Exclusion Principle for Quantum Dense Coding

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We show that the classical capacity of quantum states, as quantified by its ability to perform dense coding, respects an exclusion principle, for arbitrary pure or mixed three-party states in any dimension. This states that no two bipartite states which are reduced states of a common tripartite quantum state can have simultaneous quantum advantage in dense coding. The exclusion principle is robust against noise. Such principle also holds for arbitrary number of parties. This exclusion principle is independent of the content and distribution of entanglement in the multipartite state. We also find a strict monogamy relation for multi-port classical capacities of multi-party quantum states in arbitrary dimensions. In the scenario of two senders and a single receiver, we show that if two of them wish to send classical information to a single receiver independently, then the corresponding dense coding capacities satisfy the monogamy relation, similar to the one for quantum correlations.

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I. INTRODUCTION

Quantum correlations play an important role in quantum communication protocols [1]. Specifically, entangled states have been used to transfer classical bits encoded in a quantum state beyond the classical limit (quantum dense coding) [2], for transferring an unknown quantum state by using just two bits of classical communication (quantum teleportation) [3], and for preparing a known quantum state at a remote location (remote state preparation) [4]. Such protocols were initially introduced for the case of a single sender and a single receiver, and have also been experimentally realized [5]. However, for a fruitful application of such communication schemes, it is of vital importance to consider an information transmission network that involves several senders and receivers.

Study of correlations between separated physical systems is an important quantity in all areas of science. Such correlations can be classical as well as quantum. An important property of quantum correlations [6] in multipartite states is that they tend to be "monogamous" in nature [7–10], in the sense that if two physical systems are highly quantum correlated, they cannot be correlated, individually or as a whole, with any third party. Monogamy of quantum correlations, therefore, restricts the sharability of quantum correlations between three or more parts of a quantum system. Classical correlations of quantum states are certainly not monogamous, and an arbitrarily large number of physical systems can share the same amount of classical correlations with a single system.

In this paper, we address the question whether there are restrictions on our ability to send classical information through quantum states used as quantum channels in a multipartite scenario (three or more parties). As noted above, there are no such restrictions on *classical correlations* of quantum states. More precisely, for a three-party quantum state shared between Alice (A), Bob (B), and Charu (C), classical correlations between Alice and Bob, and between Alice and Charu can be both maximal. However, we show here that the *classical capacity*, as quantified by the dense coding capacity, of an arbitrary (pure or mixed) three-party quantum state of arbitrary dimensions satisfies a strict monogamy relation that can be viewed as an exclusion principle: If Alice has a quantum advantage in transferring classical information to Bob, she must necessarily have no quantum advantage in transferring the same to Charu. This result is independent of whether the quantum channel by which the quantum state of the sender is sent to the receiver in a dense coding protocol is noiseless or noisy. Note that this is stricter than the monogamy of quantum correlations (of quantum states): There exists quantum states for which Alice can have quantum correlations with Bob, and quantum correlations with Charu, i.e., the AB and the AC reduced quantum densities can both be quantum correlated, an example being the well-known three-party W state [11]. We go on to show that the exclusion principle holds for an arbitrary number of parties having an arbitrary amount of entanglement.

Within the realm of tripartite states, we connect the monogamy of dense coding capacity to the monogamy relations known for quantum correlations. In particular, in the scenario of two senders and a single receiver, we show that if Bob and Charu wish to send classical information to Alice, then the corresponding dense coding capacities obeys the monogamy relation in the same spirit as for quantum correlations. We subsequently generalize the monogamy relation to a multi-port scenario, involving multi-port channel capacities of multi-party (more than three-party) quantum states. We also establish a relation between the sum of the capacities of dense coding in the AB and AC channels with the corresponding entanglements of formation [12], as well as their quantum discords [13]. This provides lower bounds to the sum of the capacities, complementary to the upper bounds obtained in the monogamy relations.

The paper is organized as follows. For completeness, we begin with a discussion of the quantum dense coding capacity in Sec. II. Next, in Sec. III, we first prove the exclusion principle for dense coding capacity in the tripartite scenario. It holds for both the noiseless and noisy cases. We subsequently consider, in Sec. IV, the multi-sender single-receiver scenario, and find a monogamy relation in that case. The case of multi-port channel capacities is considered in Sec. V and it is found that it also satisfies a strict monogamy. We present a conclusion in Sec. VI.

II. QUANTUM DENSE CODING CAPACITY

Quantum dense coding is a quantum communication protocol that uses a shared quantum state between two distant observers, and a noiseless quantum channel [14] to send classical information beyond the classical capacity of the quantum channel [2]. Let the observers, Alice and Bob, share the quantum state ϱ_{AB}. Alice wishes to use this quantum state as a channel for sending classical information to Bob. Let the Hilbert space which are in possession of Alice and Bob, and which supports the quantum state ϱ_{AB}, be $\mathcal{H}_A \otimes \mathcal{H}_B$. Suppose that Alice receives a classical message i, which is known to happen with probability p_i. She encodes this classical message in a unitary operator U_i on the Hilbert space $\mathcal{H}_A$, and applies it to her part of ϱ_{AB} to obtain $\varrho^i_{AB} = U_i \otimes \mathbb{1}_B \varrho_{AB} U_i^\dagger \otimes \mathbb{1}_B$, where $\mathbb{1}_B$ is the identity operator on the Hilbert space $\mathcal{H}_B$. She then sends her part of ϱ^i_{AB}, through a noiseless quantum channel [14] between Alice and Bob that can noiselessly transfer d_A-dimensional quantum states, to Bob. Here, $d_A = \dim \mathcal{H}_A$. After this, Bob is in possession of the quantum ensemble $\{p_i, \varrho^i_{AB}\}$, and his task is to perform a quantum measurement on this ensemble so as to obtain as much information as possible about the classical index i.

After the quantum measurement by Bob, suppose that the post-measurement quantum ensemble is $\{p_{i|m}, \varrho^{i|m}_{AB}\}_i$, and also suppose that this ensemble appears with probability q_m. The amount of classical information gained by Bob due to his measurement can be quantified by the mutual information [15] between the index i and the measurement index m, and is given by

$$I(i:m) = H(\{p_i\}) - \sum_m q_m H(\{p_{i|m}\}_i) \qquad (1)$$

bits, where $H(\cdot)$ denotes the Shannon entropy of the probability distribution in its argument. The unit of mutual information is taken here to be "bits", a result of the fact that we are using the logarithms with base 2 in this paper, for both Shannon and von Neumann entropy. Henceforth, all the entropic quantities are defined in bits.

Now Bob has to perform a measurement that maximizes his information gain, and this information is the "accessible information" defined as

$$I_{acc}(\{p_i, \varrho^i_{AB}\}) = \max I(i:m), \qquad (2)$$

where the maximization is over all measurement strategies that Bob is able to implement on his ensemble.

This maximization turns out to be hard to implement. However, an useful upper bound, called the Holevo bound [16, 17], exists, and is given by

$$\chi(\{p_i, \varrho^i_{AB}\}) = S(\overline{\varrho}_{AB}) - \sum_i p_i S(\varrho^i_{AB}), \qquad (3)$$

where $S(\cdot)$ is the von Neumann entropy of the quantum state in its argument, and $\overline{\varrho}$ is the average ensemble state $\sum_i p_i \varrho^i_{AB}$. This quantity is asymptotically achievable [18], and therefore the following quantity is termed the dense coding capacity of the quantum state ϱ_{AB}:

$$\mathcal{C}(\varrho_{AB}) = \max_{\{p_i, U_i\}} \chi(\{p_i, \varrho^i_{AB}\}). \qquad (4)$$

It is possible to perform this optimization [19, 20], and one obtains

$$\mathcal{C}_{AB} \equiv \mathcal{C}(\varrho_{AB}) = \log_2 d_A + S(\varrho_B) - S(\varrho_{AB}), \qquad (5)$$

where $\varrho_B = \mathrm{tr}_A[\varrho_{AB}]$. It is to be noted that the conditional entropy $S(\varrho_{AB}) - S(\varrho_B)$ can be of both signs. If it is positive, one may not use the shared quantum state, but use the noiseless quantum channel to transfer $\log_2 d_A$ bits of classical information. In case the conditional entropy is negative, Alice will be able to use the shared quantum state to send classical information, beyond the "classical limit" of $\log_2 d_A$ bits, to Bob. We term this as a "quantum advantage" for Alice in sending classical information to Bob. So in general, the dense coding capacity is given by $\overline{\mathcal{C}}_{AB} \equiv \overline{\mathcal{C}}(\varrho_{AB}) = \max[\log_2 d_A, \mathcal{C}(\varrho_{AB})]$, and we term $\mathcal{C}_{AB}$ as the quantum part of the dense coding capacity.

III. EXCLUSION PRINCIPLE FOR DENSE CODING CAPACITY FOR THREE-PARTY STATES

In this section, we will begin by presenting the exclusion principle for an arbitrary (pure or mixed) three-party quantum state of arbitrary dimensions.

Theorem 1: ("Exclusion Principle") *Given an arbitrary (pure or mixed) three-particle quantum state ϱ_{ABC}, no two bipartite states shared with any one of the parties can have a quantum advantage in dense coding capacity simultaneously.*

`Proof.` Let us assume the contrary and suppose that both ϱ_{AB} and ϱ_{AC} have quantum advantages in dense coding, where $\varrho_{AB} = \mathrm{tr}_C[\varrho_{ABC}]$ and $\varrho_{AC} = \mathrm{tr}_B[\varrho_{ABC}]$. Then, we have

$$\begin{aligned}\overline{\mathcal{C}}_{AB} + \overline{\mathcal{C}}_{AC} &= \mathcal{C}_{AB} + \mathcal{C}_{AC} \\ &= 2\log_2 d_A + S(\varrho_B) + S(\varrho_C) - S(\varrho_{AB}) - S(\varrho_{AC}),\end{aligned} \qquad (6)$$

with $\varrho_C = \mathrm{tr}_{AB}[\varrho_{ABC}]$, and similarly for ϱ_A and ϱ_B. Strong subadditivity of von Neumann entropy [21] for the tripartite system between A, B, and C implies that

$$S(\varrho_B) + S(\varrho_C) - S(\varrho_{AB}) - S(\varrho_{AC}) \le 0. \qquad (7)$$

Therefore, we get

$$\overline{\mathcal{C}}_{AB} + \overline{\mathcal{C}}_{AC} \le 2\log_2 d_A. \qquad (8)$$

Equality sign will be satisfied by all pure three-party states.

But, if both ϱ_{AB} and ϱ_{AC} have quantum advantages, then by definition of the dense coding capacity, $\overline{\mathcal{C}}_{AB} + \overline{\mathcal{C}}_{AC}$ must be strictly greater than $2\log_2 d_A$, contradicting our assumption. ■

Remark: Note that Theorem 1 can also be interpreted as a strict monogamy relation of the dense coding capacity: If Alice has a quantum advantage in sending classical information to Bob (i.e. if $\mathcal{C}_{AB} > \log_2 d_A$), then Alice cannot have a quantum advantage with Charu (i.e., $\mathcal{C}_{AC}$ must necessarily be strictly less than $\log_2 d_A$), so that Alice will be forced to send classical information at the classical limit rate to Charu which is equal to $\log_2 d_A$.

Corollary 1: In a tripartite quantum state ϱ_{ABC}, if ϱ_{AB} and ϱ_{AC} are two reduced quantum states through which Alice wants to send classical information to Bob and Charu, then the sum of the dense coding capacities of the reduced states ϱ_{AB} and ϱ_{AC} is bounded above by $3\log_2 d_A$. The bound can be saturated.

`Proof.` From Theorem 1, it follows that the two channels cannot have quantum advantages simultaneously. Hence there are two possibilities – (i) both of them are classical, which implies $\overline{\mathcal{C}}_{AB} + \overline{\mathcal{C}}_{AC} = 2\log_2 d_A$, and (ii) one of the channels is classical and the other quantum (i.e. has a quantum advantage). In the case (ii), without loss of generality, we assume that the AB channel is quantum. Therefore,

$$\overline{\mathcal{C}}_{AB} + \overline{\mathcal{C}}_{AC} = 2\log_2 d_A + S(\varrho_B) - S(\varrho_{AB}). \qquad (9)$$

The strong subadditivity of von Neumann entropy implies that

$$S(\varrho_B) - S(\varrho_{AB}) \leq S(\varrho_{AC}) - S(\varrho_C). \qquad (10)$$

On the other hand, the nonnegativity of quantum mutual information implies that

$$S(\varrho_{AC}) - S(\varrho_C) \leq S(\varrho_A), \qquad (11)$$

so that we have

$$S(\varrho_B) - S(\varrho_{AB}) \leq S(\varrho_A) \leq \log_2 d_A. \qquad (12)$$

Using this relation in Eq. (9), we obtain

$$\overline{\mathcal{C}}_{AB} + \overline{\mathcal{C}}_{AC} \leq 3\log_2 d_A \qquad (13)$$

The proof follows by combining the cases (i) and (ii). ■

We now generalize our findings to states of more than three parties.

Theorem 2: *For an arbitrary (pure or mixed) multi-party state $\varrho_{AB_1B_2\ldots B_N}$, shared between $(N+1)$ parties, in arbitrary dimensions, only at most a single reduced density matrix among ϱ_{AB_i} $(i = 1, 2, \ldots, N)$ can have quantum advantage in dense coding.*

`Proof.` Suppose, if possible, that $\varrho_{AB_{k_1}}$ and $\varrho_{AB_{k_2}}$ have quantum advantages in dense coding. However, in that case, the reduced states $\varrho_{AB_{k_1}}$ and $\varrho_{AB_{k_2}}$ of the tripartite quantum state $\varrho_{AB_{k_1}B_{k_2}}$ violates Theorem 1. ■

Corollary 2: In an $(N+1)$-party quantum state $\varrho_{AB_1B_2\ldots B_N}$, the sum of the dense coding capacities in the cases where A is the sender and $B_i, i = 1, 2, \ldots, N$ are the receivers, is bounded above $(N+1)\log_2 d_A$.

Until now, we have considered the situation where the quantum channel, carrying Alice's part of the states to the receivers as noiseless. It turns out that the exclusion principle holds also in a more general scenario, when the aforementioned quantum channel is noisy. This is due to the fact that the capacities will be non-increasing in the presence of noise. Therefore, the upper bound, obtained in Theorems 1 and 2 also hold for any noisy channel. Henceforth, we consider only noiseless channels.

IV. REVEIVER MONOGAMY FOR DENSE CODING CAPACITIES

For quantum correlations, to check for the status of monogamy for a particular measure, one usually considers inequalities where the sum of the Alice-Bob and Alice-Charu quantum correlations is compared with that share by Alice with the Bob-Charu pair. We now consider the status of such relations, when taken over to the case of dense coding capacities. We begin with the case where two senders (Bob and Charu) wish to send information to Alice by using the three-party quantum state ρ_{ABC}. Again the state can be either pure or mixed, and in arbitrary dimensions.

Let $\mathcal{C}_{BA}$ be the quantum part of the dense coding capacity when Bob wants to send classical information to Alice by using the reduced density state ϱ_{AB}. Let $\mathcal{C}_{CA}$ be similarly defined. Let $\mathcal{C}_{BC:A}$ be the quantum part of the dense coding capacity when Bob and Charu sends classical information to Alice by using the quantum state ϱ_{ABC}.

Theorem 3: ("Receiver Monogamy") *For an arbitrary tripartite pure or mixed quantum state ϱ_{ABC}, shared between A, B, and C in arbitrary dimensions, the dense coding capacities are such that the monogamy*

$$\mathcal{C}_{BA} + \mathcal{C}_{CA} \leq \mathcal{C}_{BC:A},$$

is satisfied, even when B and C are far apart.

`Proof:` We have $\mathcal{C}_{BA} = \log_2 d_B + S(\varrho_A) - S(\varrho_{AB})$ and and $\mathcal{C}_{CA} = \log_2 d_C + S(\varrho_A) - S(\varrho_{AC})$, where d_B and d_C are the dimensions of the Hilbert spaces in possession of Bob and Charu respectively. Now, using strong subadditivity of von Neumann entropy [21] for a tripartite system between A, B, and C, we have

$$S(\varrho_A) - S(\varrho_{AB}) + S(\varrho_A) - S(\varrho_{AC}) \leq S(\varrho_A) - S(\varrho_{ABC}) \qquad (14)$$

so that

$$\mathcal{C}_{BA} + \mathcal{C}_{CA} \leq \log_2(d_B d_C) + S(\varrho_A) - S(\varrho_{ABC}). \qquad (15)$$

However, the quantum part of the dense coding capacity of BC to A is $\mathcal{C}_{BC:A} = \log_2(d_B d_C) + S(\varrho_A) - S(\varrho_{ABC})$. Note here that for Bob and Charu to attain a dense coding capacity of $\log_2(d_B d_C)+S(\varrho_A)-S(\varrho_{ABC})$ for sending classical information to Alice, it is not necessary for Bob and Charu to come together, as the dense coding capacity is attained by local encodings [20] (cf. [22]). Hence, the theorem. ■

We now consider the relation stated in Theorem 3 in the situation when Alice is the sender, instead of being the receiver of the dense coding channels. Let us therefore compare the sum of the quantities $\mathcal{C}_{AB}$ and $\mathcal{C}_{AC}$ with the quantum part of the dense coding capacity, $\mathcal{C}_{A:BC}$, when Alice wants to send classical information to Bob and Charu (who are together) by using a shared quantum state between the three parties.

Corollary 3: A tripartite pure state $|\psi_{ABC}\rangle$ satisfies the relation $\mathcal{C}_{AB} + \mathcal{C}_{AC} \leq \mathcal{C}_{A:BC}$, only if it possesses maximal entanglement between Alice and the Bob-Charu pair.

`Proof.` For a pure three-party state $|\psi_{ABC}\rangle$, Theorem 1 implies that $\mathcal{C}_{AB} + \mathcal{C}_{AC} = 2\log_2 d_A$. The quantum part of the dense coding capacity when A is sending to the BC pair (with the latter being together) is given by $\mathcal{C}_{A:BC} = \log_2 d_A + S(\varrho_{BC})$. Therefore, the relation $\mathcal{C}_{AB} + \mathcal{C}_{AC} \leq \mathcal{C}_{A:BC}$ for the quantum parts of the capacities reduces to $\log_2 d_A \leq S(\varrho_{BC}) = S(\varrho_A)$. But the entropy of a system cannot be more than the logarithm of its dimension, i.e., $S(\varrho_A) \geq \log_2 d_A$. Therefore, $\log_2 d_A = S(\varrho_{BC}) = S(\varrho_A)$. Also, maximal local entropy for a pure bipartite state implies that it is maximally entangled. Therefore, the entanglement in the $A : BC$ bi-partition has to be maximum, if the dense coding capacities satisfies the relation in the premise of the theorem. ■

Note that in the case of three-qubit pure states, the relation $\mathcal{C}_{AB} + \mathcal{C}_{AC} \leq \mathcal{C}_{A:BC}$ is satisfied only when the state has one ebit of entanglement in its $A : BC$ partition.

Corollary 4: If a tripartite pure or mixed state ϱ_{ABC} satisfies the monogamy relation $\mathcal{C}_{AB} + \mathcal{C}_{AC} \leq \mathcal{C}_{A:BC}$, then the state should satisfy the following inequality:

$$\log_2 d_A - S(\varrho_A) \leq \sum_{i=A,B,C} S(\varrho_i) - S(\varrho_{ABC}). \qquad (16)$$

`Proof.` For an arbitrary tripartite pure or mixed state ϱ_{ABC}, the monogamy relation $\mathcal{C}_{AB} + \mathcal{C}_{AC} \leq \mathcal{C}_{A:BC}$ can be written by using Eq. (5) as

$$\log_2 d_A + S(\varrho_B) + S(\varrho_C) \leq S(\varrho_{AB}) + S(\varrho_{BC}) + S(\varrho_{AC}) - S(\varrho_{ABC}). \qquad (17)$$

Using the subadditivity of entropy [21], i.e., $S(\varrho_{AB}) \leq S(\varrho_A) + S(\varrho_B)$, and after rearrangement, we obtain the stated sufficient condition. ■

We will now derive a lower bound on the sum, $\mathcal{C}_{AB} + \mathcal{C}_{AC}$, of the quantum parts of the capacities, in terms of measures of quantum correlations. Let the entanglements of formation [12] between Alice and Bob, and between Alice and Charu be E_{AB} and E_{AC} respectively. Also, suppose that the quantum discords [13] between Alice and Bob, and between Alice and Charu are D_{AB} and D_{AC} respectively.

Corollary 5: The sum of the quantum parts of the capacities, $\mathcal{C}_{AB}$ and $\mathcal{C}_{AC}$, of a tripartite pure state $|\psi_{ABC}\rangle$ is bounded below by $D_{AB} + D_{AC} = E_{AB} + E_{AC}$.

`Proof.` In case of a pure tripartite state $|\psi_{ABC}\rangle$, Koashi and Winter [10] have found a relation between the bipartite entanglement of formation and bipartite quantum discord, which reads $E_{AB} = D_{AC} + S(\varrho_{A|C})$, where $S(\varrho_{A|C}) = S(\varrho_{AC}) - S(\varrho_C)$ is the conditional entropy. By using Eq. (5), one obtains $\mathcal{C}_{AB} = D_{AB} - E_{AC} + \log_2 d_A$, and the quantum part of the capacity between A and C is $\mathcal{C}_{AC} = D_{AC} - E_{AB} + \log_2 d_A$. The sum of these two quantities will then give

$$\mathcal{C}_{AB} + \mathcal{C}_{AC} = D_{AB} + D_{AC} - E_{AB} - E_{AC} + 2\log_2 d_A. \qquad (18)$$

Moreover, the sum of the entanglements of formation of AB and AC are bounded above by $2\log_2 d_A$ [12], i.e. $E_{AB} + E_{AC} \leq 2\log_2 d_A$. This immediately implies that

$$\mathcal{C}_{AB} + \mathcal{C}_{AC} \geq D_{AB} + D_{AC} = E_{AB} + E_{AC}. \qquad (19)$$

To obtain the last equality, we use Theorem 1 in Eq. (18) which leads to $D_{AB} + D_{AC} - E_{AB} - E_{AC} = 0$. ■

V. MONOGAMY OF MULTI-PORT DENSE CODING CAPACITIES

In this section, we generalize the strict monogamy relations to an arbitrary number of parties for the case of multi-port capacities. Let us consider a situation where there are N observers, whom we call Alices (A_1, A_2, ..., A_N), and who share an N-party quantum state $\varrho_{A_1 A_2 \ldots A_N}$. Let $\mathcal{C}_{A_1 A_2 \ldots A_{N-2} A_{N-1}}$ denote the quantum part of the "distributed" or "multi-port" dense coding capacity in the case when all Alices except A_{N-1} and A_N are senders, and A_{N-1} is the receiver. Let $\mathcal{P}^N_{N-1}$ denote a periodic shift operator that takes $N-1$ elements from the ordered periodic collection $A_1 A_2 \ldots A_N$, so that $\mathcal{P}^N_{N-1} A_1 A_2 \ldots A_{N-2} A_{N-1} = A_2 A_3 \ldots A_{N-1} A_N$, $(\mathcal{P}^N_{N-1})^2 A_1 A_2 \ldots A_{N-2} A_{N-1} = A_3 A_4 \ldots A_N A_1$, etc. Therefore, we can visualize the N Alices as situated on different points in a ring. We suppose that they are ordered and we assume that the ordering has been performed in the clockwise direction. Any consecutive $N-2$ of them are acting as senders, and they are trying to send classical information to the Alice who is situated just beside them in a clockwise direction.

Theorem 4: ("Strict Monogamy for Multi-port Capacities") *For an arbitrary pure or mixed quantum state $\varrho_{A_1 A_2 \ldots A_N}$ in arbitrary dimensions, the quantum parts* *of the distributed dense coding capacities satisfy the following strict monogamy relation:*

$$\sum_{j=0}^{N-1} \mathcal{C}_{(\mathcal{P}^N_{N-1})^j A_1 A_2 \ldots A_{N-2} A_{N-1}} \leq (N-2) \sum_{j=1}^{N} \log_2 d_{A_j}, \quad (20)$$

where d_{A_j} is the dimension of the Hilbert space in possession of A_j.

`Proof.` The quantum part of the distributed dense coding capacity $\mathcal{C}_{A_1 A_2 \ldots A_{N-2} A_{N-1}}$ is given by [20]

$$\mathcal{C}_{A_1 A_2 \ldots A_{N-2} A_{N-1}} = \sum_{i=1}^{N-2} \log_2 d_{A_i} + S(\varrho_{A_{N-1}}) - S(\varrho_{A_1 A_2 \ldots A_{N-2} A_{N-1}}), \quad (21)$$

in which the senders are allowed to perform unitary encoding. Here, $\varrho_{A_{N-1}} = \mathrm{tr}_{A_1 A_2 \ldots A_{N-2} A_N} \varrho_{A_1 A_2 \ldots A_N}$ and $\varrho_{A_1 A_2 \ldots A_{N-2} A_{N-1}} = \mathrm{tr}_{A_N} \varrho_{A_1 A_2 \ldots A_N}$. Below, the local densities are defined similarly. Using Eq. (21), we have

$$\sum_{j=0}^{N-1} \mathcal{C}_{(\mathcal{P}^N_{N-1})^j A_1 A_2 \ldots A_{N-2} A_{N-1}} = (N-2) \sum_{j=1}^{N} \log_2 d_{A_j} + \sum_{j=1}^{N} S(\varrho_{A_j}) - \sum_{j=0}^{N-1} S(\varrho_{(\mathcal{P}^N_{N-1})^j A_1 A_2 \ldots A_{N-2} A_{N-1}}). \quad (22)$$

To prove the nonpositivity of the last line in the above equation (Eq. (22)), we will need the strong subadditivity of von Neumann entropy involving N parties, which we now establish, for completeness. We have

$$\sum_{j=1}^{N} S(\varrho_{A_j}) - \sum_{j=0}^{N-1} S(\varrho_{(\mathcal{P}^N_{N-1})^j A_1 A_2 \ldots A_{N-2} A_{N-1}}) = -\sum S(\varrho_{R_j | (\mathcal{P}^N_{N-1})^j A_1 A_2 \ldots A_{N-2} A_{N-1}}) \equiv \mathcal{Q}(\varrho_{A_1 A_2 \ldots A_N}), \quad (23)$$

where R_j is the observer which is left out from the N Alices in the collection $(\mathcal{P}^N_{N-1})^j A_1 A_2 \ldots A_{N-2} A_{N-1}$, and $S(\varrho_{R_j | (\mathcal{P}^N_{N-1})^j A_1 A_2 \ldots A_{N-2} A_{N-1}})$ is the conditional entropy defined as $S(\varrho_{(\mathcal{P}^N_{N-1})^j A_1 A_2 \ldots A_{N-2} A_{N-1}}) - S(\varrho_{R_j})$. Since the conditional entropies are convex, $\mathcal{Q}(\varrho_{A_1 A_2 \ldots A_N})$ is also a convex function. Moreover $\varrho_{A_1 A_2 \ldots A_N}$ can be written in a spectral decomposition as $\sum p_k |K\rangle\langle K|$. So, $\mathcal{Q}(\varrho_{A_1 A_2 \ldots A_N}) \leq \sum p_k \mathcal{Q}(|K\rangle\langle K|)$. However, $\mathcal{Q}(\varrho_{A_1 A_2 \ldots A_N}) = 0$ for pure states. Therefore,

$$\sum_{j=1}^{N} S(\varrho_{A_j}) - \sum_{j=0}^{N-1} S(\varrho_{(\mathcal{P}^N_{N-1})^j A_1 A_2 \ldots A_{N-2} A_{N-1}}) \leq 0.$$

Hence the theorem. ■

Remark: Theorem 4 implies that not all groups of $N-2$ senders can get a quantum advantage in sending classical information to the corresponding receiver. They must respect the monogamy relation, given in Eq. (20). There are N such sender groups and at most $N-1$ sender groups can have quantum advantages. In other words, if $N-1$ sender groups have quantum advantages, the Nth sender group must necessarily have no quantum advantage in sending classical information to their intended receiver. In this sense, the monogamy for multi-port capacities is again strict.

VI. CONCLUSION

Usually, *quantum* correlations are expected to obey monogamy. However, in this paper, we have found that *classical* capacity of a quantum channel obeys an extreme form of monogamy, which we refer as an exclusion principle. Specifically, we have shown that in a tripartite scenario, if Alice, Bob, and Charu share an arbitrary tripartite (pure or mixed) state in arbitrary dimensions, and Alice wishes to send classical information, encoded in a quantum state, to Bob and Charu independently, then quantum protocols can give advantage over classical ones either in the Alice-Bob protocol or in the Alice-Charu protocol. This is also true for an arbitrary number of parties in arbitrary dimensions. This exclusion principle is independent of the shared entanglement between the parties. The principle also holds in the case when the quantum channel carrying the post-encoding quantum states from the sender to the receiver is noisy. In the opposite scenario, where Bob and Charu are the senders, we find that the dense coding capacity also follow the usual monogamy relation of quantum correlations. We subsequently proved that a strict monogamy holds for the case when there are an arbitrary number of senders and a single receiver in arbitrary dimensions. This has potential applications in quantum networks, involving several senders and several receivers.

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