A possible way to experimentally prepare the state ρ is by preparing an $n+1$ qubit *pure* state $\frac{1}{\sqrt{2}}\left(|a\rangle \otimes |W\rangle + |b\rangle \otimes |\overline{W}\rangle\right)$ where $|a\rangle$ and $|b\rangle$ are orthogonal states of the $n+1$th qubit, and subsequently tracing out the $n+1$th qubit. In fact, very recently, this four qubit pure state for $n = 3$ has been experimentally prepared by using spontaneous parametric down converted photons [22].

Conclusion. We have proposed a plausible notion of genuine n-party classical correlations in a multipartite quantum state, and demonstrated by an explicit example that it is possible for a state to be genuinely n-party entangled without it having genuine n-party classical correlation – at least for odd n (we expect the same to occur for even n but don't have an example as yet).

One possible reaction to this is to dispute the soundness of our definition of classical correlations – however, it is not at all straightforward to come up with a reasonable notion instead (e.g., there is no ready-to-use entropic correlation measure for n-party systems, otherwise we could try to follow in the steps of [23]), apart from insisting that quantum correlations should imply classical correlations. One should be cautious, however, with this intuition, too: in [24] examples of asymptotically large bipartite states are presented with almost no mutual information (i.e., classical correlation) compared to almost maximal entanglement of formation. See also the recent preprint [25] (cf. [26]), where a different way of separating quantum from classical correlations is explored, with similar results.

Hence, for the moment, we have to be prepared to accept the paradoxical statement that quantum correlations can exist without accompanying classical correlations. What are the consequences? One is to the Bell's inequalities, most of which – as already remarked – are expressed in terms of n-party correlators. For bipartite systems, there already exist Bell inequalities that use statistics beyond correlators [17, 27, 28, 29]! However, in multipartite systems, such inequalities are absent. The example provided in this paper leads to the need for new Bell inequalities, based on concepts different than classical correlations to detect multiparticle nonclassicality.

The boundary between the classical and the quantum worlds is a long-standing and arguably hard problem in the foundations of physics. There are many ways of looking at this boundary. We believe that the existence of a state that has quantum correlations and yet has vanishing classical correlators is a splendid example to deal with the question, e.g., by using "local realism" to characterise the classical world.

Along with its direct effects in the science of quantum information, the fact can have important consequences in the physics of phase transitions [30], where so far the usual method to detect a phase transition is to look at the scaling of classical and quantum correlations in the system; see [23] for an indication that most recently condensed matter physicists are abandoning correlators in favour of universal, entropic, measures of correlation. These tools, however, are restricted to bipartite correlations. The existence of states with vanishing classical correlations but with non-vanishing quantum correlations opens up the possibility of phase transitions that are detectable by quantum correlations only.

Finally let us mention here that important examples exist where two-body correlations are not enough to describe the important phases/properties of the system. And then researchers have resorted to many-body parameters. This, for instance, is the case in the Affleck-Kennedy-Lieb-Tasaki system, where a certain "string order" is necessary (see e.g. [31], and references therein). The concept of "localizable entanglement" from quantum information, has been a very successful one in describing many-body systems, and again it depends on all the particles of the system [32]. Note also that the usual intractability of many-body parameters of condensed matter systems may change in foreseeable future, with the advent of experimentally realizable quantum simulators: Many-body quantum correlated states are being realized in several physical systems, ranging from ion traps to down-converted photons in several laboratories around the globe.

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